MOMENTS OF PHASE-SPACE DENSITY, ENTROPIES OF A MULTIPARTICLE SYSTEM & COINCIDENCE PROBABILITIES

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(i) MOMENTS OF PHASE-SPACE DENSITY $\langle D^{\ell} \rangle$.

(ii) COINCIDENCE PROBABILITIES $C_{\ell+1}$ & RENYI ENTROPIES $H_{\ell+1}$.

(iii) RELATION OF $C_{\ell+1}$ TO $\langle D^{\ell} \rangle$.

(iv) DISCRETIZATION & MEASUREMENT OF <u>EXPERIMENTAL</u> COINCIDENCE PROBABILITIES $C_{\ell+1}^{EXP}$.

(v) "OPTIMAL" DISCRETIZATION AND ESTIMATE OF $\langle D^{\ell} \rangle$ & $C_{\ell+1}$ FROM $\underline{C_{\ell+1}^{EXP}}$.

(vi) CONCLUSIONS.

PHASE-SPACE DISTRIBUTION OF M PARTICLES

$$W(\vec{x}_1, \ldots \vec{x}_M ; \vec{K}_1, \ldots, \vec{K}_M) \equiv W(X, K)$$

$$\int W \, dX dK = 1$$

PHASE-SPACE DENSITY:

$$D(X, K) = M \, W(X, K)$$

MOMENTS OF THE DENSITY:

$$\langle D^\ell \rangle = \int dX \, dK \left[D(X,K)\right]^\ell W(X,K) = M^\ell \int \left[W(X,K)\right]^{\ell+1} dX dK$$

PROPOSAL: TO ESTIMATE $\langle D^\ell \rangle$

FROM THE MEASURED COINCIDENCE

PROBABILITIES OF THE OBSERVED EVENTS.

RENYI ENTROPIES & COINCIDENCE PROB.

RELATION TO MOMENTS OF PHASE-SPACE DISTRIBUTION

$$\underline{H_e \equiv \frac{1}{1-e} \log C_e} \qquad \underline{H_1 = S} \geq H_2 \;!$$

COINCIDENCE PROBABILITY $\rightarrow C_e = \mathrm{Tr}\,[\rho^e]$ ρ - DENSITY MATRIX OF THE SYSTEM

$$C_e = \sum_{\text{states}} [p_i]^e = \langle p^{e-1} \rangle$$

IN DIAGONAL REPRESENTATION of ρ

[DENSITY MATRIX IN MOMENTUM REPRESENTATION IS - GENERALLY - NOT DIAGONAL]

TAKE THE PHASE-SPACE DISTRIBUTION:

$$\boxed{W(X,K) = \frac{1}{(L_x L_y L_z)^M}\, G\!\left(\frac{X-\bar{X}}{L}\right) F(K)}$$

THEN

$$\Longrightarrow C_{e+1} = (2\pi)^{3Me} \langle W^e \rangle \left[1 - \delta_e\right]^M$$

$$\delta_1 \equiv 0 \qquad \delta_e \simeq O(L^{-2}) \qquad e > 1$$

EXPERIMENTAL

COINCIDENCE PROBABILITIES C_ℓ^{exp}

C_ℓ^{exp} IS THE PROBABILITY TO FIND
- AMONG N OBSERVED EVENTS -
ℓ IDENTICAL ONES.

THIS CAN BE MEASURED BY SIMPLY COUNTING THE # OF IDENTICAL EVENTS IN THE SAMPLE

$$C_\ell^{exp} = \frac{N_\ell}{N_{tot}}$$

N_ℓ = # OF IDENTICAL ℓ-PLETS

$N_{tot} = N(N-1)\cdots(N-\ell+1)/\ell! =$
= TOTAL # OF ℓ-PLETS

TO MEASURE C_ℓ^{exp} ONE HAS TO <u>DISCRETIZE</u> THE MOMENTUM DISTRIBUTION.
THE RESULT OBVIOUSLY DEPENDS ON DISCRETIZATION.

