The Evolution of the Ideas of Spontaneous Symmetry Breaking in Particle Physics

The multiple solutions of QED

G. S. Guralnik

Brown University

Prague Symposium, October 2012

- *“What was front-line theoretical particle physics like in the early 60’s?”*
  Many “modern” tools were available in some form.
- Schwinger Action Principle and Green’s Function methods - mostly from Harvard
- Feynman Path Integral - although probably even less known than Schwinger’s methods.
- Calculation methods were primitive: Mostly coupling constant perturbation theory applied to quantum field theory.
- Renormalization theory was well developed - deep concern about the 4 fermion interaction not being renormalizable thus throwing doubt on our understanding of weak interactions.
- There was some experimentation with summation techniques of subsets of graphs with hope of understanding this problem.
- *S* Matrix theory was king. Feynman Graphs were OK, but generally not much faith in field theory

- We had the beginnings of the Standard Model.
- Interactions based on vector Yang-Mills (or Shaw -Salam) interactions 1954. Sakurai 1960
- A very fundamental contribution was the V-A theory of the weak interactions by Sudarshan and Marshak (1957) which was "publicized by Feynman and Gell-Mann" (Feynman 1963)
- Group Theory — flavor SU(3)(Gell-Mann). The Ω^- needed to fill in the baryon decuplet (10 particles) was found in 1963, The Gell-Mann Zweig quark (ace) ideas existed but were far from completely accepted.
- Color SU(3) Wally Greenberg 1963. With this the Quark model began to make sense in terms of field theory dynamics as this idea established a reasonable way to construct fermion out of 3 quarks.

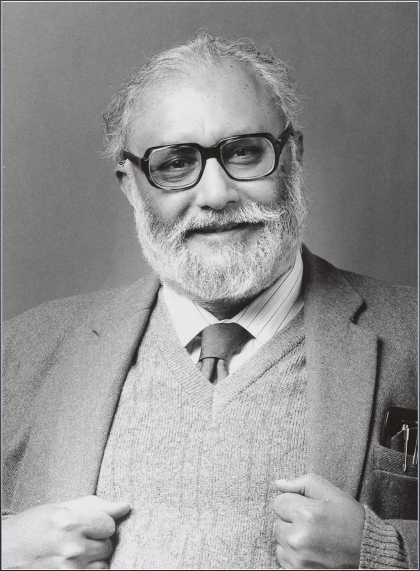
- Nambu launched the study of spontaneous symmetry breaking of an internal group through his work on the BCS model (1960) and the Nambu, Jona-Lasinio model with interaction

$$g\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2\right]$$

- This interaction in itself was disturbing (as was the four fermion interaction to describe weak processes) because perturbation theory in g produces a series of increasingly divergent terms that cannot be renormalized. NJL studied this model by imposing a constraint that seemed to be inconsistent with its symmetry and then formulated a new (not coupling constant perturbation theory) leading order approximation. Their results included a zero mass pseudoscalar (now called the Nambu-Goldstone) boson. NJL argue that the massless particle is an exact requirement.

- This was a real breakthrough!
- We understood little beyond coupling constant perturbation theory about how to actually solve a QFT. Attempts had been made to re-sum these series but until NJL there was little or no realization how to proceed to find different solutions.

- After the NJL papers, J. Goldstone wrote (1960) his famous paper where he examined a two (real) component scalar field theory with a quartic self interaction.
- This interaction has a conserved charge symmetry which is dynamically broken by requiring that the vacuum expectation of the scalar field is a non-vanishing number. Goldstone shows that in the leading approximation that there is a solution consistent with the breaking that has two scalar particles - one with mass zero and the other with square mass equal to minus twice the bare mass squared.
- In 1962 Goldstone Salam and Weinberg convincingly proved that the spontaneous breaking of a continuous global symmetry in a relativistic theory requires associated zero mass excitations.
- THIS IS NOT A GOOD THING! THERE IS ONLY ONE OBSERVED ZERO MASS PARTICLE!.



