Experimental checks of low energy QCD precise predictions using $\pi^+\pi^-$, $K^+\pi^-$ *and* $K^-\pi^+$ atoms

Leonid Nemenov

JINR Dubna

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Outline

- *Low-energy QCD precise predictions*
- *Lattice calculation precise predictions*
- *Results on the ππ scattering lengths measurement*
- *ππ earch for $K^+\pi^-$ and $K^-\pi^+$ atoms*
- *Status of the long-lived ππ atom states observation. Prospects of the Lamb-shift measurement.*
- *New prospects of DIRAC at SPS CERN*

DIRAC collaboration

CERN
Geneva, Switzerland

Czech Technical University
Prague, Czech Republic

Institute of Physics ASCR
Prague, Czech Republic

Nuclear Physics Institute ASCR
Rez, Czech Republic

INFN-Laboratori Nazionali di Frascati
Frascati, Italy

University of Messina
Messina, Italy

KEK
Tsukuba, Japan

Kyoto University
Kyoto, Japan

Kyoto Sangyou University
Kyoto, Japan

Tokyo Metropolitan University
Tokyo, Japan

IFIN-HH
Bucharest, Romania

JINR
Dubna, Russia

SINP of Moscow State University
Moscow, Russia

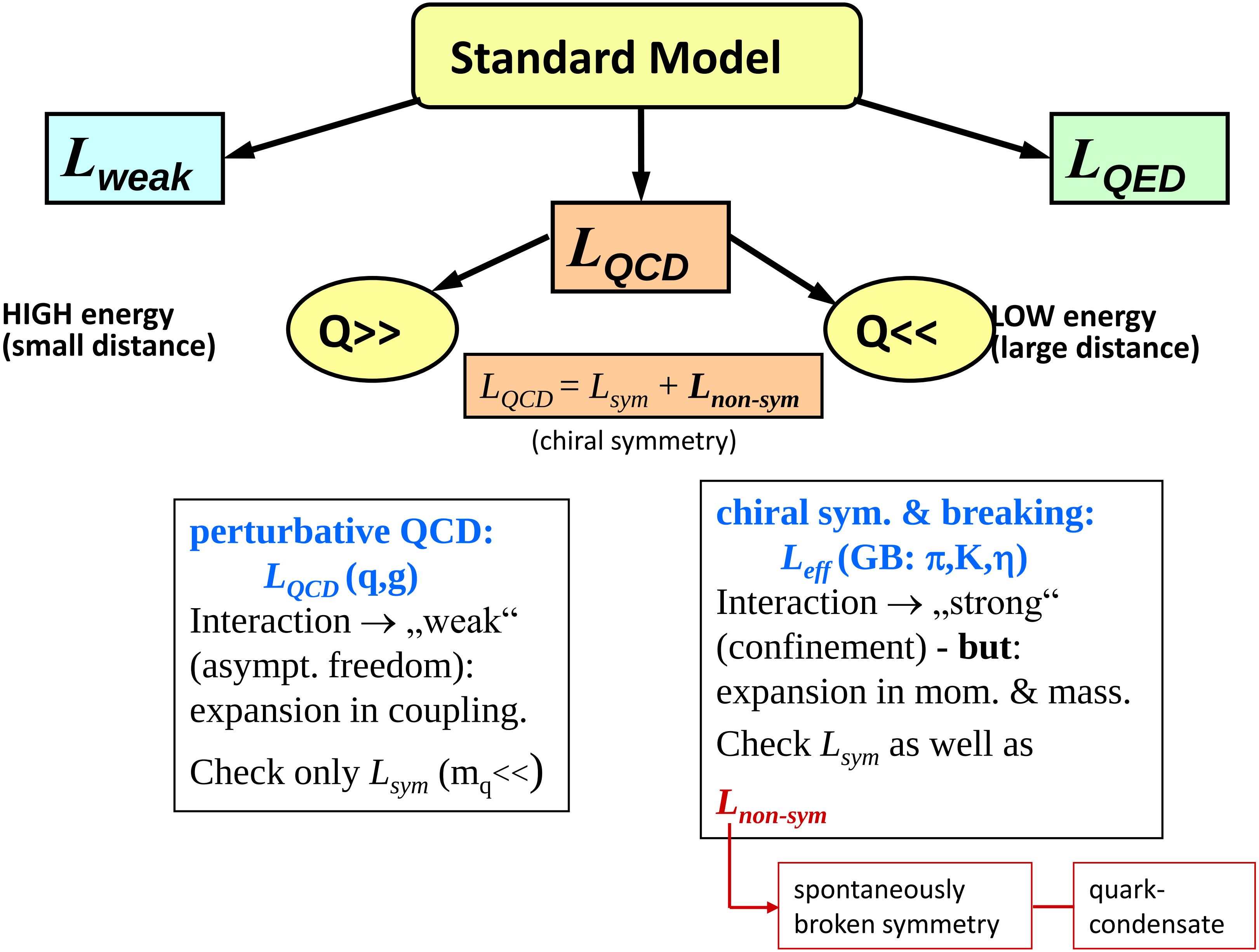
IHEP
Protvino, Russia

Santiago de Compostela University
Santiago de Compostela, Spain

Bern University
Bern, Switzerland

Zurich University
Zurich, Switzerland

Theoretical motivation



Standard Model

L_{weak}

L_{QCD}

L_{QED}

HIGH energy
(small distance)

Q>>

Q<<

LOW energy
(large distance)

$L_{QCD} = L_{sym} + \mathbf{L}_{\textbf{non-sym}}$

(chiral symmetry)

perturbative QCD:
L_{QCD} **(q,g)**
Interaction → „weak“
(asympt. freedom):
expansion in coupling.

Check only L_{sym} (m_q<<)

chiral sym. & breaking:
L_{eff} **(GB: π,K,η)**
Interaction → „strong“
(confinement) **- but**:
expansion in mom. & mass.
Check L_{sym} as well as

$\mathbf{L}_{\textbf{non-sym}}$

spontaneously
broken symmetry

quark-
condensate

Theoretical status

In ChPT the effective Lagrangian, which describes the ππ interaction, is an expansion in (even) terms:

$$L_{eff} = \underset{\text{(tree)}}{L^{(2)}} + \underset{\text{(1-loop)}}{L^{(4)}} + \underset{\text{(2-loop)}}{L^{(6)}} + \cdots$$

Colangelo et al. in 2001, using ChPT (2-loop) & Roy equations:

$$\left.\begin{aligned} a_0 &= 0.220 \pm 2.3\% \\ a_2 &= -0.0444 \pm 2.3\% \end{aligned}\right\} \quad a_0 - a_2 = 0.265 \pm 1.5\%$$

These results (precision) depend on the low-energy constants (LEC) ***l*** $_3$ and ***l*** $_4$:
Lattice gauge calculations from **2006** provided values for these ***l*** $_3$ and ***l*** $_4$.

Because ***l*** $_3$ and ***l*** $_4$ are sensitive to the quark condensate,
precision measurements of ***a*** $_0$, ***a*** $_2$ are a way
to study the **structure** of the QCD vacuum.

ππ scattering

ChPT predicts s-wave scattering lengths:

$$a_0 = 0.2220 \pm 0.005\,(2.3\%) \qquad a_2 = -0.0444 \pm 0.0010\,(2.3\%) \quad a_0 - a_2 = 0.265 \pm 0.004\,(1.5\%)$$

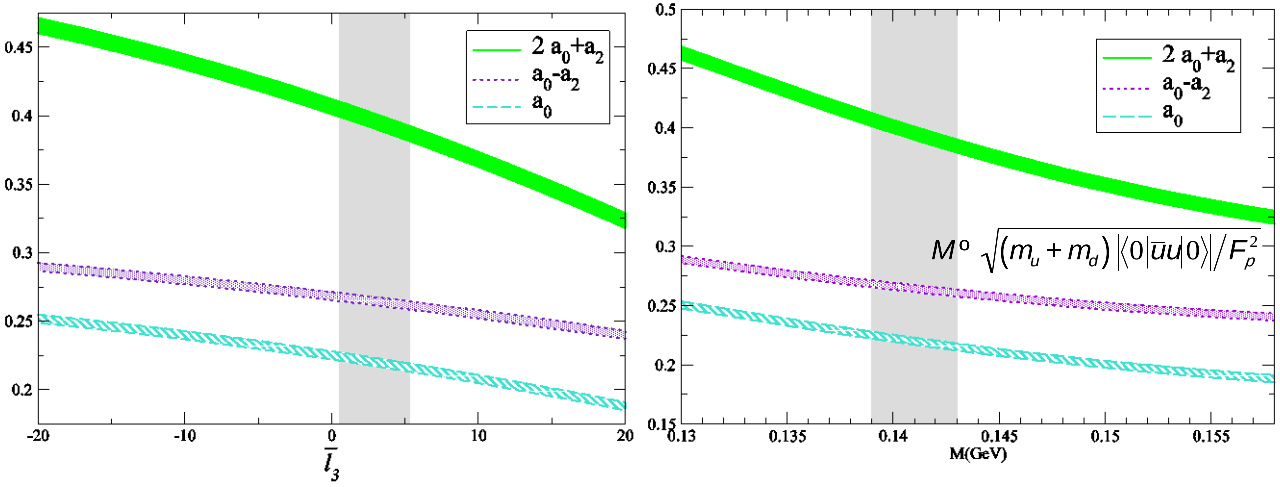
The expansion of M_π^2 in powers of the quark masses starts with the linear term:

$$M_\pi^2 = (m_u + m_d)B - \left[(m_u + m_d)B\right]^2 \frac{\bar{l}_3}{32\pi^2 F^2} + O\left((m_u + m_d)^3\right)$$

where $B = \frac{1}{F_\pi^2}\left|\langle 0|\bar{q}q|0\rangle\right|$ is the quark condensate, reflecting the property of QCD vacuum.

The estimates indicate values in the range $0 < \bar{l}_3 < 5$

Measurement of $\bar{l}_3 \Rightarrow$ improved the value of $(m_u + m_d)\left|\langle 0|\,\bar{u}u\,|0\rangle\right|$



$$\text{e.g.:}\ a_0 - a_2 = 0.260 \pm 3\% \Rightarrow \quad 1 < \bar{l}_3 < 11 \ \text{or}\ 1.00 < M / M_\pi < 1.06$$

E865: $a_0 = 0.216 \pm 6\% \Rightarrow -4 < \bar{l}_3 < 12 \ \text{or}\ 0.98 < M / M_\pi < 1.06$

(BNL)

Lattice calculations of $\bar{l}_3, \bar{l}_4$

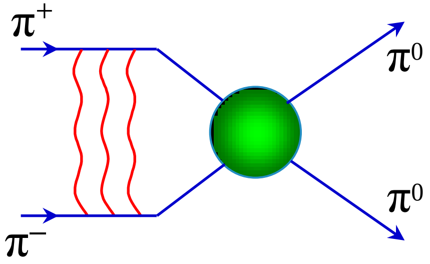
- 2006 $\bar{l}_3, \bar{l}_4$ First lattice calculations
- 2012 10 collaborations: 3 USA, 5 Europe, 2 Japan
- J.Gasser, H.Leutwyler Model calculation(1985)
  $\bar{l}_3$=2.6±2.5 $\Delta\bar{l}_3/\bar{l}_3 \approx 1$
- Lattice calculations in 2012-2013 will obtain $\Delta\bar{l}_3/\bar{l}_3 \approx 0.1$ or $\Delta\bar{l}_3 \approx$ 0.2-0.3
- To check the predicted values of $\bar{l}_3$ the experimental relative errors of ππ-scattering lengths and their combinations must be at the level (0.2–0.3)%

Pionium lifetime

Pionium ($A_{2\pi}$) is a hydrogen-like atom consisting of π^+ and π^- mesons:

E_B=-1.86 keV, r_B=387 fm, $p_B \approx 0.5$ MeV

The lifetime of $\pi^+\pi^-$ atoms is dominated by the annihilation process into $\pi^0\,\pi^0$:



$$\Gamma = \frac{1}{\tau} = \Gamma_{2\pi_0} + \Gamma_{2\gamma} \quad \text{with} \quad \frac{\Gamma_{2\gamma}}{\Gamma_{2\pi_0}} \approx 4 \times 10^{-3}$$

$$\Gamma_{1S,2\pi^0} = R\left|a_0 - a_2\right|^2 \quad \text{with} \quad \frac{\Delta R}{R} \approx 1.2\%$$

$$\tau = (2.9 \pm 0.1) \times 10^{-15}\, s$$

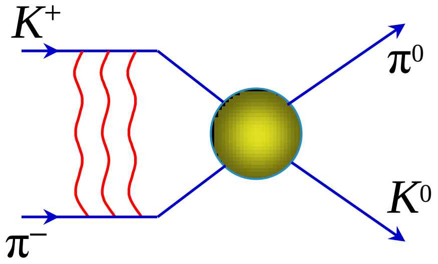
a_0 and a_2 are the ππ S-wave scattering lengths for isospin I=0 and I=2.

If $$\frac{\Delta\tau}{\tau} = 4\% \quad \Rightarrow \quad \frac{\Delta\left|a_0 - a_2\right|}{\left|a_0 - a_2\right|} = 2\%$$

$K^+\pi^-$ and $K^-\pi^+$ atoms lifetime

$K\pi$-atom ($A_{K\pi}$) is a hydrogen-like atom consisting of K^+ and π^- mesons:

E_B= -2.9 keV r_B = 248 fm $p_B \approx$ 0.8 MeV



The $K\pi$-atom lifetime (ground state 1S), $\tau=1/\Gamma$ is dominated by the annihilation process into $K^0\pi^0$:

$$A_{K^+\pi^-} \rightarrow \pi^0 K^0 \qquad A_{\pi^+K^-} \rightarrow \pi^0 \overline{K}^0$$

$$\Gamma_{1S,K^0\pi^0} = R_K \left|a_{1/2} - a_{3/2}\right|^2 \quad \text{with} \quad \frac{\Delta R_K}{R_K} \approx 2\%^{**}$$

(**) J. Schweizer (2004)

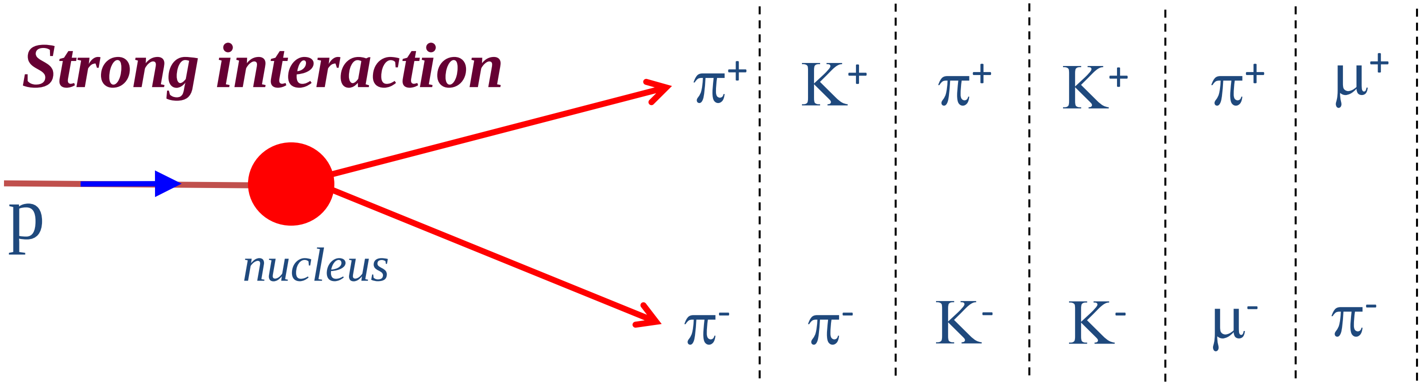
From Roy-Steiner equations: $a_0^{1/2} - a_0^{3/2} = 0.269 \pm 0.015$

$$\downarrow$$

$$\tau = (3.7 \pm 0.4) \cdot 10^{-15}\ s$$

If $$\frac{\Delta\Gamma}{\Gamma} = 20\% \quad \Rightarrow \quad \frac{\Delta\left|a_{1/2} - a_{3/2}\right|}{\left|a_{1/2} - a_{3/2}\right|} = 10\%$$