ESTIMATE OF $\langle W^\ell \rangle$ FROM $C^{exp}_{\ell+1}$

CONSIDER THE PHASE-SPACE DISTRIBUTION OF THE GENERAL FORM

$$(*) \quad W(X,K)\,d^{3M}X\,d^{3M}K = \underbrace{\frac{d^{3M}X}{(L_x L_y L_z)^M}\, G\!\left(\frac{X-\bar{X}}{L}\right)}_{X\ \text{DISTRIBUTION}} \underbrace{F(K)\,d^{3M}K}_{\text{MOMENTUM DISTRIBUTION}}$$

$$\int G(u)\,u\,d^{3M}u = 0 \rightarrow \langle X \rangle = \bar{X}$$

$$\int G(u)\,u^2\,d^{3M}u = 1 \rightarrow \langle X^2 \rangle = L_x^2$$

$$\underline{\int G(u)\,d^{3M}u = 1}$$

USING $(*)$ <u>ONE CAN CALCULATE</u>

(i) $\underline{\langle W^\ell \rangle} = \int dX\,dK\,[W(X,K)]^{\ell+1}$

THEY OBVIOUSLY DEPEND ON <u>L's</u>, ON <u>$G(u)$</u> & ON <u>$F(K)$</u>.

(iii) EXPERIMENTAL COINCIDENCE PROBABILITIES $\underline{C^{EXP}_{\ell+1}}$

THEY DEPEND ON <u>SIZE OF THE BINS</u> & ON <u>$F(K)$</u>.

MOMENTUM DISTRIBUTION

<u>AND COMPARE (i) WITH (ii)</u>

RESULT: A (SIMPLIFIED) DO-LIST

(i) SELECT BINS IN MOMENTUM: $\Delta_x, \Delta_y, \Delta_z$

(ii) CHOOSE BIN SIZES TO SATISFY ("OPTIMAL" BINS)

$$\boxed{\omega \equiv \Delta_x \Delta_y \Delta_z = \frac{2\pi g_{\ell+1}}{L_x L_y L_z}} \qquad g_{\ell+1} = \int d^{3M}u \left[G(u)\right]^{\ell+1}$$

(iii) $$(2\pi)^{3M\ell} \langle W^{\ell} \rangle = C^{exp}_{\ell+1}(\omega)\, \underline{\underline{\Phi}}$$

$$\underline{\underline{\Phi}} = \frac{\sum_{bins} \frac{1}{\omega^M} \int_\omega d^{3M}k \left[F(k)\right]^{\ell+1}}{\sum_{bins} \left[\frac{1}{\omega^M} \int_\omega d^{3M}k\, F(k)\right]^{\ell+1}} = \frac{\sum_{bins} \left\langle \left[F(k)\right]^{\ell+1} \right\rangle_\omega}{\sum_{bins} \left[\langle F(k) \rangle_\omega\right]^{\ell+1}}$$

(i) SIZE OF "OPTIMAL" BINS DEPENDS ON L $\rightarrow$

$\rightarrow$ INFORMATION ON <u>VOLUME</u> IS NECESSARY

(ii) FOR GAUSSIANS $2\pi g_2 \simeq 1.77$; $2\pi g_3 = 2.09$; $2\pi g_4 \simeq 2.23$

$L \sim 1\,\mathrm{fm} \rightarrow \Delta \sim 400\,\mathrm{MeV}$; $L \sim 5\,\mathrm{fm} \rightarrow \Delta \sim 100\,\mathrm{MeV}$

(iii) $\Phi \sim 1$ IF BINS ARE SMALL ENOUGH

(iv) NOTE: THE RESULT DEPENDS ONLY ON THE PRODUCT <u>$\Delta_x \Delta_y \Delta_z$</u>.

CONCLUSIONS

1) EXPERIMENTALLY MEASURED COINCIDENCE PROBABILITIES OF EVENTS PRODUCED IN H.E. COLLISIONS CAN GIVE AN ESTIMATE OF MOMENTS OF PHASE-SPACE DENSITY AT FREEZE-OUT. INFORMATION ON THE VOLUME OF THE SYSTEM IS NECESSARY, HOWEVER.

2) MOMENTS OF PHASE-SPACE DENSITY AT FREEZE-OUT ARE CLOSELY RELATED TO RENYI ENTROPIES AND THUS GIVE DIRECT INFORMATION ON THE ENTROPY OF THE SYSTEM. THE RELATION IS PARTICUCARLY SIMPLE IN THE LIMIT OF INFINITE VOLUME.

3) $$H_l^{EXP} \leq H_l \leq S \quad !$$

STRICT LOWER LIMIT ON ENTROPY CAN BE OBTAINED!