Abdus Salam

- THERE SHOULD NEVER HAVE BEEN ANY DOUBT THAT INTERACTING FIELD THEORIES HAVE SOLUTIONS IN ADDITION TO THOSE PRODUCED BY COUPLING CONSTANT PERTURBATION!
- The 1952 work of Dyson, showing that there is a singularity at zero coupling in QED, was an early indicator that perturbation theory was not the whole story and indeed that the perturbation expansion is asymptotic.
- We can get insight to help understand the ideas of the solution space of QFT and also spontaneous symmetry breaking by examining simple differential equations (zero dimensional QFT). Consider,

$$g\,\frac{d^3y}{dJ^3}+m^2\,\frac{dy}{dJ}=J\,.$$

- *“How many solutions?”*
  Three! They can be found from an integral representation (zero dimensional Feynman path integral for quartic interacting scalar field theory):

$$Z = \int e^{-g\frac{\phi^4}{4} - \frac{m^2}{2}\phi^2 + J\phi}\,\mathcal{D}\phi$$

  The J derivative of Z satisfies the original differential equation when the integral is evaluated over COMPLEX paths which do not contribute at the end points. It is easy to show that there are 3 allowed independent paths in the complex plane.
- Associated with the three integration paths, the integral has 3 stationary points that correspond to the three solutions of the original differential equation.

- These are ($J = 0$) located at

$$\phi = 0, \; \phi = \pm\sqrt{\frac{-m^2}{g}} \; .$$

It is easy to expand around these stationary phase points to discover asymptotic expressions for each of the three solutions. The path along the real axis corresponds to the stationary point at $\phi = 0$ and is the familiar solution found by perturbation expansion around $g = 0$.
- The perturbative solution vanishes at $J = 0$ is regular in g at $g = 0$.
- The other solutions break reflection symmetry and are singular at $g = 0$

- The Nambu-Goldstone Theorem - (more than 2 spacetime dimensions:
- Roughly: If a *charge* associated with a *conserved current* in a relativistic field theory does not *destroy* the vacuum ⇒ the theory has zero mass excitations.
- The Nambu-Goldstone Theorem can be *shown* to be true using exact results of QFT without use of *any* perturbation techniques in a manifestly covariant theory.
- Generally, in an interacting theory, you can not break a symmetry using a coupling constant based perturbative expansion around zero coupling strength.
- The symmetry breaking solutions show singularities as the coupling vanishes. You often have to be very careful to pick these up
- *"What is the Nambu-Goldstone Theorem good for?"*
- NOTHING IN OUR WORLD - BUT (MAYBE?) TO PROVE THE PHOTON IS MASSLESS.

- This is where I come in: After Bjorken gave a talk (1962) at Harvard, my thesis advisor, Walter Gilbert (Nobel Laureate Chemistry 1980), suggested that I look at Bjorken's proposed model of E&M — a variant of the Nambu-Jona-Lasinio model with interaction

$$g\,(\bar{\psi}\,\gamma^{\mu}\,\psi)\,(\bar{\psi}\,\gamma_{\mu}\,\psi)\,.$$

- The current is required to have non-vanishing vacuum expectation.
- The symmetry that is broken is Lorentz symmetry — relativistic invariance.

- I showed that BJ's basic conclusion that this theory is equivalent to QED is correct. Refined calculation shows that the Lorentz symmetry breaking is trivial and does not manifest itself in a physically observable way.
- This is a surprise since in *coupling constant perturbation theory* this interaction leads to hopelessly divergent results.
- In fact, this calculation provides results in a different phase corresponding to symmetry breaking boundary conditions and is an entirely different solution than the non-existent coupling constant perturbation expansion.