Coulomb pairs and atoms



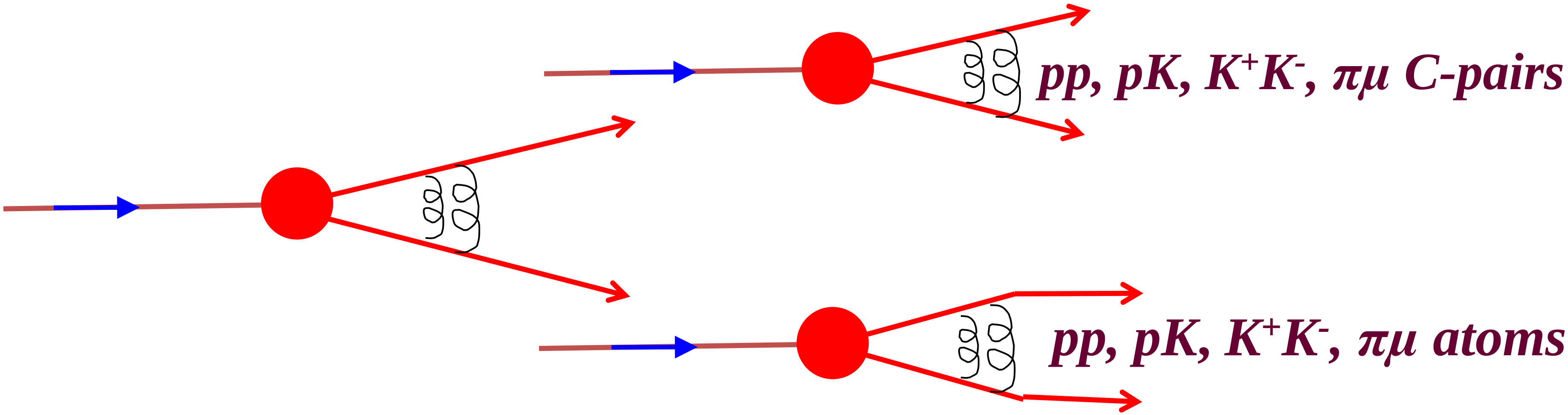
Strong interaction

p

nucleus

π^+ π^- | K^+ π^- | π^+ K^- | K^+ K^- | π^+ μ^- | μ^+ π^-

For small Q there are Coulomb pairs :

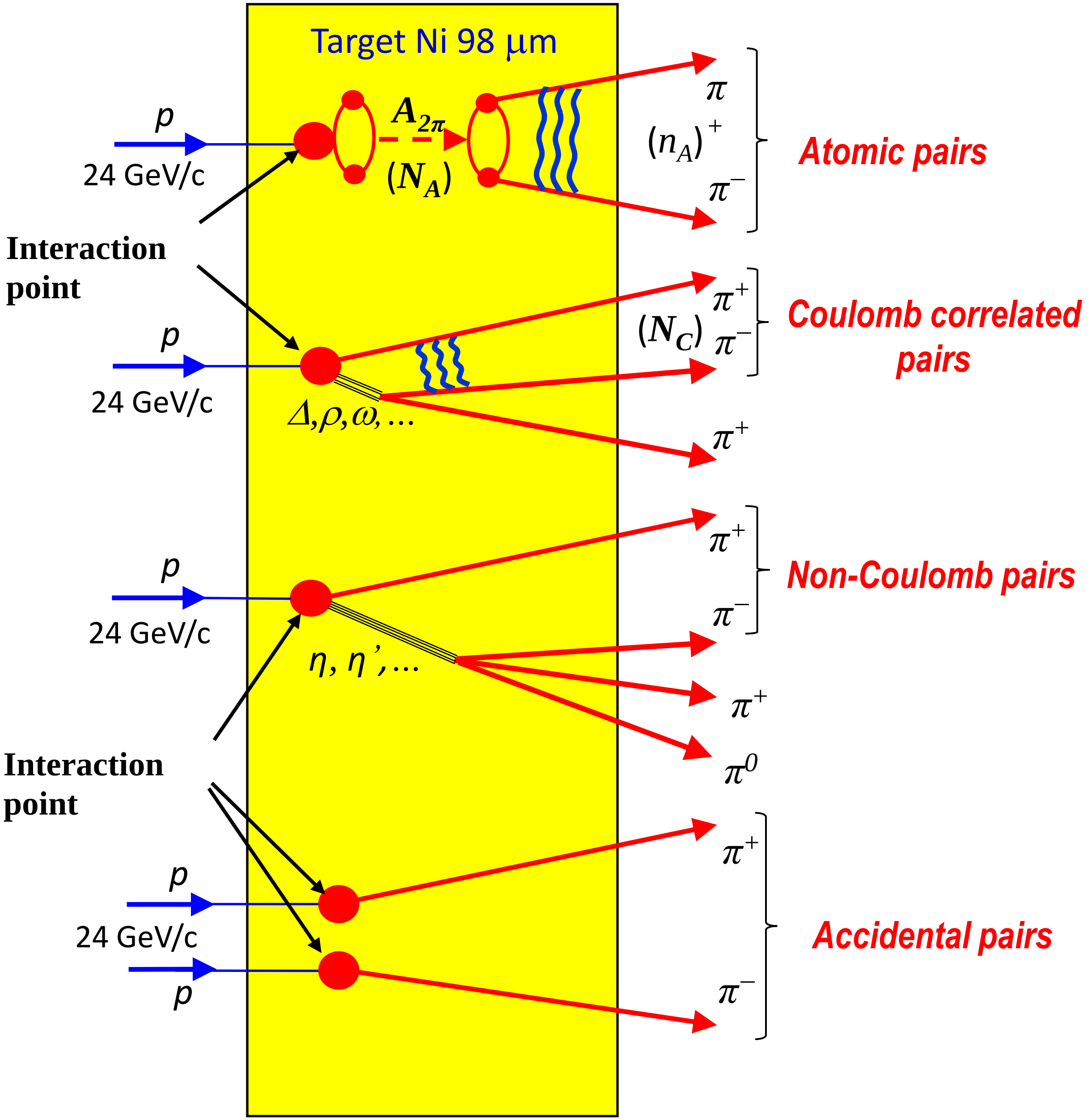


pp, pK, K^+K^-, $\pi\mu$ C-pairs

pp, pK, K^+K^-, $\pi\mu$ atoms

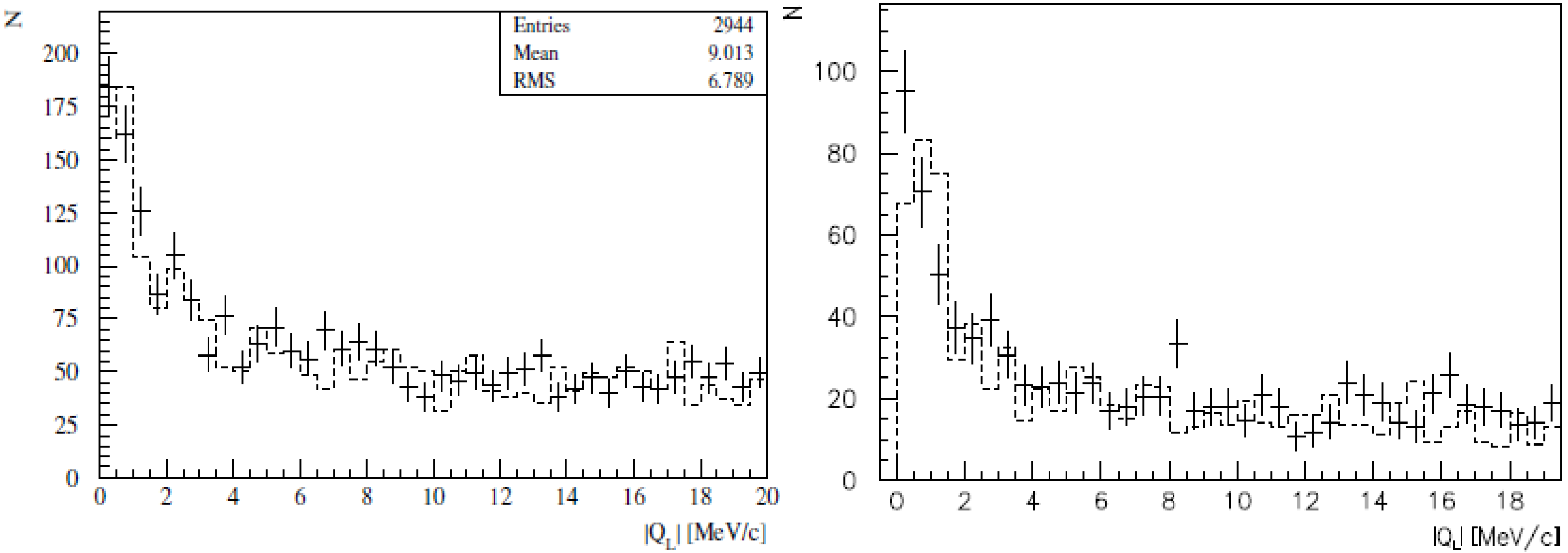
The production yield strongly increases for smaller Q

Method of $A_{2\pi}$ observation and measurement



Production of $A_{2\pi}$ in Beryllium target

Distribution over $|Q_L|$ of $\pi^+\pi^-$ pairs collected in 2010 (left) and in 2011 (right) with Beryllium target with the cut $Q_T < 1$ MeV/c. Experimental data (points with error bars) have been fitted by a sum of the simulated distribution of "Coulomb" and "non-Coulomb" pairs (dashed line).



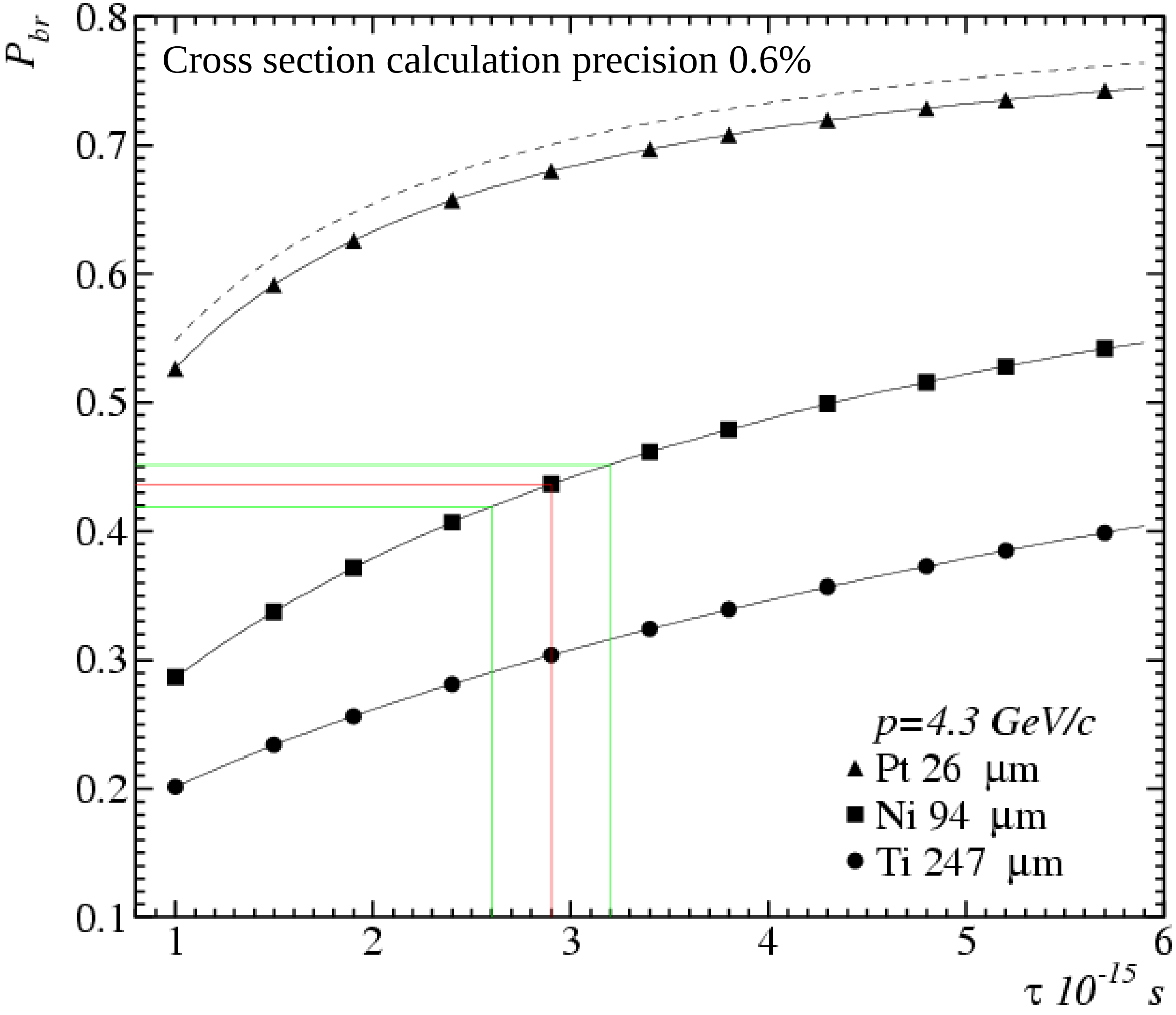
Produced atom numbers normalized on the proton flux:

$N_{A_{2\pi}}/p = (5.1 \pm 0.5) \times 10^{-14}$ *(2010)*

$N_{A_{2\pi}}/p = (5.9 \pm 0.5) \times 10^{-14}$ *(2011)*

Break-up probability

Solution of the transport equations provides one-to-one dependence of the measured break-up probability (P_{br}) on pionium lifetime τ