- This is the direct analog to the multiple solutions of the ordinary differential equation previously discussed.
- Despite the fact that Schwinger had argued that there was no dynamical reason for the photon to have zero mass, I thought from the arguments made for the Bjorken model that I could construct a symmetry breaking argument that would require massless photons in conventional E&M. This argument was wrong and, fortunately, Coleman detected this in my (1963) thesis presentation.
- I removed the offending chapter in the final version.

- Somewhat before my thesis was finished I had discussed a related project with Gilbert. He made the observation that the action of a massless *scalar* particle (B) and a massless *vector* particle (A^λ) with the simple "interaction"

$$g A^\lambda \left(\partial_\lambda B - g A_\lambda\right)$$

  produces a free spin 1 field with mass g^2.
- This can be *anticipated* by *counting* degrees of freedom and noting that g carries the dimension of mass (this model has a conserved current and a trace of gauge invariance).

- I told David Boulware about this and he spoke to Gilbert and they wrote a paper on this (Boulware, Gilbert; 1962)-BG.
- Thus, at this time, the 2-dimensional Schwinger model (E&M in 2 dimensions) showed that gauge theories need not have zero mass and the BG model in 4 dimensions *confirmed* this again.
- It is an *easy* step from the BG model to the lowest approximation used in the Guralnik, Hagen and Kibble (GHK) paper. They are essentially the same! With hindsight, all the ingredients for the GHK paper were ready to be mixed at Harvard in 1962!

- During my time at Harvard, I was talking with Dick Hagen an undergraduate friend at MIT and then a Physics graduate student at MIT and already my co-author on our first physics research paper.

A Working Afternoon: Young Baron May of Oxford, Guralnik and Hagen 1961

- In 1963 Hagen took a postdoctoral position at the University of Rochester (and is still there).
- We continued collaborating
- He became interested in complicated expensive and unreliable but beautiful machinery.
- Also on how to minimize living costs
- I thought he might be thinking of becoming an experimentalist

- I went to Imperial College (after being rejected by CERN) at the beginning of 1964 with a new NSF postdoctoral fellowship and the certainty that *something interesting* happened with gauge theories and *symmetry breaking*.
- IC was probably the best High Energy Theory place in the world at that time and I met a fantastic bunch of physicists there. The ones I interacted with the most were Tom Kibble, Ray, Streater, John Charap, and to a lesser degree Paul Matthews and Abdus Salam.

Salam and Kibble -like the Harvard and MIT crowd, the IC people were very serious

- I quickly learned that while Harvard was relatively safe field theory ground, protected by Schwinger's large (but indifferent) umbrella, the idea that there was even such a thing as symmetry breaking in field theory was not universally accepted - even at IC where Salam with Goldstone and Weinberg had already published their nice paper on these ideas.

- Ray Streater (an axiomatic or constructive field theorist) stated that his community did not believe that symmetry breaking was possible.
- A lot of arguing and construction of a free model *convinced* him that the axioms were too restrictive. Later. he published a paper on this, which amusingly got a lot more attention than the paper I published in PRL giving the simple free example and a significant example of the Nambu-Goldstone theorem in gauge theory but with an incomplete analysis of the resulting gauge structure.

- This entire line of analysis was just plain wrong in the radiation gauge - the only case that matters for physics! In that case something very subtle happens - CHARGE LEAKS OUT OF ANY VOLUME NO MATTER HOW BIG -
- THERE IS NO TIME INDEPENDENT CHARGE AND THERE IS NO NAMBU GOLDSTONE THEOREM!
- CONSEQUENTLY THERE IS NO CONSTRAINT OF ANY KIND ON ANY MASS OF THE THEORY.

- I caught this error in a couple of days but through a series of unlikely events, this paper was received by P.R.L. on June 1 1964 and published!
- This is actually an important paper! It is the first in the series that led to the prediction of the "Higgs Boson"
- Ironically, Peter Higgs sent a paper to Physics Letters -received July 27, 1964 and in it he makes the exact same subtle error.
- As can happen with wrong proofs - he comes to a different but unjustified conclusion- but more to every ones liking.
- Further this paper did not provide the path to his future papers!
- Despite this, it is a very famous paper!