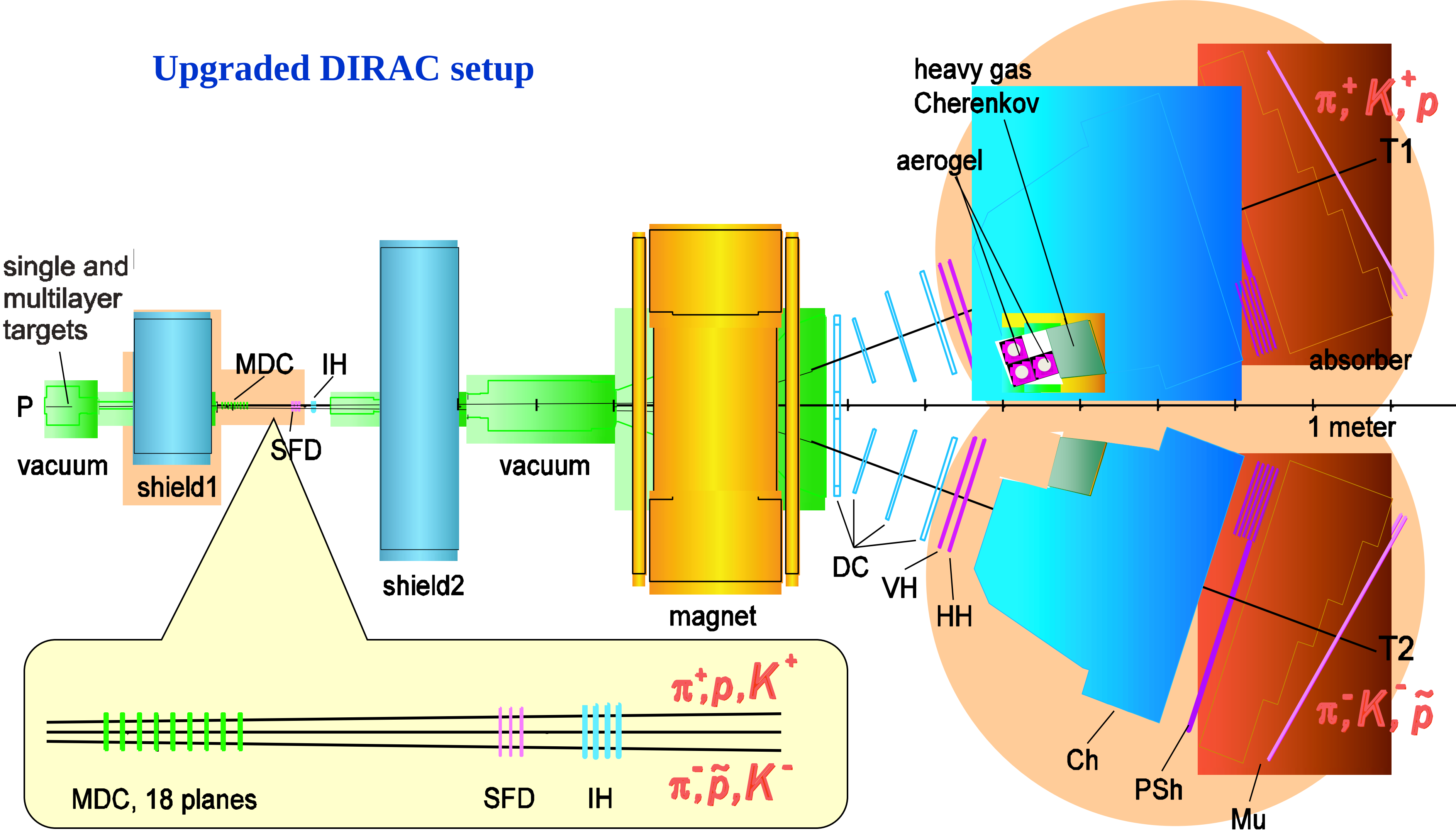

Cross section calculation precision 0.6%

ll targets have the same hickness in radiation engths $6.7*10^{-3}\ X_0$

There is an optimal target material for a given lifetime

DIRAC setup

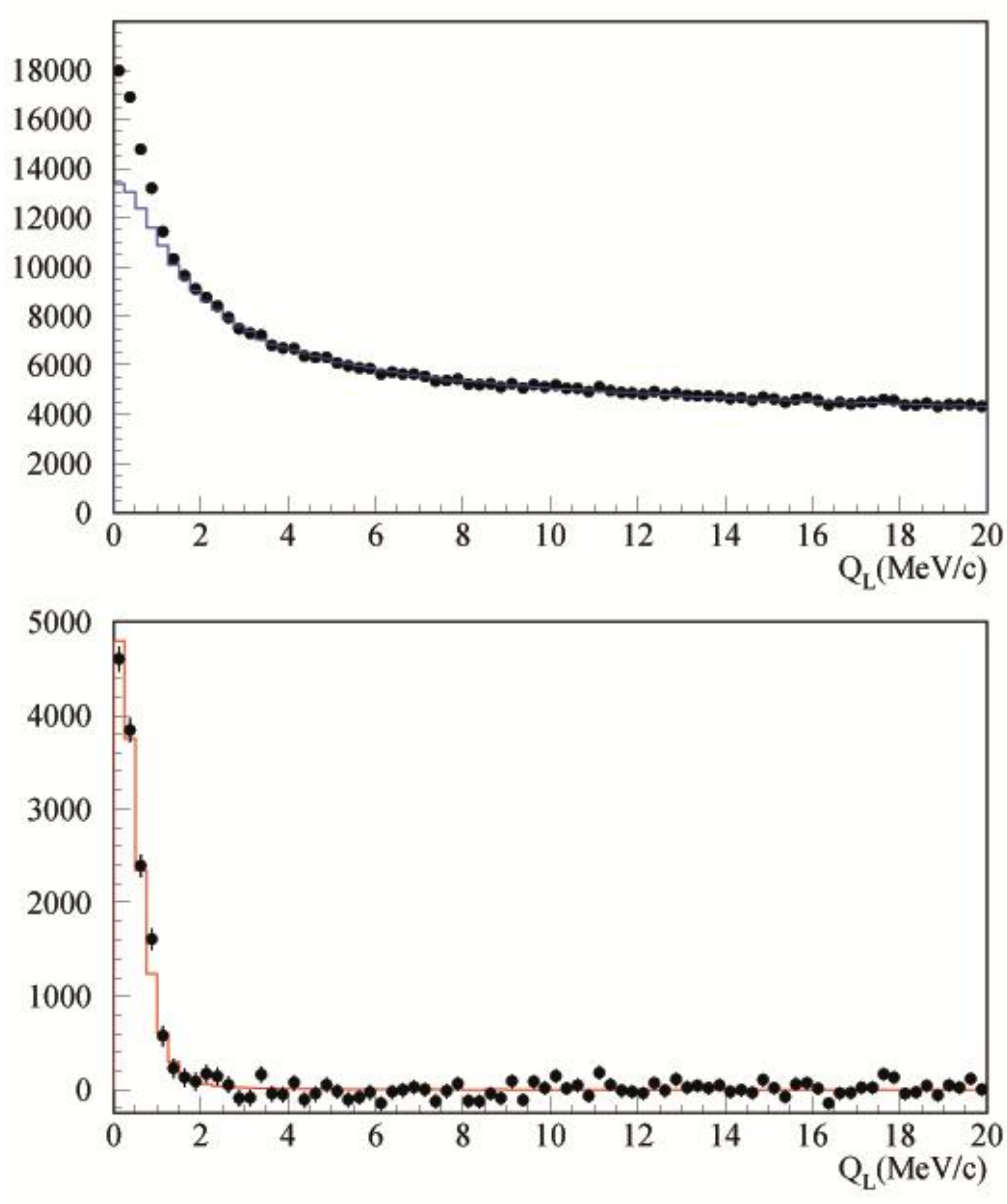
Upgraded DIRAC setup



🟧 **Modifided parts**

MDC - *microdrift gas chambers*, SFD - *scintillating fiber detector*, IH – *ionization hodoscope*. DC - *drift chambers* , VH – *vertical hodoscopes*, HH – *horizontal hodoscopes*, Ch – *nitrogen Cherenkov* , PSh - *preshower detectors* , Mu - *muon detectors*

DIRAC results with GEM/MSGC



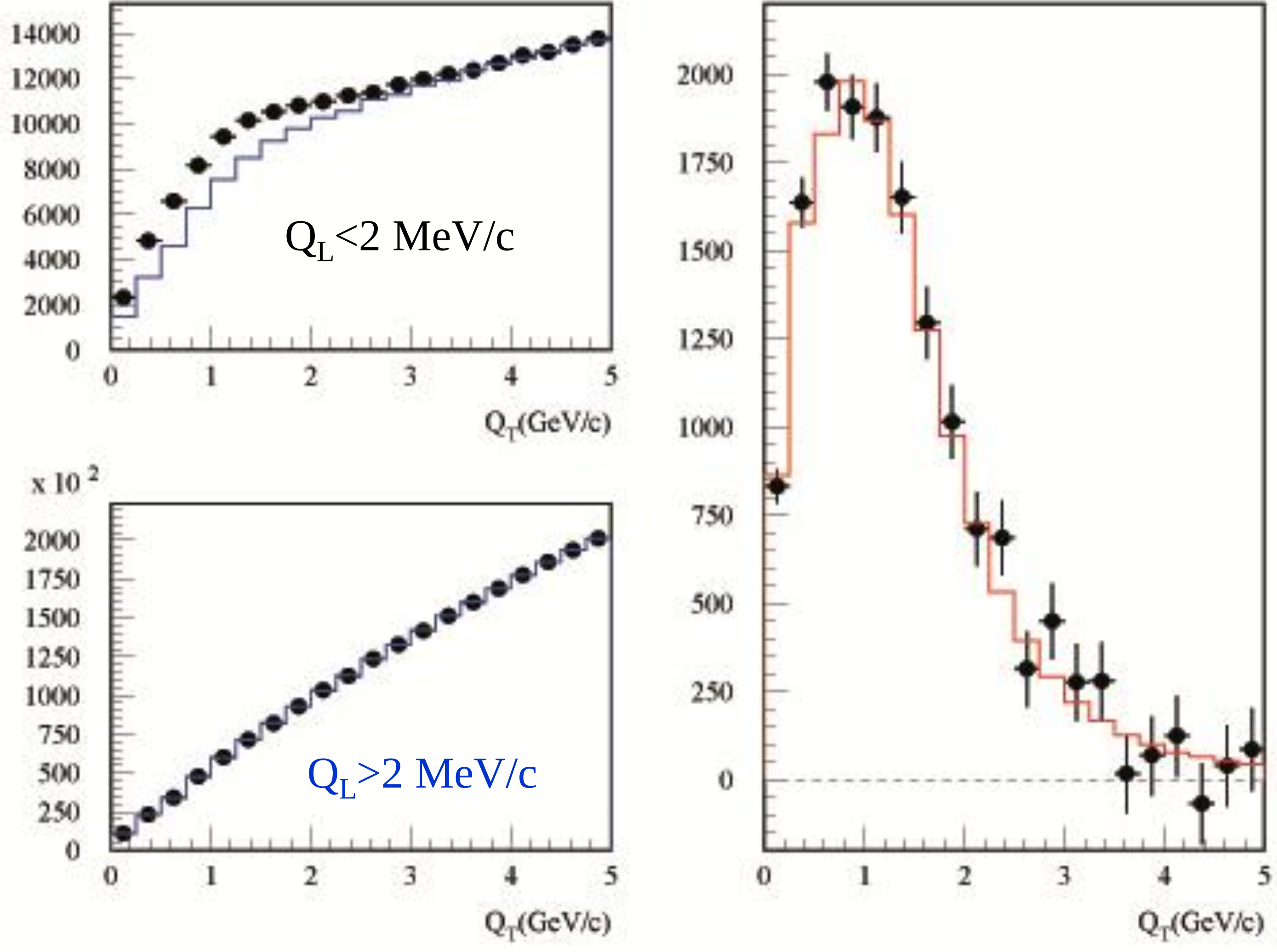
Q_L distribution

←All events

←After background subtraction

DIRAC results with GEM/MSGC

Q_T distribution



←After background subtraction for Q_L<2MeV/c

Comparition with other experimental results

$K\rightarrow 3\pi$:

2009 **NA48/2** (EPJ C64, 589)

...without constraint between a_0 and a_2:

$$\Rightarrow a_0 - a_2 = 0.2571 \pm 1.9\%|_{stat} \pm 1.0\%|_{syst} \pm 0.5\%|_{ext} = \ldots \pm 2.2\%$$

and 3.4% theory uncertainty

...with ChPT constraint between a_0 and a_2:

$$\Rightarrow a_0 - a_2 = 0.2633 \pm 0.9\%|_{stat} \pm 0.5\%|_{syst} \pm 0.7\%|_{ext} = \ldots \pm 1.3\%$$

and 2% theory uncertainty

Ke4:

2010 **NA48/2** (EPJ C70, 635)

...without constraint between a_0 and a_2:

$$\Rightarrow a_0 = 0.2220 \pm 5.8\%|_{stat} \pm 2.3\%|_{syst} \pm 1.7\%|_{theo} = \ldots \pm 6.4\%$$

$$\Rightarrow a_2 = -0.0432 \pm 20\%|_{stat} \pm 7.9\%|_{syst} \pm 6.5\%|_{theo} = \ldots \pm 22\%$$

...with ChPT constraint between a_0 and a_2:

$$\Rightarrow a_0 = 0.2206 \pm 2.2\%|_{stat} \pm 0.8\%|_{syst} \pm 2.9\%|_{theo} = \ldots \pm 3.7\%$$

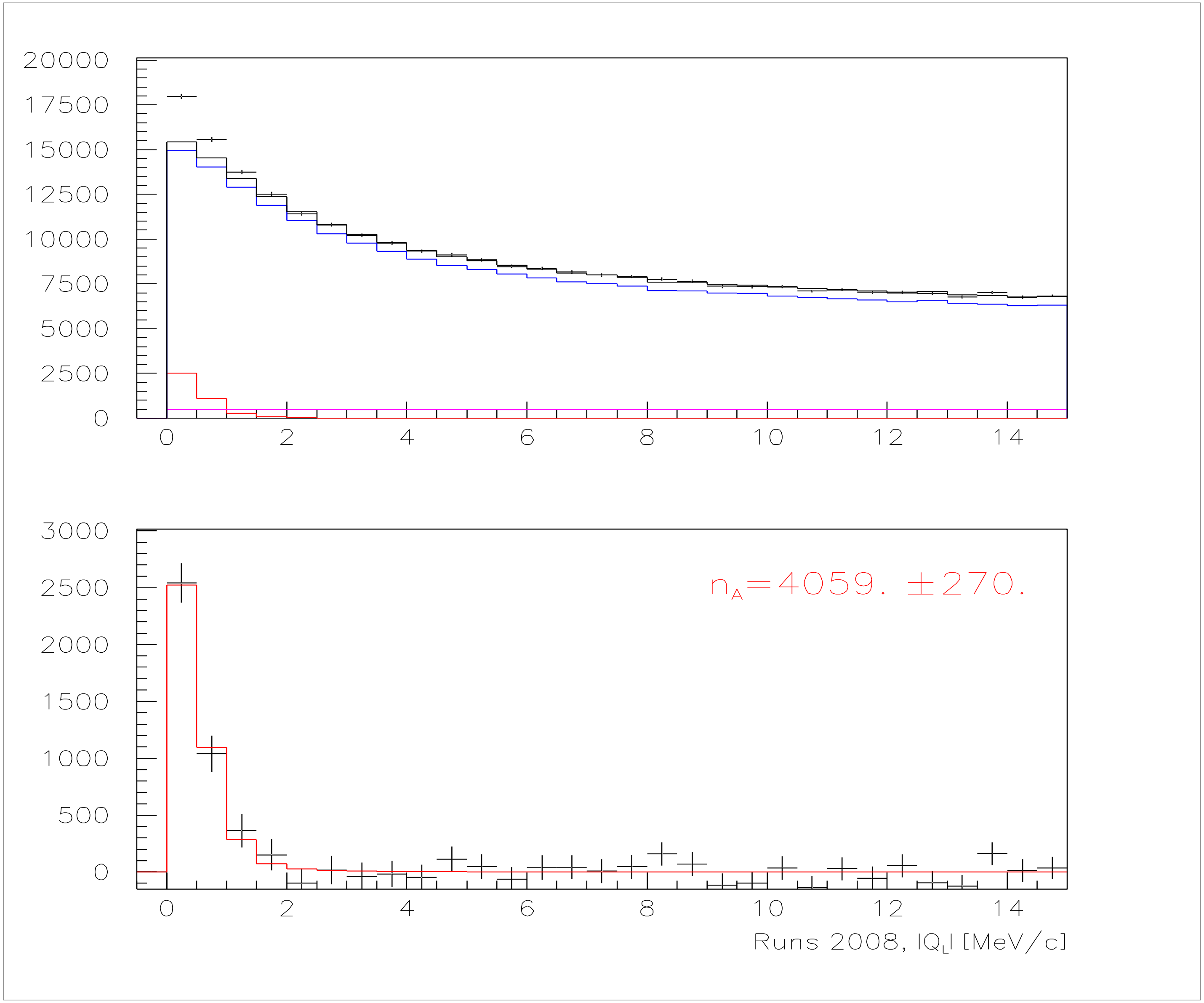
Published results on $\pi\,\pi$ *atom*: lifetime & scattering length

DIRAC data	τ_{1s} (10^{-15}s)					$\lvert a_0 - a_2 \rvert$					Reference
	value	stat	syst	theo*	tot	value	stat	syst	theo*	tot	
2001	2.91	$^{+0.45}_{-0.38}$	$^{+0.19}_{-0.49}$		$\left[^{+0.49}_{-0.62}\right]$	0.264	$^{+0.017}_{-0.020}$	$^{+0.022}_{-0.009}$		$\left[^{+0.033}_{-0.020}\right]$	PL B 619 (2005) 50
2001-03	3.15	$^{+0.20}_{-0.19}$	$^{+0.20}_{-0.18}$		$\left[^{+0.28}_{-0.26}\right]$	0.2533	$^{+0.0078}_{-0.0080}$	$^{+0.0072}_{-0.0077}$		$\left[^{+0.0106}_{-0.0111}\right]$	PL B 704 (2011) 24

* theoretical uncertainty included in systematic error

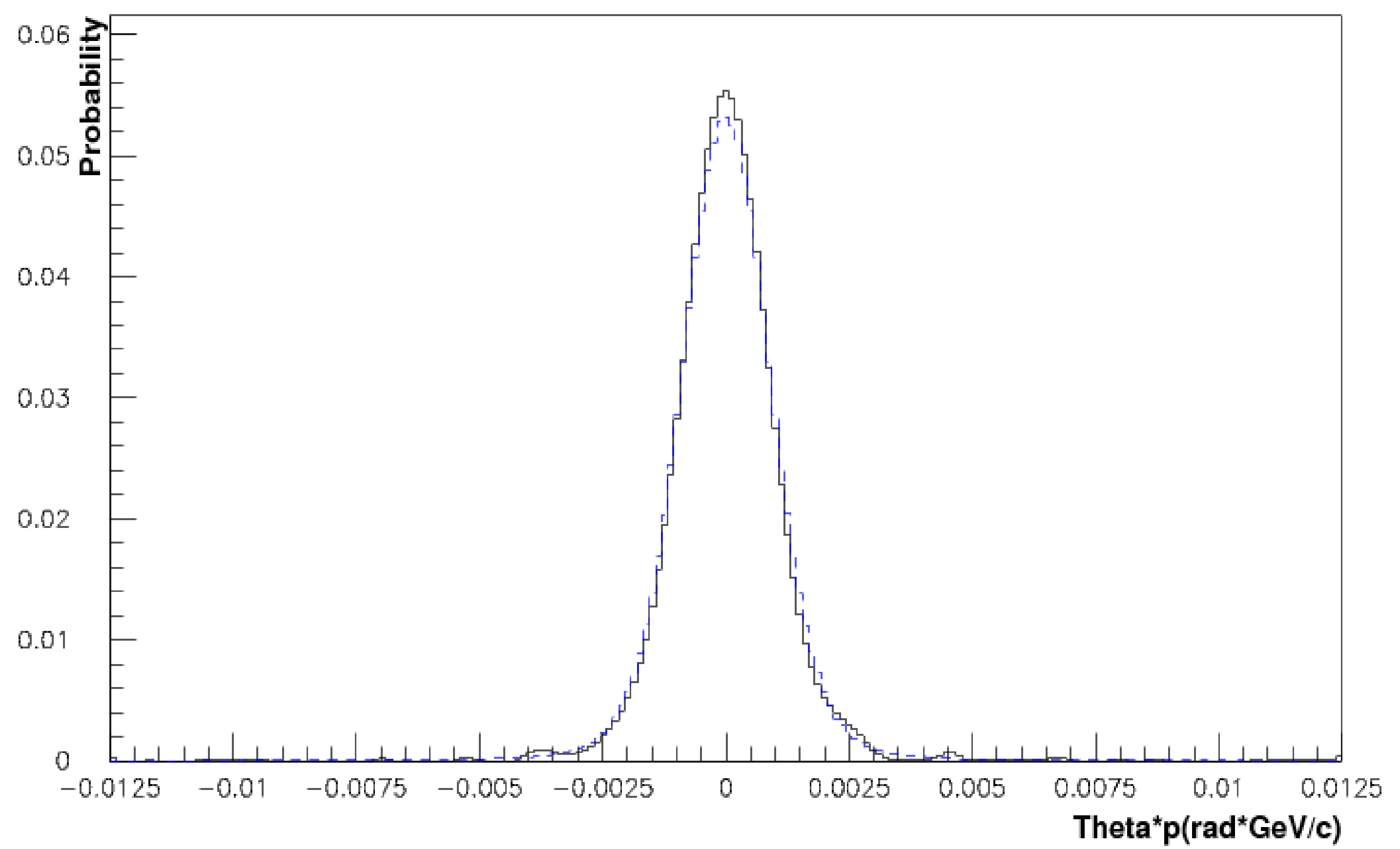
NA48	K-decay	$a_0 - a_2$					Reference
		value	stat	syst	theo	tot	
2009	$K_{3\pi}$	0.2571	± 0.0048	± 0.0029	0.0088		EPJ C64 (2009) 589
2010	K_{e4} & $K_{3\pi}$	0.2639	± 0.0020	± 0.0015			EPJ C70 (2010) 635

New data on $\pi\pi$ atom production



Multiple scattering measurement

100 mkm Ni target



Reconstructed and simulated (blue) MS distributions

$\pi^+\pi^-$ data

Statistics for measurement of $|a_0 - a_2|$ scattering length difference and expected precision

Year	n_A	δ_{stat} (%)	Δ_{syst} (%)	δ_{syst} (%) MS	δ_{tot} (%)
2001-2003	21000	3.1	3.0	2.5	4.3
2008-2010 *	24000	3.0	3.0	2.5	4.3
2001-2003 2008-2010	45000	2.1	3.0 (2.1)	2.5 (1.25)	3.7 (3.0)

* There is 40% of data with a higher background whose implication is under investigation.

πK scattering

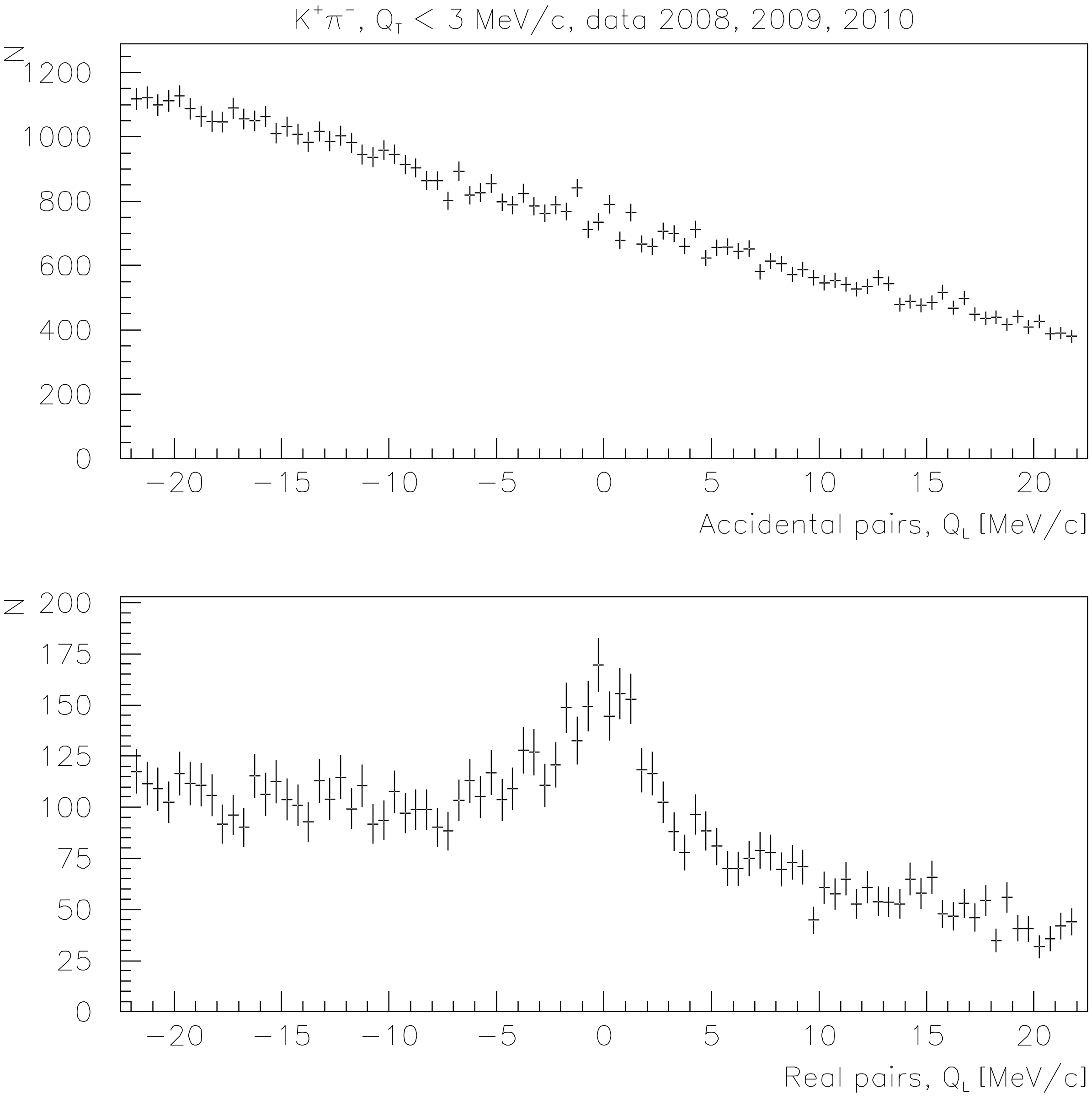
What new will be known if πK scattering length will be measured?

The measurement of the s-wave πK scattering lengths would test our understanding of the chiral $SU(3)_L \times SU(3)_R$ symmetry breaking of *QCD* (u, d and s quarks), while the measurement of $\pi\pi$ scattering lengths checks only the $SU(2)_L \times SU(2)_R$ symmetry breaking (u, d quarks).

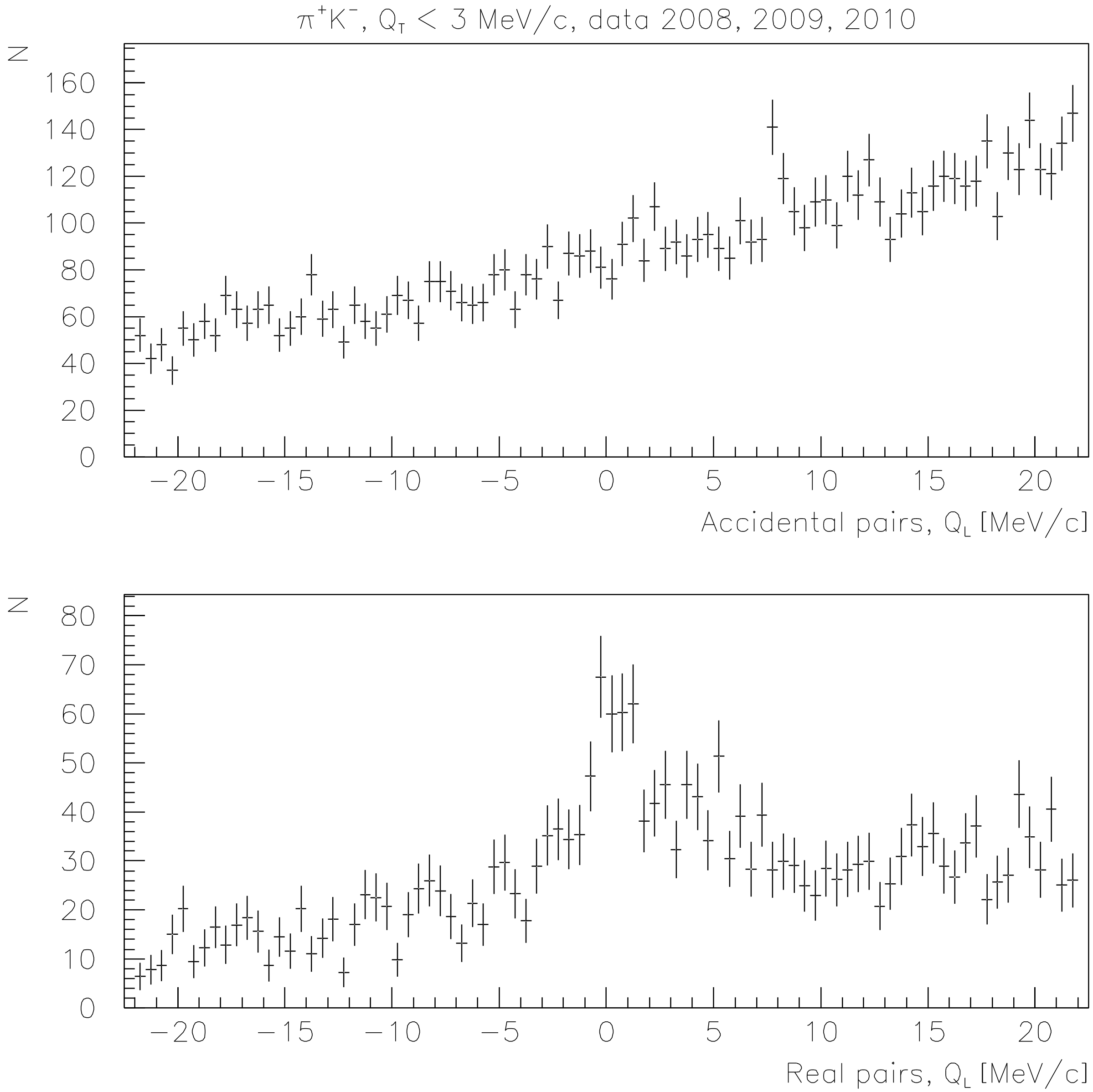
This is the principal difference between $\pi\pi$ and πK scattering!

Experimental data on the πK low-energy phases are absent

Q_L distribution for $K^+\pi^-$ pairs



Q_L distribution for π^+K^- pairs

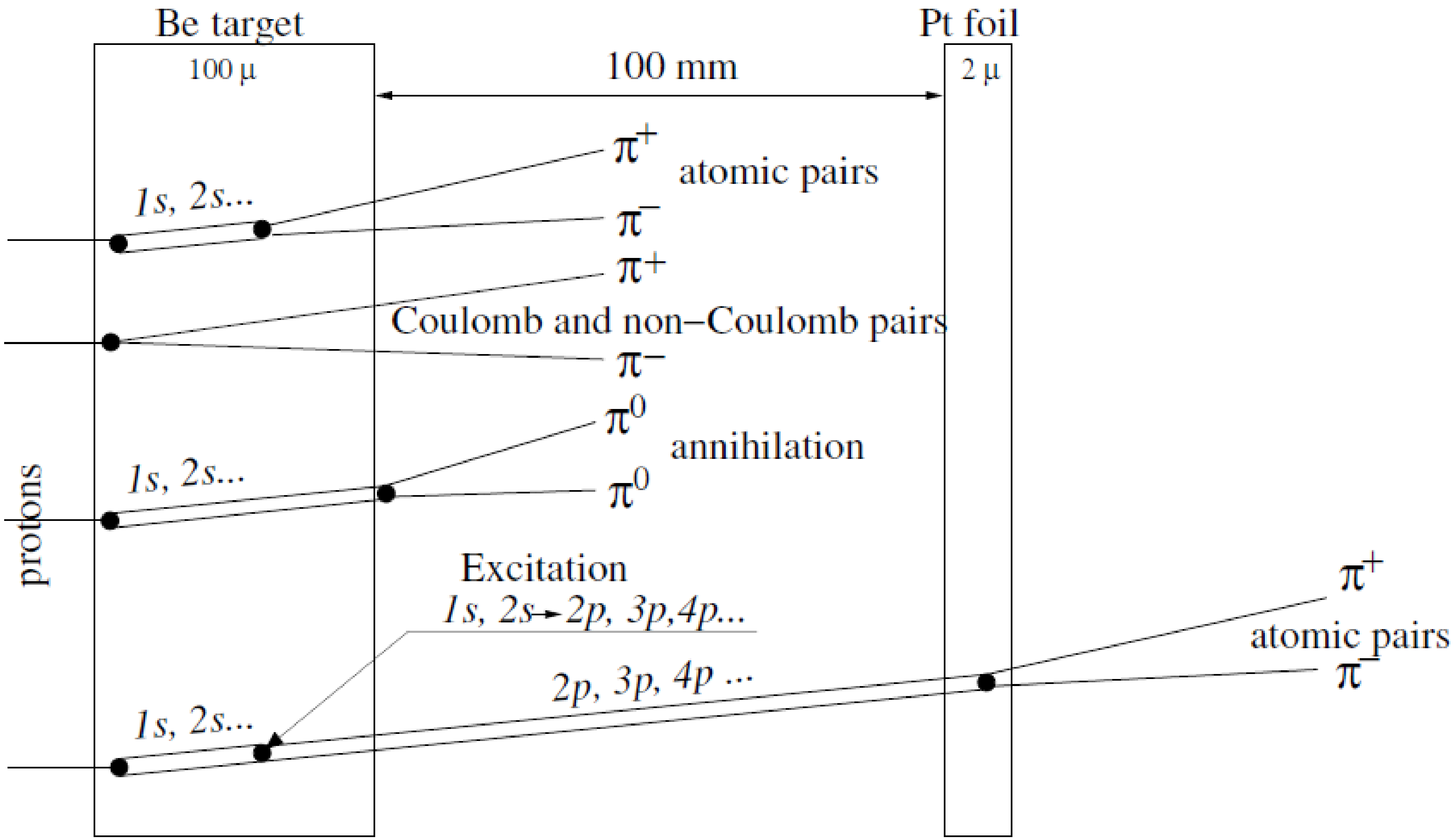


Long-lived $\pi^+\pi^-$ atoms

The observation of $\pi\pi$ atom long-lived states opens the future possibility to measure the energy difference between ***ns*** and ***np*** states $\Delta E(ns\text{-}np)$ and the value of $\pi\pi$ scattering lengths $|2a_0+a_2|$.

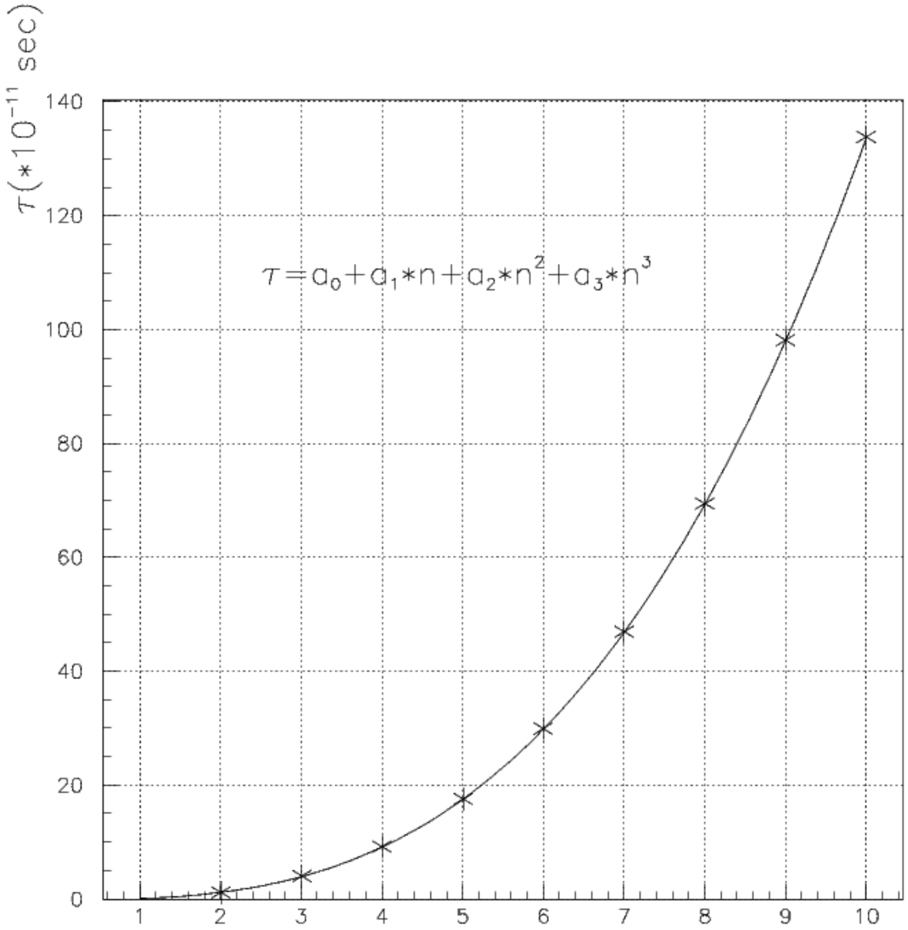
If a resonance method can be applied for the $\Delta E(ns\text{-}np)$ measurement, then the precision of $\pi\pi$ scattering length measurement can be improved by one order of magnitude relative to the precision of other methods.