- The understanding of my oversight in this paper (which, incidentally, was also caught by Dave Boulware) was the final key to the GHK understanding of why symmetry breaking in a gauge theory, does not require massless particles. There is no Nambu-Goldstone theorem in physical gauges, while in manifestly covariant gauges, the theorem requires only that the gauge variant modes to be massless.

- I now show a general example justifying the above statements.
- This explanation was the motivation for my paper with Hagen and Kibble.
- The following exact observations are valid for QED: There is an asymmetric conserved tensor current,

$$
\begin{aligned}
J^{\mu\nu} &= F^{\mu\nu} - x^{\nu} J^{\mu} \\
\partial_{\mu} J^{\mu\nu} &= 0 \\
\Rightarrow \; Q^{\nu} &= \int d^3x \left[F^{0\nu} - x^{\nu} J^{0} \right] \\
\text{and } \frac{d}{dt} Q^{\nu} &= 0 \, .
\end{aligned}
$$

- Pick the gauge $\vec{\nabla} \cdot \vec{A} = 0$ (a very natural gauge in operator QED) so that we only deal with physical excitations.

- By the commutation relations it is easily seen that this requires

$$\langle 0 | \left[Q^k, A^l(\vec{x}, t) \right] | 0 \rangle = (\text{non-zero constant})\,.$$

- However, direct calculation using spectral representations show that this expression is time-dependent for $e \neq 0$!
- *"What went wrong?"*
  The radiation gauge is not explicitly Lorentz invariant, and we cannot use *causality* arguments to prove that the *commutator* above is *confined* to a local region of space-time.

- This means that, even though $\partial_0 J^{0\,0} + \partial_k J^{0\,k} = 0$, we cannot neglect surface integrals of $J^{0\,k}$. It follows that our weird charge leaks out of any volume!
- This leads us, at once, to consider the proof of the Nambu-Goldstone theorem.
- *"What have we learned?"*
  The theorem is true for a manifestly covariant theory, i.e., a theory where $\partial_\mu J^\mu = 0$ and *surface terms* vanish *fast enough* so that

$$\langle 0| \left[\int d^3x (\partial_\mu J^\mu), \text{(local operator)} \right] |0\rangle =$$
$$= \frac{d}{dt} \langle 0| \left[\int d^3x J^0, \text{(local operator)} \right] |0\rangle .$$

- That is to say

$$Q = \int d^3x J^0$$

  has a zero mass particle in its spectrum.
- This includes electromagnetism with the special charge introduced above if you re-gauge to a manifestly covariant gauge.
- However, in this case, you can demonstrate exactly that the zero mass particles are gauge excitations.
- Note that these are very general statements: The Nambu-Goldstone theorem need not require physical zero mass states in any gauge theory (and it does not).
- This is because these theories are made to be relativistic by introducing extra gauge degrees of freedom.
- Indeed, the Goldstone bosons are always nonphysical.

- THESE VERY GENERAL STATEMENTS CONSTITUTE THE ESSENCE OF THE “HIGGS MECHANISM”!
- THIS IS AN EXACT OBSERVATION!
- ONLY GHK HAVE THE EXACT MODEL INDEPENDENT MECHANISM!

- We can see an approximate example of the failure of the Nambu-Goldstone theorem by looking at the action

$$L = -\frac{1}{2} F^{\mu\nu} (\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \phi^\mu \partial_\mu \phi + \\ + \frac{1}{2} \phi^\mu \phi_\mu + i e_0 \phi^\mu \mathbf{q} \phi A_\mu$$

$$\mathbf{q} = \sigma_2$$

$$\phi = (\phi_1, \phi_2)$$

$$\phi_\mu = (\phi_1^\mu, \phi_2^\mu)$$

- This is the Lagrangian for scalar electrodynamics. It is a very non-trivial interacting theory characterized by a conserved current. It is renormalizable in the coupling constant expansion with an induced ϕ^4 interaction. No other non-trivial ϕ^n interaction can be added to it and keep it renormalizable.
- We do not put in counter terms in accord with the conventions that Schwinger used!