Method to observe long-lived $A_{2\pi}$ by means of a breakup foil of Platinum



$A_{2\pi}$ lifetime, τ, in np states

M. Pentia

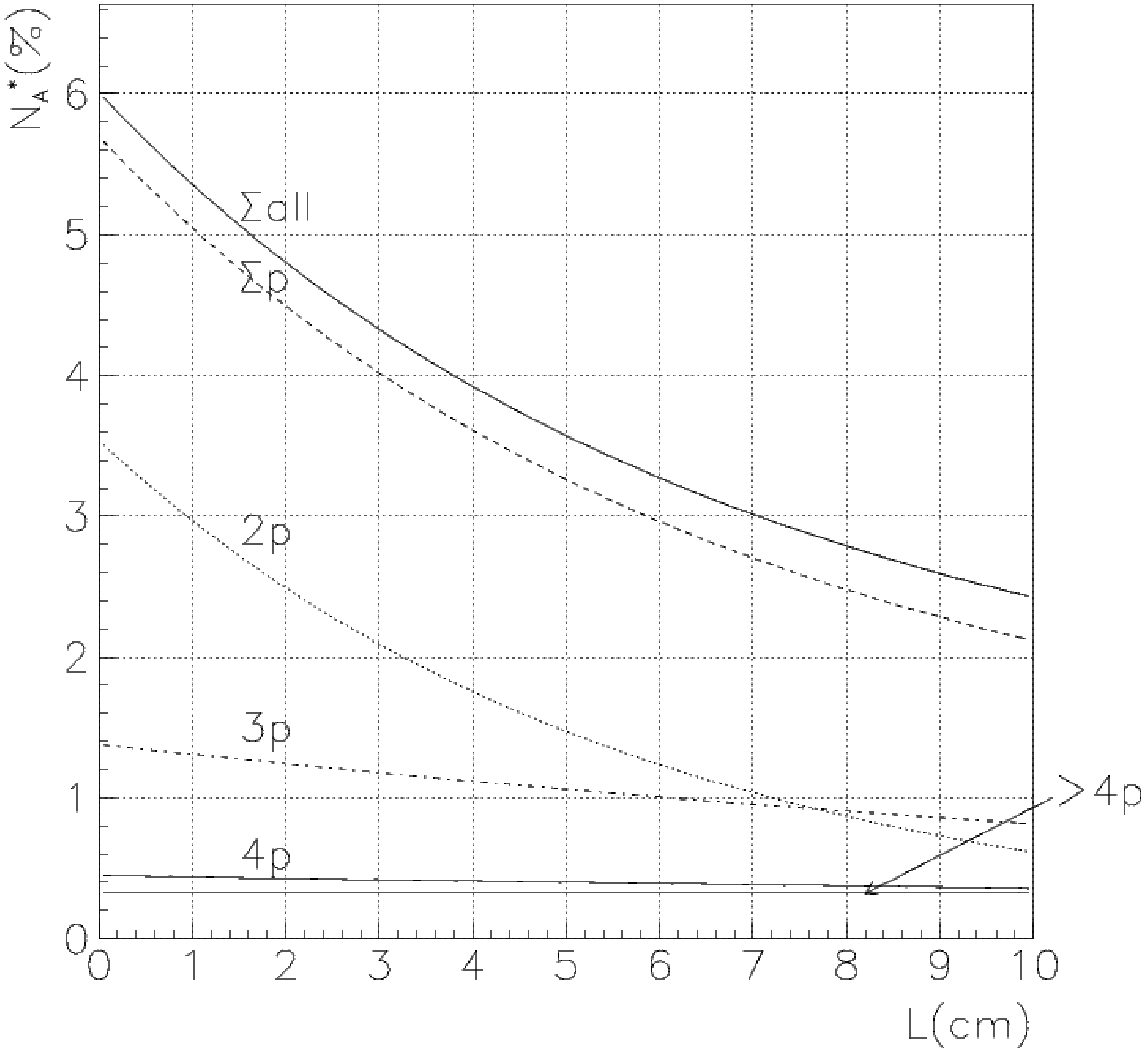


n_H	$\tau_H \bullet 10^8$ s	$\tau_{2\pi} \bullet 10^{11}$ s	Decay length $A_{2\pi}$ in L.S. cm for γ=16.1
2p	0.16	1.17	5.7
3p	0.54	3.94	19
4p	1.24	9.05	44
5p	2.40	17.5	84.5
6p	4.1	29.9	144
7p		46.8*	226
8p		69.3*	335

*** - extrapolated values**

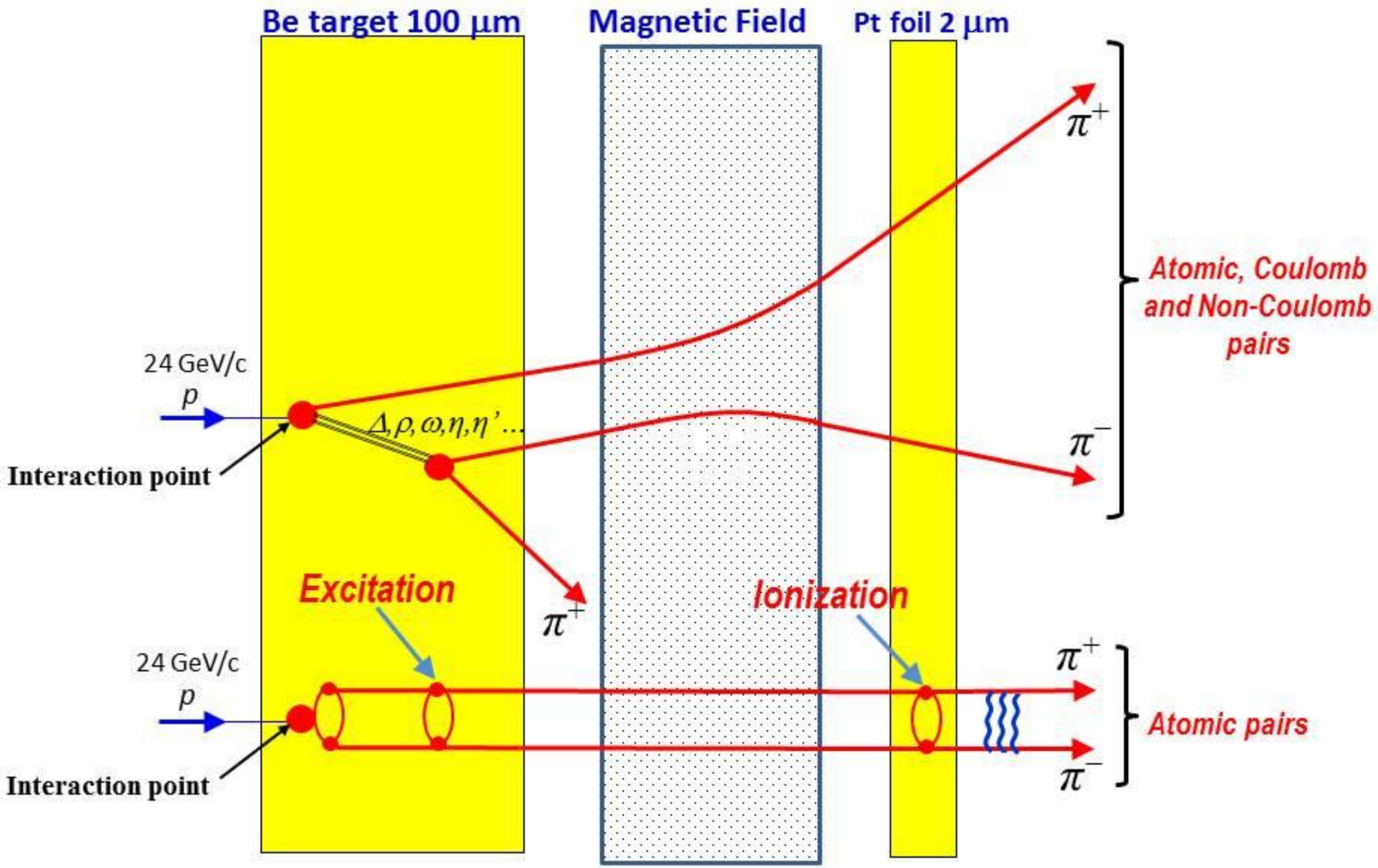
“Long-lived $A_{2\pi}$” yield and quantum numbers

L. Afanasev; O. Gorchakov (DIPGEN)



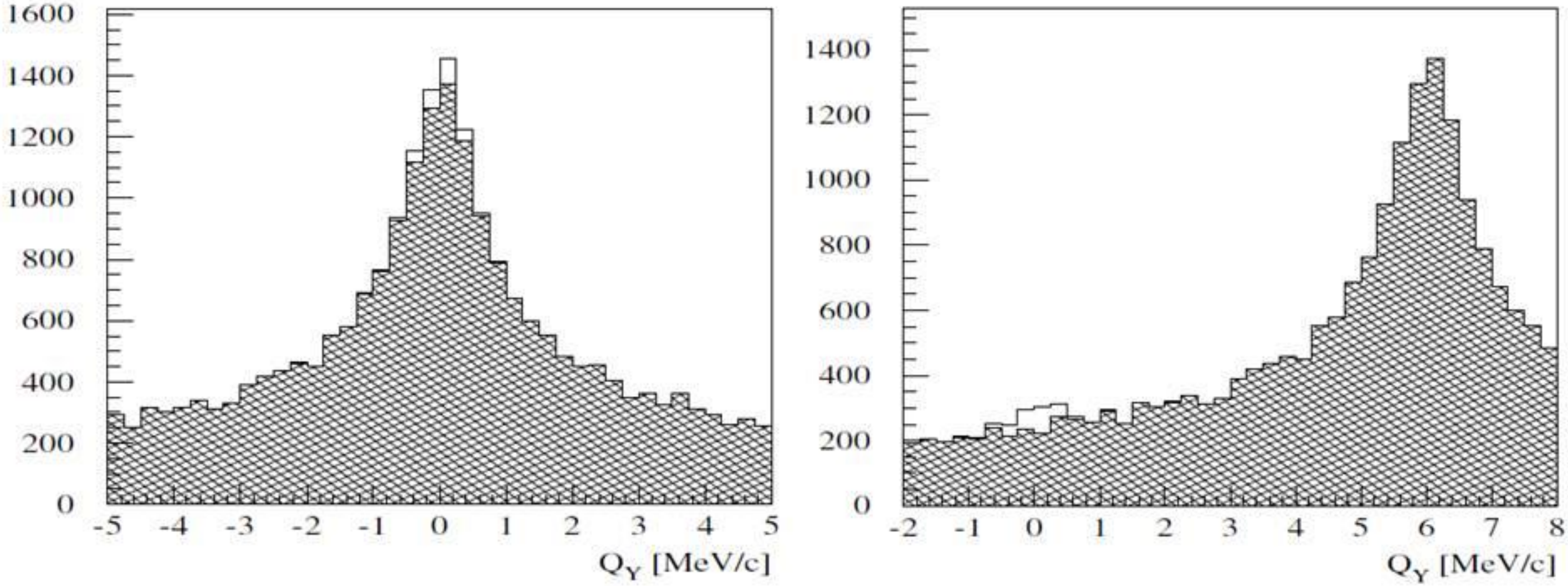
Atomic pairs from “long-lived $A_{2\pi}$” breakup in 2µm Pt.

Method for observing long-lived $A_{2\pi}$ with breakup Pt foil



Simulation of long-lived $A_{2\pi}$ observation

V. Yazkov



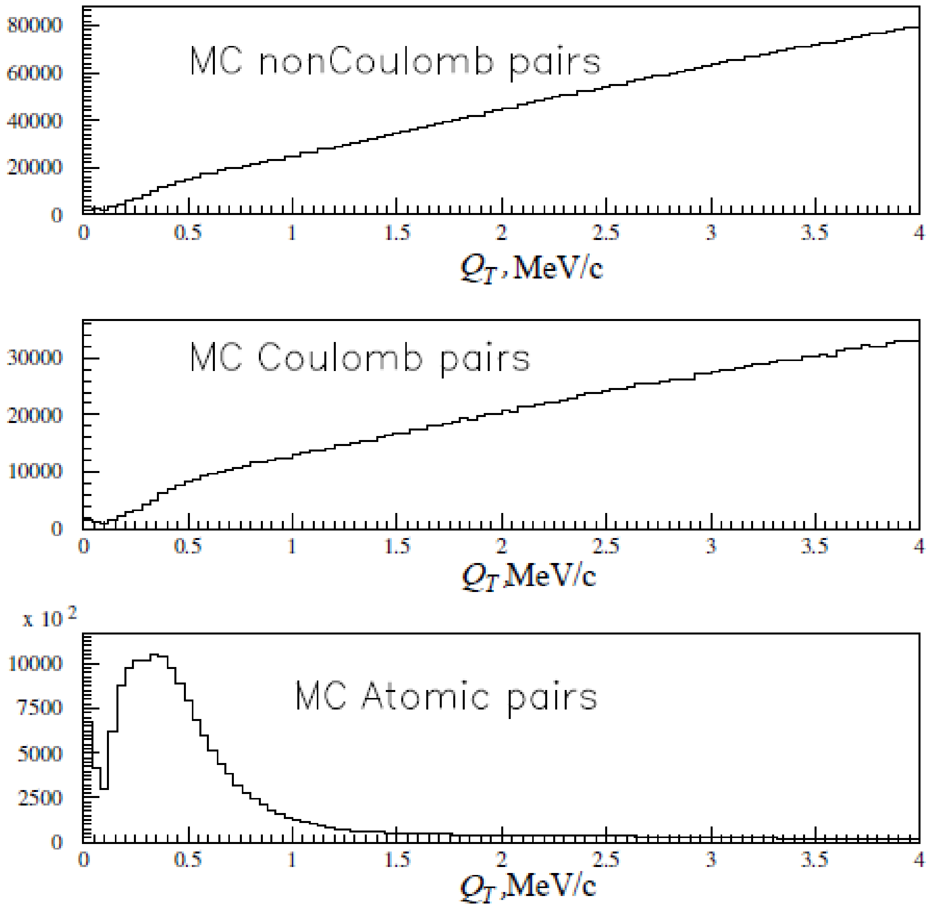
Without magnet

With magnet after Be target

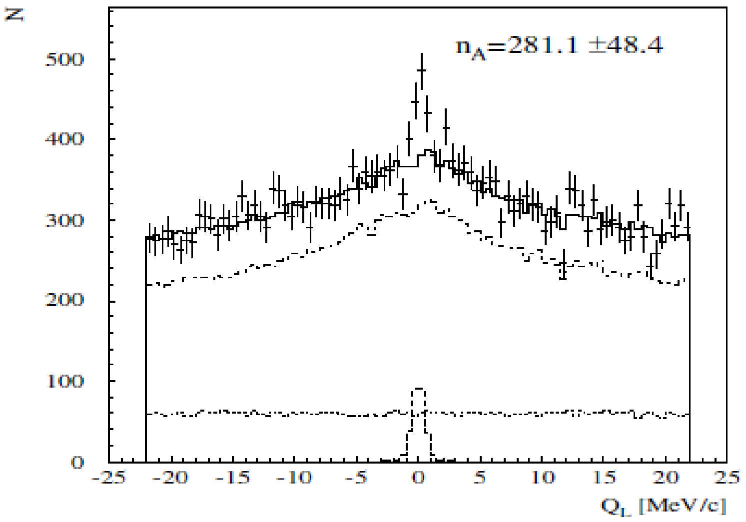
Simulated distribution of $\pi^+\pi^-$ pairs over Q_Y with criteria: $|Q_X| < 1$ MeV/c, $|Q_L| < 1$ MeV/c. "Atomic pairs" from long-lived atoms (light area) above background (hatched area) produced in Beryllium target.

Simulation of $\pi^+\pi^-$ pairs from Beryllium target and ”atomic pairs” from Platinum foil

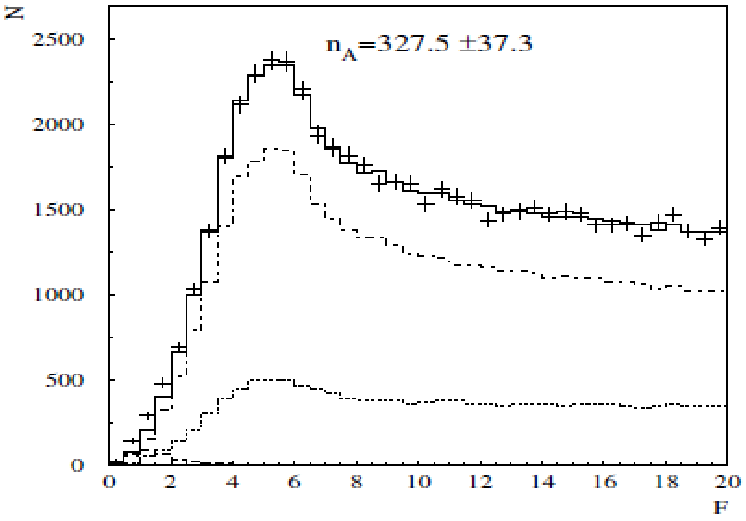
Distributions of reconstructed values of Q_T for non-Coulomb, Coulomb pairs and pairs from metastable atom



Simulation of extraction the long-lived $A_{2\pi}$ signal



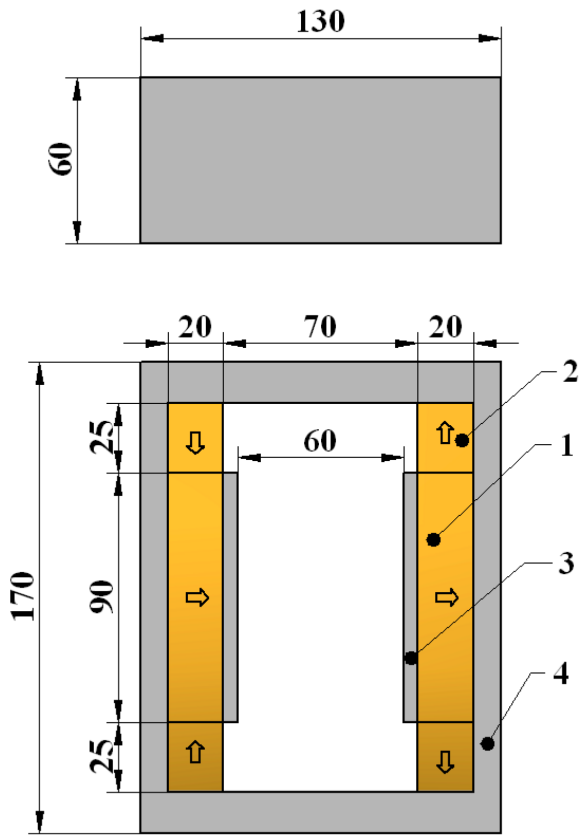
Simulated distribution of $\pi^+\pi^-$ pairs over Q_L, with criterion $Q_T < 1$ MeV/c. "Experimental data" (points with error bars) are fitted by the sum of "atomic pairs" from long-lived states (dashed line), "Coulomb pairs" (by dotted-dashed line), "non-Coulomb pairs" (dotted line). The background sum is shown by the solid line.



Simulated distribution of $\pi^+\pi^-$ pairs over F, with criterion $Q_T < 2$ MeV/c. "Experimental data" (points with error bars) are fitted by the sum of "atomic pairs" from long-lived states (dashed line), "Coulomb pairs" (dotted-dashed line), "non-Coulomb pairs" (dotted line). The background sum is shown by the solid line.

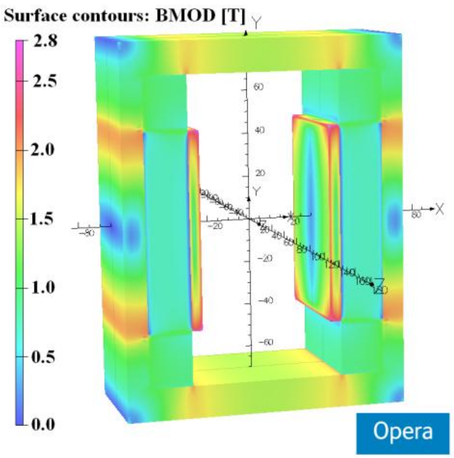
Magnet design

Layout of the dipole magnet (arrows indicate the direction of magnetization)

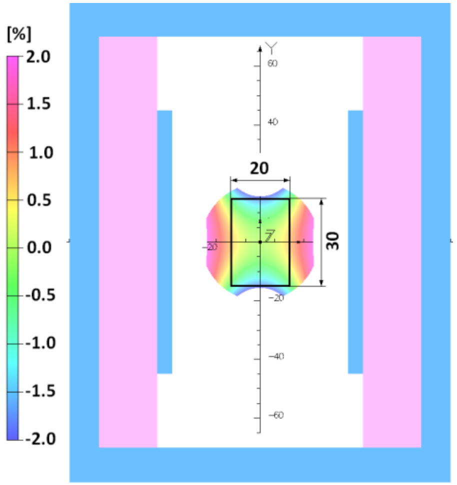


1- PM block Sm2Co17
2- PM block Sm2Co17
3- Pole AISI 1010
4- Return yoke AISI 1010

Opera 3D model with surface field distribution

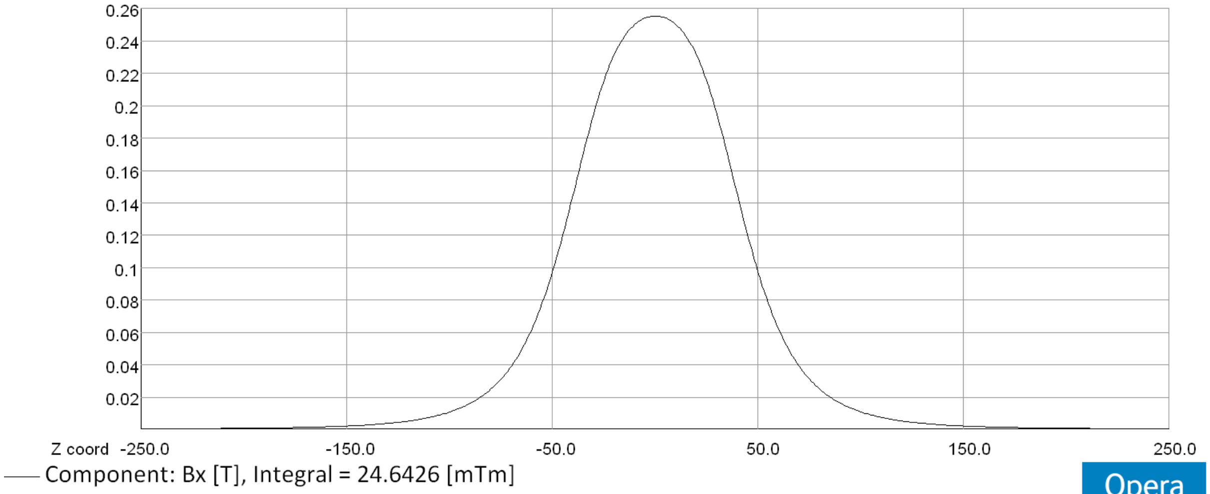


Integrated horizontal field homogeneity inside the GFR X×Y = 20 mm × 30 mm: $\Delta\int B_x dz / \int B_x(0,0,z)dz$ [%]

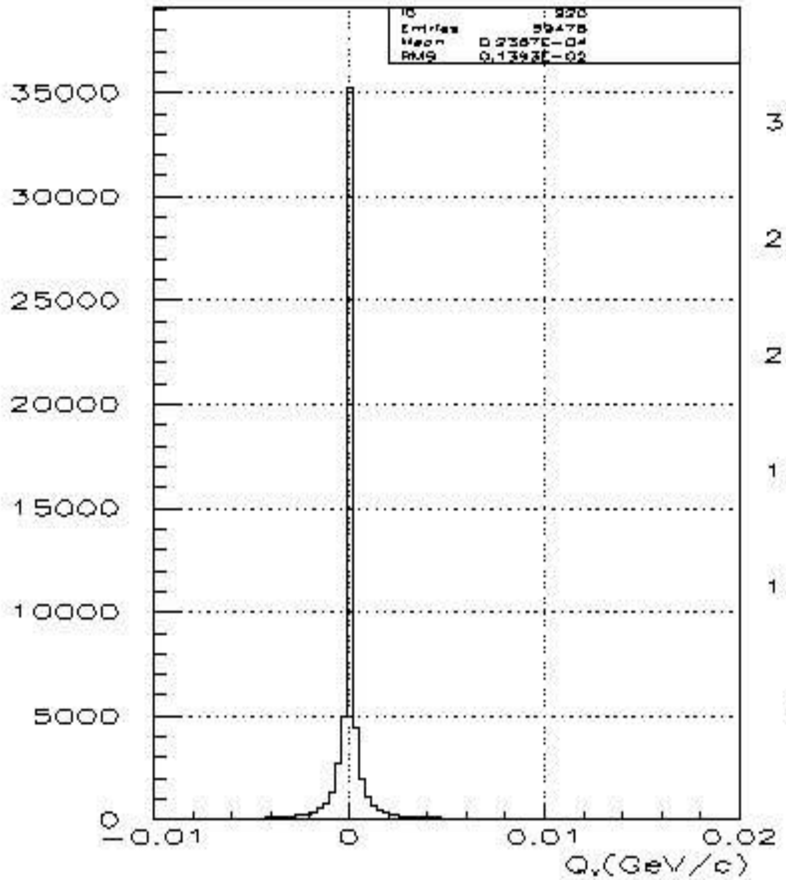


Horizontal field distribution along z-axis at X=Y=0 mm

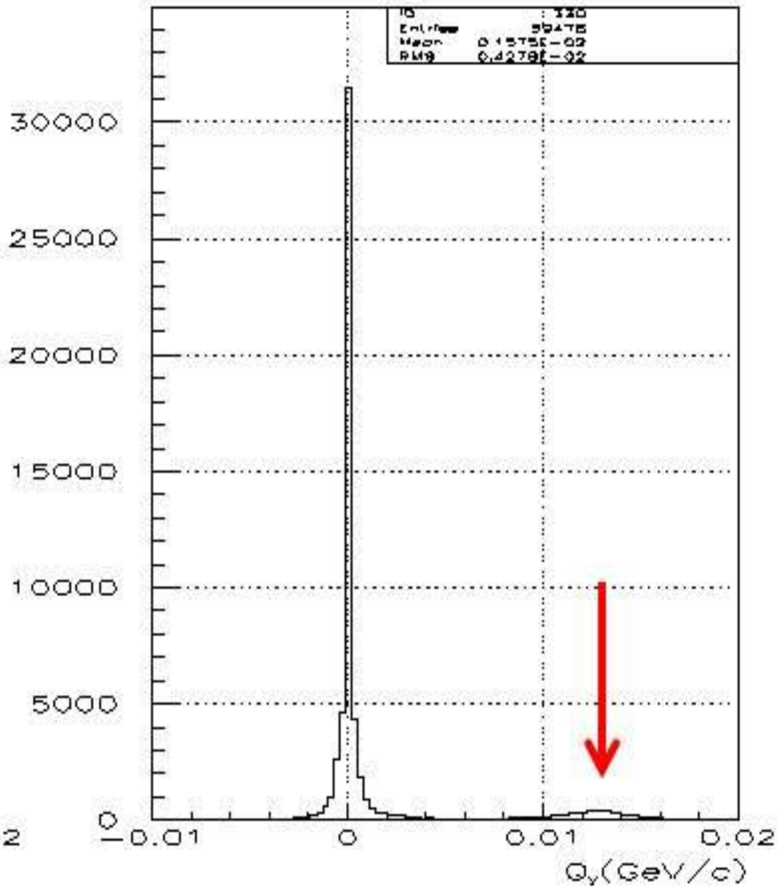
$\int B_x(0,0,z)dz = 24.6\times10^{-3}$ [T×m]