- We want to look for solutions other than the coupling constant expansion. It was very natural for us to put in a source for ϕ and order an iterative expansion by the number of derivatives with respect to the source. This generates what is now known as a loop expansion. A variant of this method is a variant of what was used in my thesis to study the Bjorken model.
- In our GHK paper we only consider the lowest order symmetry breaking solution to this Lagrangian

- To us it was obvious that under further iteration the theory was renormalizable and could be understood from the resummation of Feynman graphs.
- This was not a radical revelation - this seemed far less interesting than actually what we found in the leading order and we did not pursue it.
- Technically this is much simpler than the Yang Mills (Shaw -Salam) case.

- The leading approximation is obtained by replacing $i\,e_0\,\phi^\mu\,\mathbf{q}\,\phi\,A_\mu$ in the Lagrangian by $\phi^\mu\,\eta\,A_\mu$. (The result is essentially the Boulware-Gilbert action with an extra scalar field)
- This "reduced Lagrangian" results in the linearized field equations:

$$\begin{aligned} F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu\ ; \\ \partial_\nu F^{\mu\nu} &= \phi^\mu\,\eta\ ; \\ \phi^\mu &= -\partial^\mu\phi - \eta\,A^\mu\ ; \\ \partial_\mu\phi^\mu &= 0\ . \end{aligned}$$

- These equations are soluble, since they are (rotated) free field equations. The diagonalized equations for the physical degrees of freedom are:

$$\begin{aligned} (-\partial^2 + \eta_1^2)\,\phi_1 &= 0\ ; \\ -\partial^2\phi_2 &= 0\ ; \\ (-\partial^2 + \eta_1^2)\,A_k^T &= 0\ . \end{aligned}$$

- For convenience, we have made the assumption that η_1 carries the full value of the vacuum expectation of the scalar field (proportional to the expectation value of ϕ_2). The superscript ^{T} denotes the transverse part. The two components of A_k^T and the one component of ϕ_1 form the three physical components of a massive spin-one field while ϕ_2 is a spin-zero field.
- As previously mentioned, the Nambu-Goldstone theorem is not valid, so there is no resulting massless particle.
- If the theorem were valid, ϕ_1 would be massless.
- It is very important to realize that it is an artifact of the lowest order approximation for the above action that ϕ_2 is massless. The excitation spectrum of this field is NOT CONSTRAINED by any theorem!

- A BIG FUSS HAS BEEN MADE ABOUT THIS IN TALKS, NEWSPAPERS AND EVEN SOCIAL MEDIA BY A FEW PHYSICISTS.
- They claim we do not have The Boson!
- What can I say? - There is a degree of freedom and the equation is above.
- It has been claimed that we do not have The Boson because what we display has zero mass!
- THIS IS MISLEADING AND BASICALLY AN INCORRECT EVALUATION.

- TO BE VERY SPECIFIC:
- Our Lagrangian and hence our equations of motion are superficially different from that of Higgs.
- GHK and Higgs (and EB) only examine the leading approximation in their PRL papers.
- Because these papers start from different Lagrangians GHK had mass zero for The Boson in leading order
- The Boson in the Higgs paper has leading order mass determined by an arbitrary quartic interaction coupling. It can be any value including zero.

- With enough iterations, the answers generated by each Lagrangian will be the same because of induced quartic interactions.
- The mass of the “Higgs Boson” will be arbitrary because it is intrinsically left undetermined because of the necessity of renormalization.
- This is not any surprise - this mass is not constrained by any symmetry or theorem and this is what field theories do.
- This is not any surprise - this is why the theory says nothing about the Higgs boson mass
- This is why it was not clear where to look experimentally - only other considerations suggested a reasonable range in the actual standard model.