Q_x and Q_y distributions for e^+e^- pair

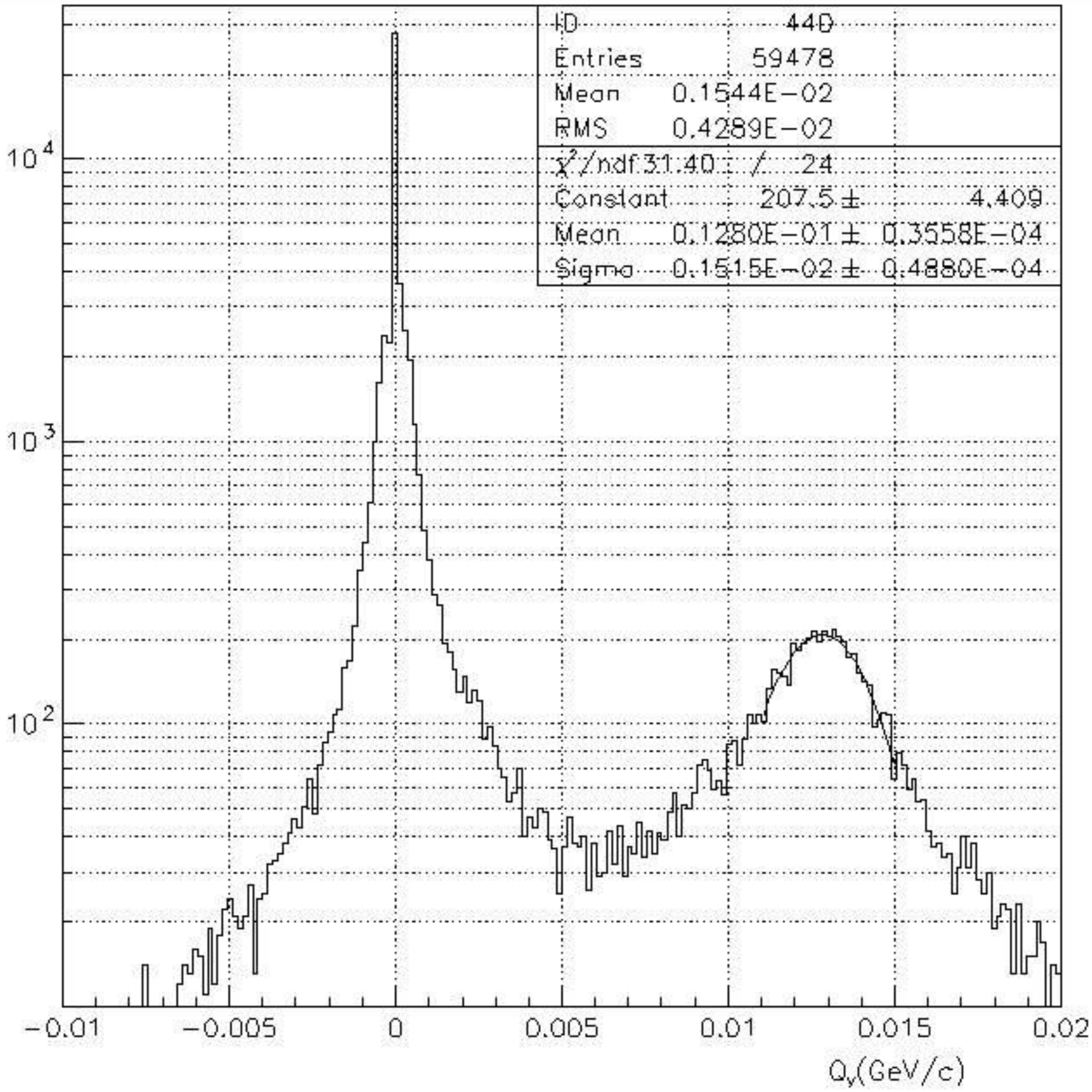


Q_x distribution

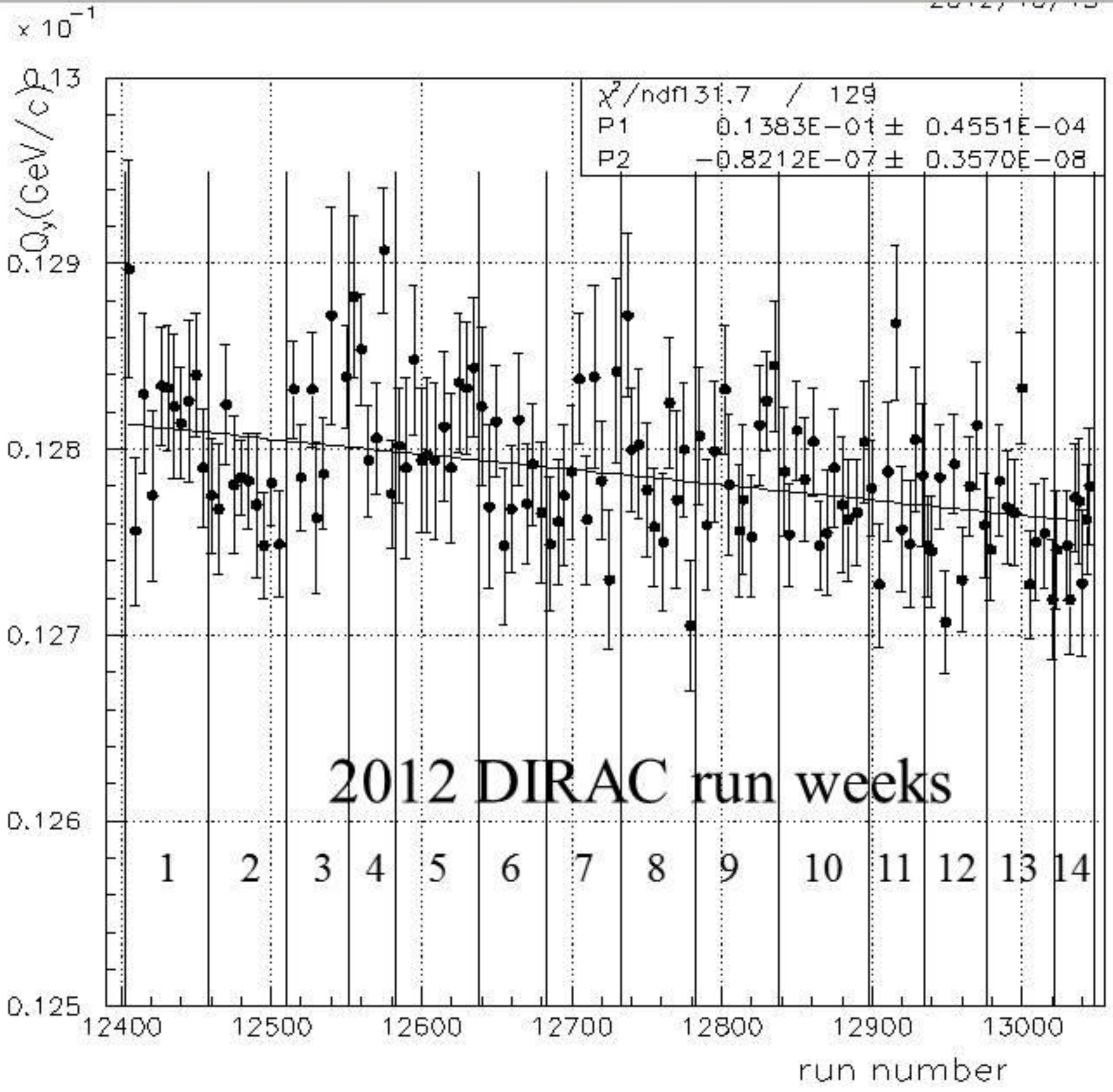


Q_y distribution

Q_y distribution for e^+e^- pair



Magnetic field stability measured by Q_y of the e^+e^- pair

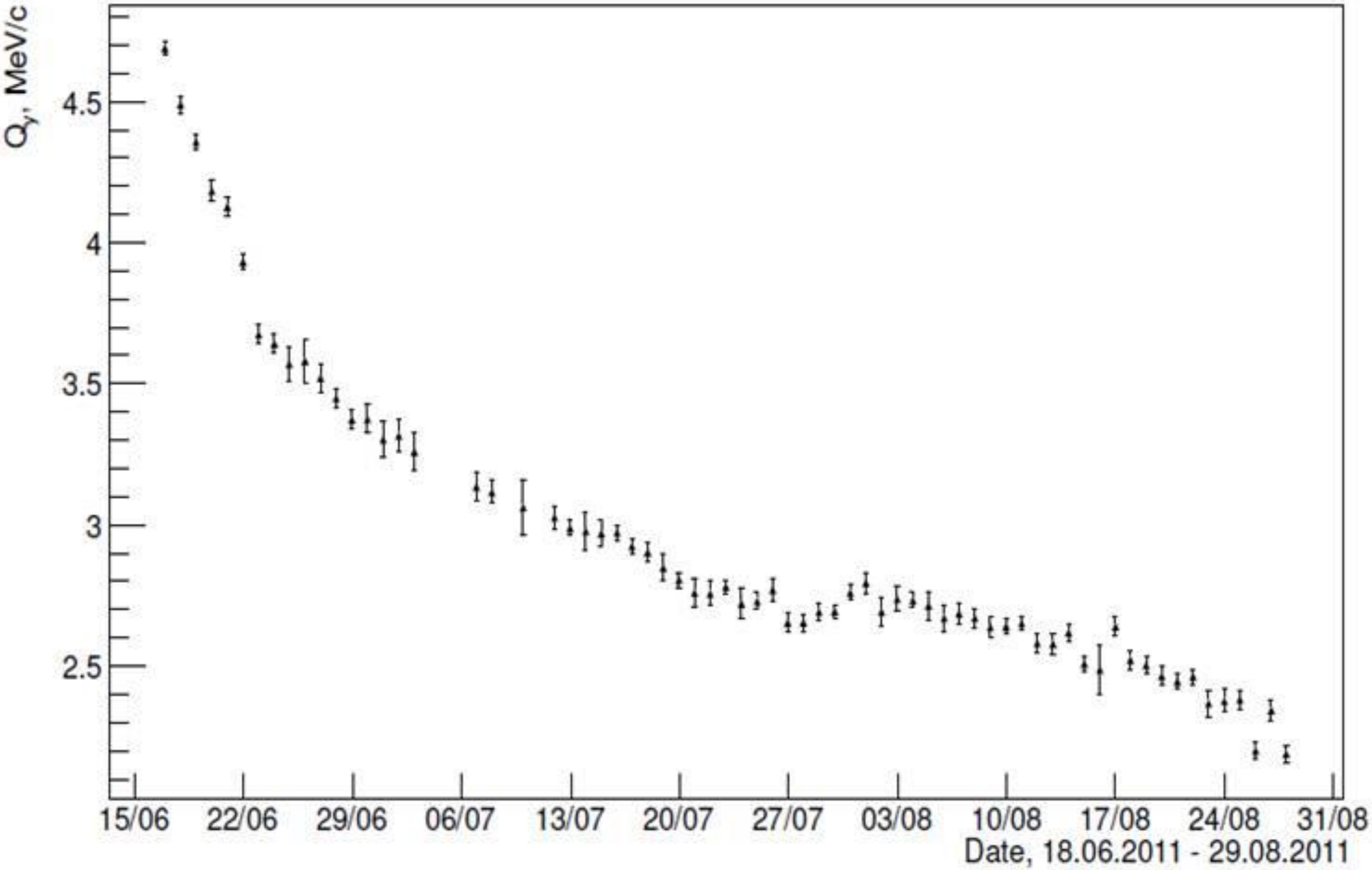


Sm_2Co_{17}

$$\frac{\Delta Q_y}{Q_y} = \frac{5}{1281}$$

$$= 0.4\%$$

Degradation of the old magnet in June-August 2011



Nd-Fe-B

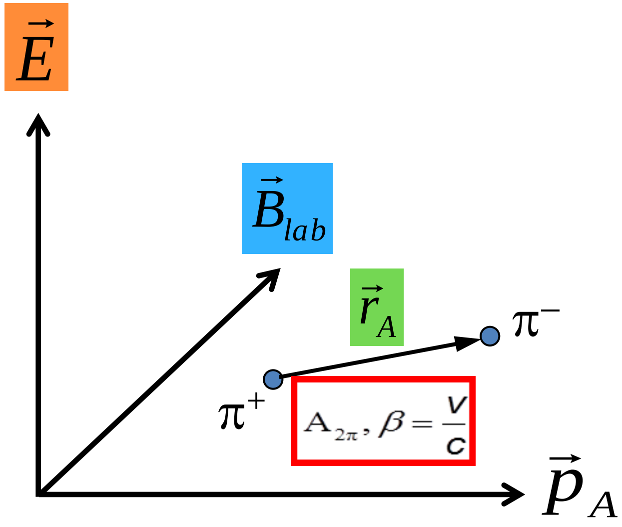
$$\frac{\Delta Q_\gamma}{Q_\nu} > 50\%$$

The position of second peak in Q_Y distributions of e^+e^- pairs versus dates.

Lamb shift measurement with external magnetic field

See: L. Nemenov, V. Ovsiannikov, Physics Letters B 514 (2001) 247.

Impact on atomic beam by
external magnetic field $\mathbf{B}_{lab}$ and Lorentz factor $\boldsymbol{\gamma}$



$\vec{r}_A$ …. relative distance between π^+ and π^- in $A_{2\pi}$ system

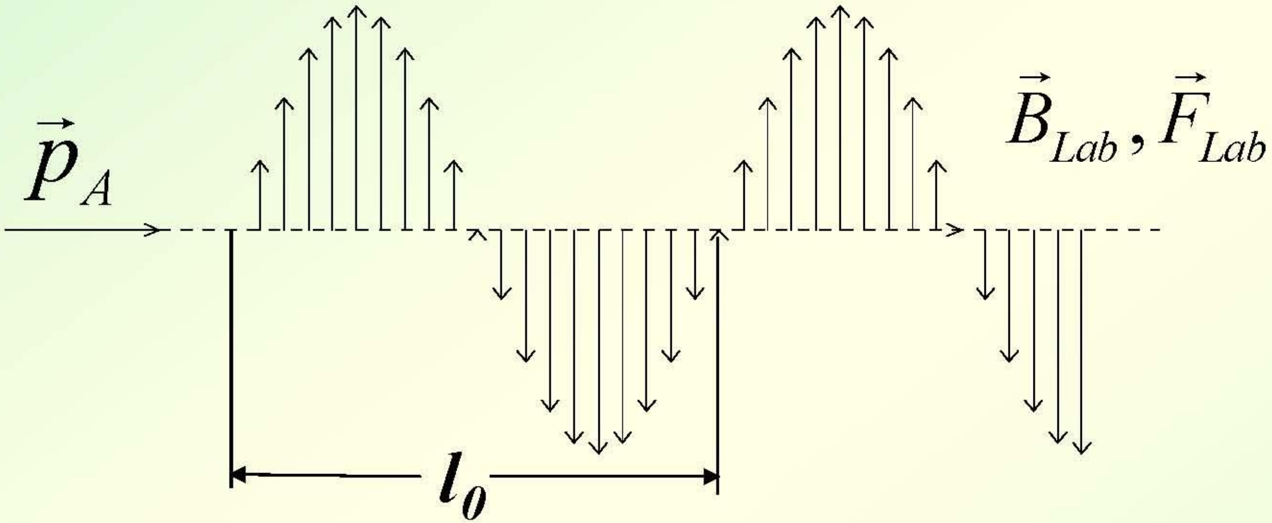
$\vec{B}_{lab}$ …. laboratory magnetic field

$\vec{E}$ …electric field in $A_{2\pi}$ system

$$\boxed{\left|\vec{E}\right| = \beta\gamma\, B_{lab} \approx \gamma\, B_{lab}}$$

1.9 Resonant enhancement of the annihilation rate of $A_{2\pi}$

L.Nemenov, V.Ovsiannikov, E.Tchaplyguine, Nucl. Phys. (2002)

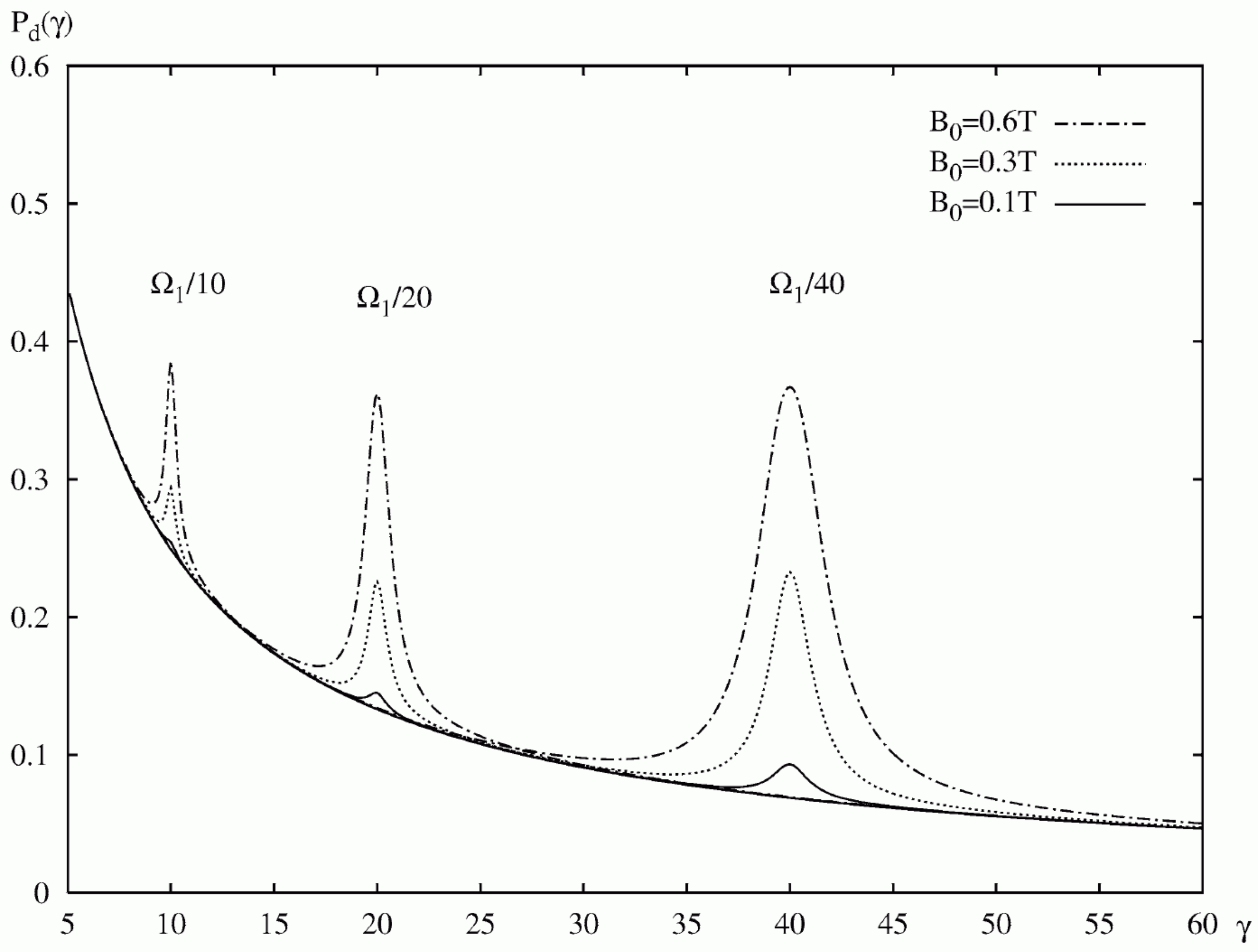


In Lab. System: $T_{Lab} = \dfrac{l_0}{\beta c}, \qquad \omega_{Lab} = \dfrac{2\pi}{T_{Lab}}$

In CM System: $\tilde{\omega} = \gamma \omega_{Lab}, \qquad \tilde{\vec{F}} = \gamma \vec{F}_{Lab} \cdot \cos \tilde{\omega} t, \qquad \tilde{\Omega} = \dfrac{E_{2p} - E_{2s}}{\hbar}$

at resonance: $\tilde{\Omega} = \tilde{\omega} = \gamma_{res} \cdot \omega_{Lab} \qquad \Rightarrow \qquad \gamma_{res} = \dfrac{\tilde{\Omega}}{\omega_{Lab}}$

Resonant enhancement



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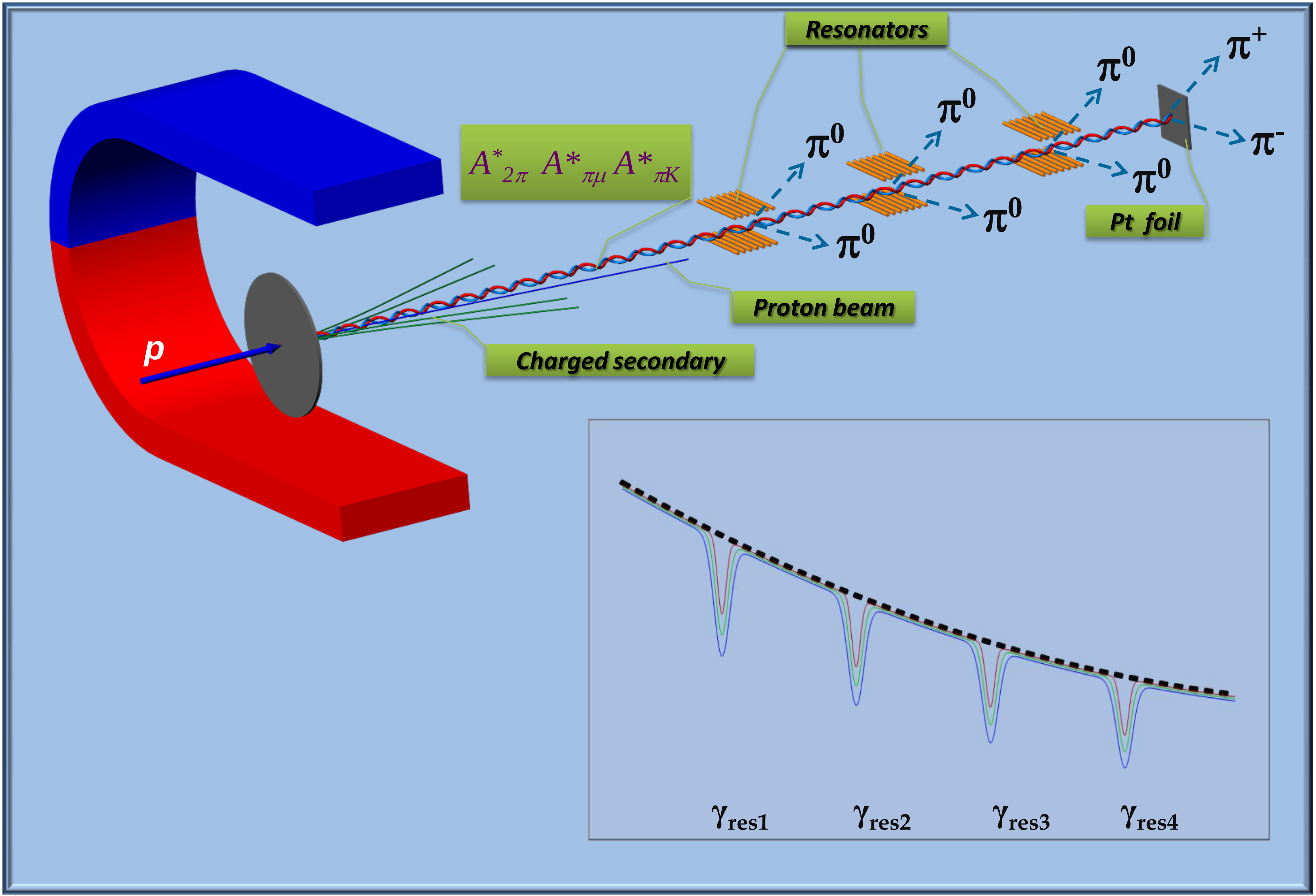
Resonant method