- At this stage, it might be thought that we have written down an interesting, but possibly totally uncontrolled, approximation. There is no *a priori* reason to believe that this is even a meaningful approximation.
- The main result, that the massless spin-one field and the scalar field unite to form a spin-one massive excitation, could be negated by the next iteration of this approximation. However, this approximation meets an absolutely essential criterion that makes this unlikely. While the symmetry breaking removes full gauge invariance, current-conservation, which is the fundamental condition, is still respected. This is clear from the above linearized equations of motion.
- We can directly demonstrate that the mechanism, described earlier in this note for the failure of the Nambu-Goldstone theorem, applies in this approximation.

- The internal consistency and the consistency with exact results gives this approximation credence as a leading order of an actual solution. It is, in fact, not hard to make this the leading order of a well defined approximation scheme. This, of course, has now been worked out in detail.
- This solution of the action describes a 3-degree of freedom spin 1 particle and a 1-degree of freedom spin 0 particle.
- The Nambu-Goldstone theorem does not apply, even though the current

$$J^\mu = i\, e_0\, \phi^\mu\, \mathbf{q}\, \phi = \phi^\mu \cdot \eta$$

is conserved.

SUMMARY

The GHK paper addresses two major issues in detail:

- It shows exactly why the gauge theories do not intrinsically require zero mass particles.
- This emphatically does not depend on a model, but is a consequence of the “leakage” of appropriate charges out of any surface
- This is a fully quantum mechanical proof.

AND

- We demonstrate this in a (non-coupling constant) self consistent leading order (non-perturbative) approximation of scalar electrodynamics.

In Contradistinction to Englert and Brout and Higgs:

- GHK fully explains why the Goldstone boson is a gauge only excitation in manifestly covariant formulations and is not present in the physical radiation gauge. This is exact!
- EB touches, without analysis, the Nambu Goldstone boson issue but does not fully construct the lowest order approximation. They do not address the "Higgs Boson"

- Higgs does not write down the full solution to his equations.
- Check this out! He writes down the EM equations in arbitrary gauge
- He then writes down a solution - but not the complete solution.
- He fails to observe the “zero mass” excitations which are obvious solutions to his equations! They turn out to be pure gauge but this needs proof!
- Quantum mechanics requires that the zero mass modes must be present in manifestly relativistic gauges!

- The all important question was to explain why spontaneous symmetry breaking solutions of field theory could describe real world physics.
- A required zero mass physical particle means that spontaneous symmetry breaking is useless for physics.
- Higgs does not address this question.
- His PL paper is wrong
- His PRL paper fails to address the issue and can not because he misses some of the solutions.

There are Two More Papers in this series

- Gauge Invariance and the Goldstone Theorem, Gerald S. Guralnik, “Proceedings of seminar of unified theories of elementary particles”, July 1965
- This paper has much of the detailed discussion of items mentioned in this talk and is a significant extension in detail of the GHK paper.
- This is the writeup of my standard speech at the time. It was given at Edinburgh with Peter Higgs in the audience on November 23, 1964.
- It has recently been republished:“Gauge Invariance and the Goldstone Theorem” Jul 2011. 12 pp. Published in Mod.Phys.Lett. A26 (2011) 1381-1392 e-Print: arXiv:1107.4592 [hep-th]

- Peter Higgs in 1965 submitted to Physical Review the paper: Peter Higgs, “Spontaneous Symmetry Breakdown without Massless Bosons”, Physical Review **145**,1156 (1966). He has the correct ingredients in this paper. Much of it is similar to my Edinburgh talk and he acknowledges conversations with me. Higgs adds one element to this paper. He displays the tree graphs (not included in the PRL papers) as contributions to the next order calculation.

“Gang of five” Sakurai Prize 2010