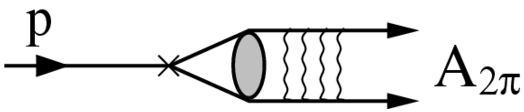


$A_{2\pi}$ and $A_{\pi K}$ production

$$\frac{d\sigma_{nlm}^{A}}{d\vec{P}} = (2\pi)^3 \frac{E}{M} \left|\psi_{nlm}^{(C)}(0)\right|^2 \frac{d\sigma_s^0}{dp_1 dp_2} \overline{\propto} \frac{d\sigma}{dp_1} \cdot \frac{d\sigma}{dp_2}$$

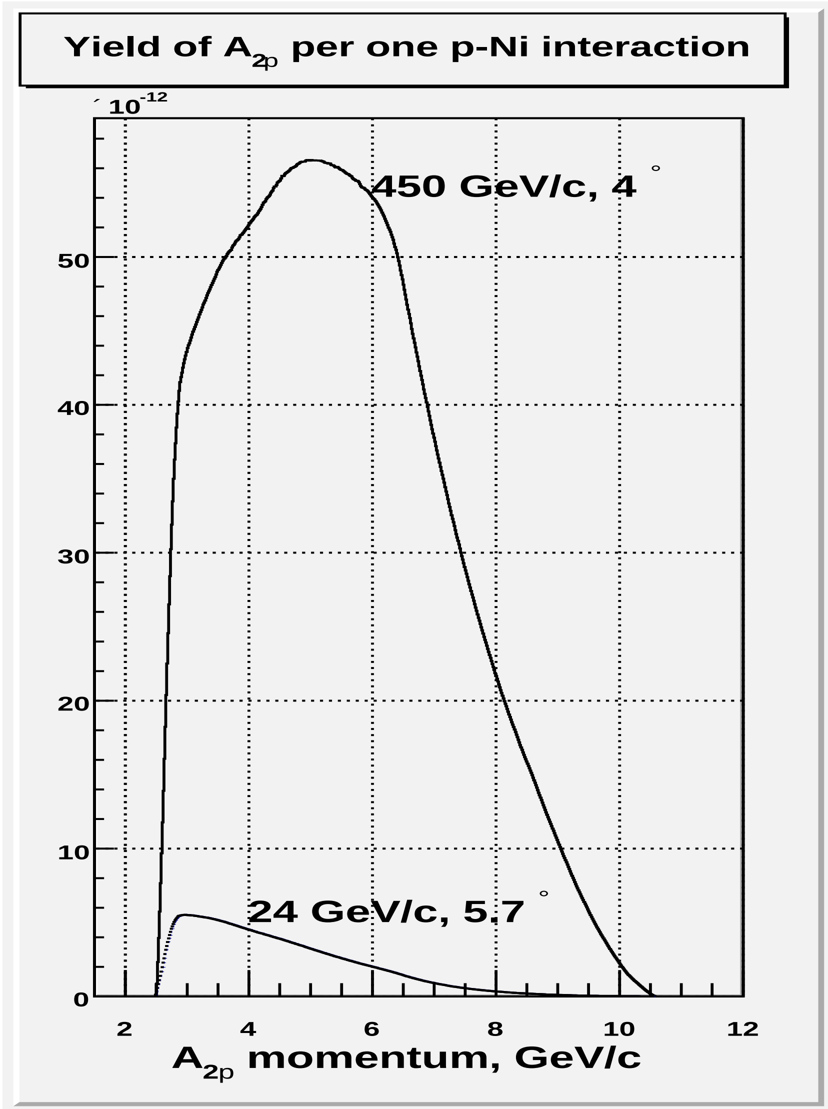
for atoms $\vec{\mathrm{v}}_1 = \vec{\mathrm{v}}_2$ *where* $\vec{\mathrm{v}}_1, \vec{\mathrm{v}}_2$ *– velocities of particles in the L.S. for all types of atoms*

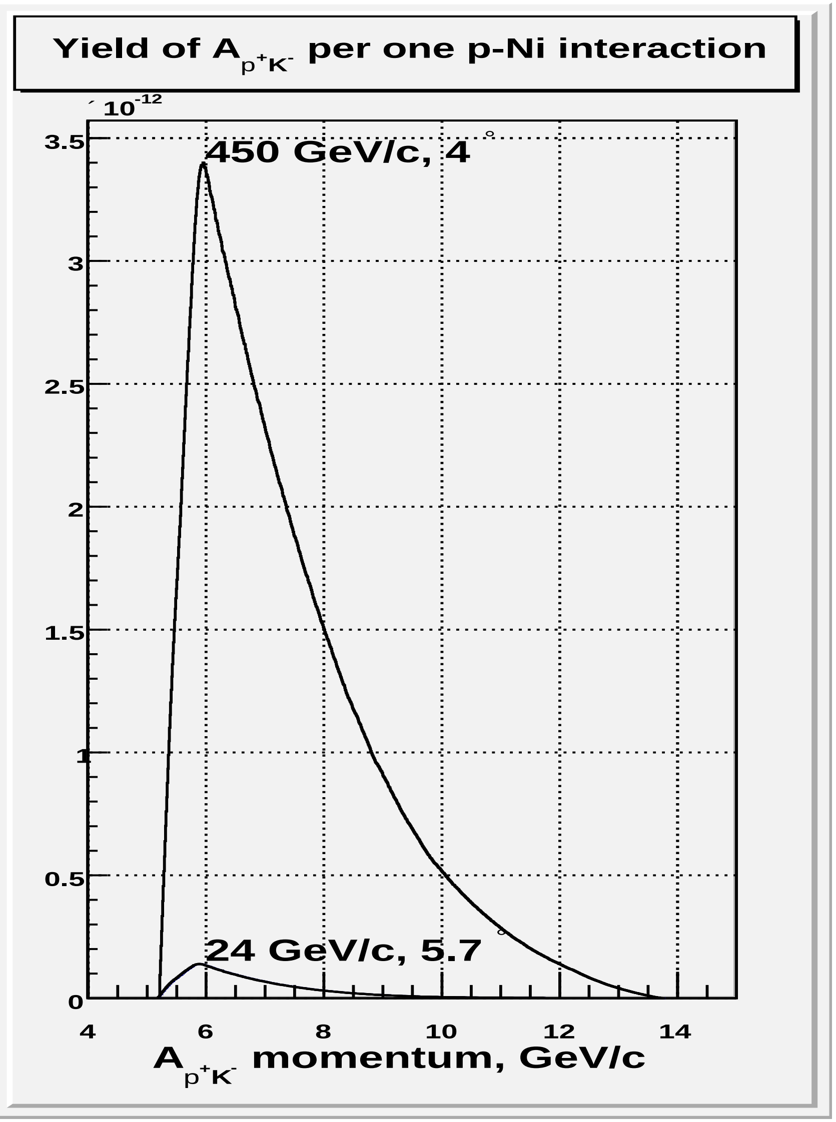
for $A_{2\pi}$ *production* $\vec{p}_1 = \vec{p}_2$



for $A_{\pi K}$ *production* $\vec{p}_\pi = \frac{m_\pi}{m_K} \vec{p}_K$

$A_{2\pi}$ and $A_{\pi K}$ production on PS and SPS at CERN





$A_{2\pi}$ and $A_{\pi K}$ production on PS and SPS at CERN

	Yield ratio
$\pi^+\pi^-$ atoms	17
π^+K^- atoms	35
$K^+\pi^-$ atoms	27

The ratio of $\pi^+\pi^-$, π^+K^- and $K^+\pi^-$ atom yields at the proton momenta 450 GeV/c and angle 4° to the yields at the proton momenta 24 GeV/c and angle 5.7°.

Conclusion

- DIRAC experiment on PS CERN measured $\pi^+\pi^-$ atom lifetime with precision about 9%. With the existing additional statistic the lifetime will be measured with precision about 6% and $\pi\pi$ scattering lengths with accuracy about 3%.
- The existing statistics allows to measure the Kπ cross section production.
- The statistics obtained in 2011 and 2012 allows to observe the long-lived $\pi^+\pi^-$ atoms

Conclusion

The same setup on SPS CERN will allow:

- To measure the Kπ atom lifetime with precision better than 10% and to perform the first measurement of Kπ scattering lengths
- To measure the Lamb shift of $\pi^+\pi^-$ atom.

Thank you for your attention

πK scattering lengths

I. ChPT predicts s-wave scattering lengths:

$$a_0^{1/2} = 0.19 \pm 0.2 \qquad a_0^{3/2} = -0.05 \pm 0.02$$

V. Bernard, N. Kaiser,
U. Meissner. – 1991

$\mathcal{L}^{(2)}, \mathcal{L}^{(4)}$ *and* 1 – *loop*

$$a_0^{1/2} - a_0^{3/2} = 0.23 \pm 0.01$$

A. Rossel. – 1999

J. Bijnens, P. Talaver. – April 2004

$\mathcal{L}^{(2)}, \mathcal{L}^{(4)}, \mathcal{L}^{(6)}$ *and* 2 - *loop*

II. Roy-Steiner equations:

$$a_0^{1/2} - a_0^{3/2} = 0.269 \pm 0.015$$

P.Büttiker et al. – 2004

Production, annihilation and breakup of long-lived $A_{2\pi}$

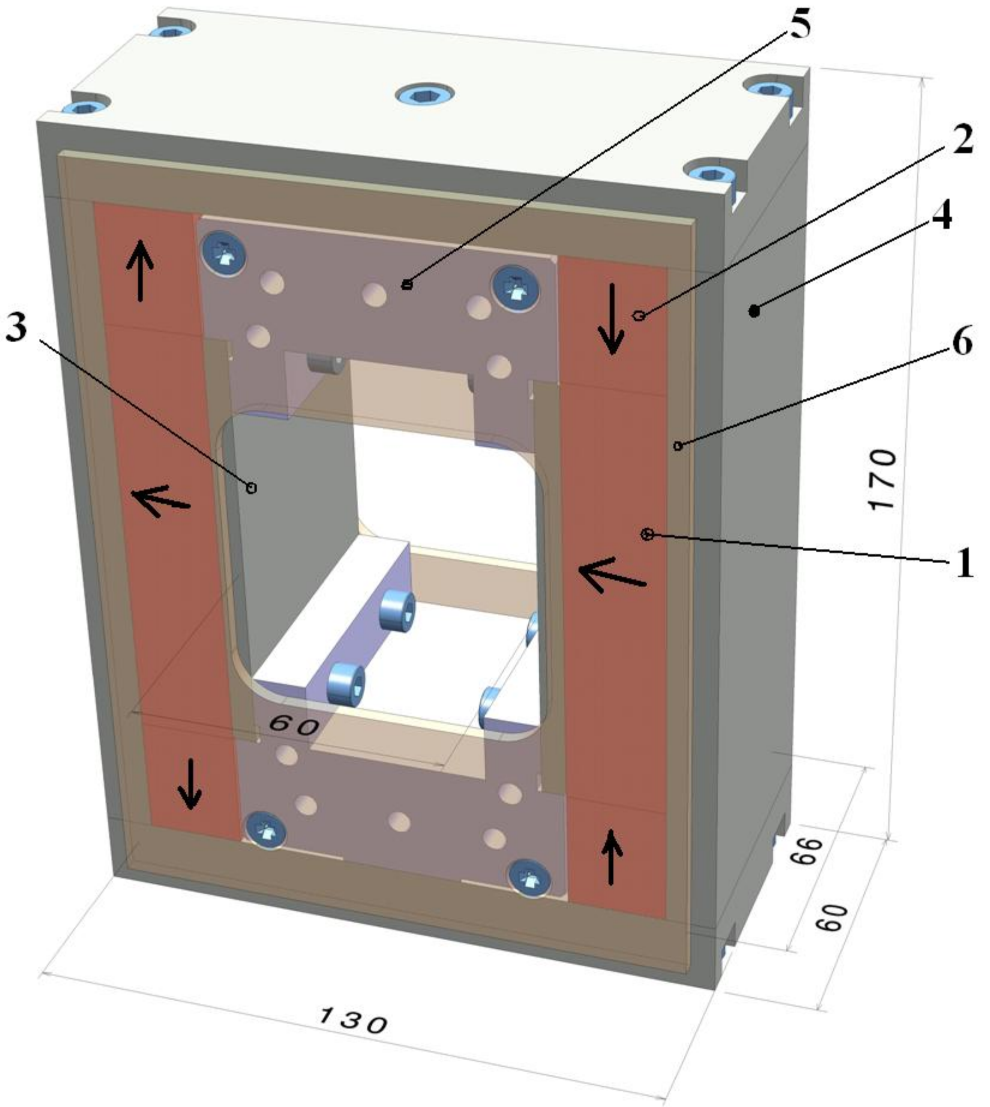
Relative populations (‰) of $A_{2\pi}$ long-lived states at the Be target exit as a function of principal quantum number n and orbital momentum l

n l	2	3	4	5	6	7	8
1	417	148	48	18	7	3	1
2	0	117	49	20	9	4	1
3	0	0	45	21	10	4	2
4	0	0	0	20	10	5	2
5	0	0	0	0	10	5	2
6	0	0	0	0	0	4	2
7	0	0	0	0	0	0	2

Breakup probability of $A_{2\pi}$ in np states for different thicknesses of Platinum foils ($A_{2\pi}$ momentum $P_{\mathrm{A}} = 4.5$ GeV/c and $A_{2\pi}$ ground-state lifetime $\tau = 3 \times 10^{-15}$ s)

Thickness (µm)	2p	3p	4p	5p	6p	7p
0.1	0.0251	0.0520	0.0858	0.1327	0.2035	0.3219
0.2	0.0559	0.1175	0.1978	0.3001	0.4185	0.5392
0.5	0.1784	0.3595	0.5537	0.7176	0.8323	0.9043
1.0	0.4147	0.6895	0.8553	0.9324	0.9667	0.9828
1.5	0.6084	0.8526	0.9446	0.9765	0.9889	0.9944
2.0	0.7422	0.9244	0.9743	0.9895	0.9951	0.9975
3.0	0.8844	0.9739	0.9918	0.9967	0.9985	0.9992

Mechanical structure



1) Permanent block Type 1 (Sm2Co17)

2) Permanent block Type 2 (Sm2Co17)

3) Pole (AISI 1010)

4) Return yoke (AISI 1010)

5) Central insert (stainless steel)

6) Cover plates (aluminum)

The lifetime of $A_{2\pi}$ in electric field

L. Nemenov, V. Ovsiannikov (P. L. 2oo1)

$$M = \frac{3F\hbar^2}{\mu_l}\delta_{m,0},$$

F – strength of electric field in $A_{2\pi}$ c.m.s.

$$F = \beta\gamma B_L,$$ B_L in lab. syst.

→ m must be 0

$$\xi = \frac{2M}{\Omega_1},$$

$$\Omega_1(n=2) = \frac{E_{2s} - E_{2p}}{\hbar}$$

$$\xi(2s-2p) = \xi_0 \gamma B_L \qquad \xi_0 \sim \frac{1}{E_{2s} - E_{2p}} \qquad \xi_n = \frac{\xi_0}{8} n^3 \gamma B_L$$

$$\tau_n^{eff} = \frac{\tau_n}{1 + 120\xi_n^2}$$

CONCLUSION: the lifetimes for long-lived states can be calculated using only one parameter → E_{2s}-E_{2p}.

The probability W(m=0) of $A_{2\pi}$ to have m=0 on $\vec{F}$ will be calculated by L. Afanasev. The preliminary value is W (m=0) ≈ 50%.

Period **L** of resonators for $\gamma=16$

Transition	2S-2P	3S-3P	4S-4P	5S-5P
L, μ	*40*	*125*	*320*	*525*