$k_\perp$ factorization and quark production from the CGC

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Outline

- Introduction
- Quark production in pA collisions in the CGC framework
- Numerical estimate on violation of $k_\perp$ factorization
  - Single Quark production
  - Quark Pair production
- Conlusion

- HF, Gelis, Venugopalan, hep-ph/0504047 and in progress

Introduction

- Collinear factorization : one large scale $\sqrt{s} \sim \boldsymbol{p}_\perp \gg \Lambda_{\rm QCD}$

$$d\sigma = \int \hat{\sigma} \;\; x_1 G(x_1) \;\; x_2 G(x_2)$$

- $\boldsymbol{k}_\perp$ factorization : two large scales $\sqrt{s} \gg \boldsymbol{p}_\perp \gg \Lambda_{\rm QCD}$

Collins Ellis 1991, Catani Ciafaloni Hautmann 1991

Levin Ryskin Shabelsky Shuvaev 1991

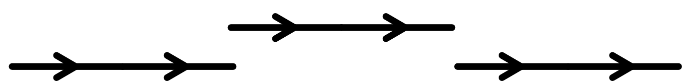
$$d\sigma = \int \frac{|\mathcal{M}|^2}{\boldsymbol{k}_{1\perp}^2 \boldsymbol{k}_{2\perp}^2} \;\; \phi_1(x_1, \boldsymbol{k}_{1\perp}) \;\; \phi_2(x_2, \boldsymbol{k}_{2\perp}) \;\; \delta(\boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp} - \boldsymbol{q}_\perp)$$

- Useful devise to include the intrinsic $\boldsymbol{k}_\perp$
- Extension to include rescattering corrections in pA ?

$\Rightarrow$ CGC provides a natural framework

Introduction

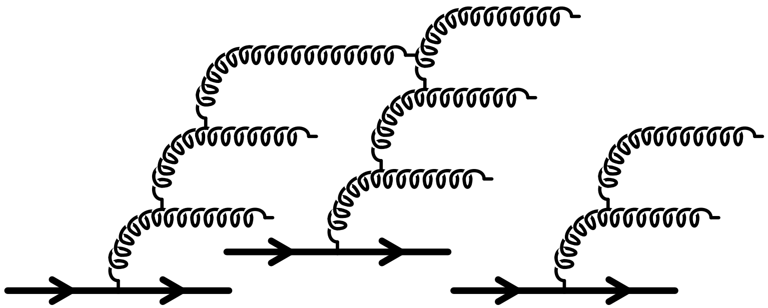
Particle procution from the CGC in pA collisions



- Nucleus at high energy

Introduction

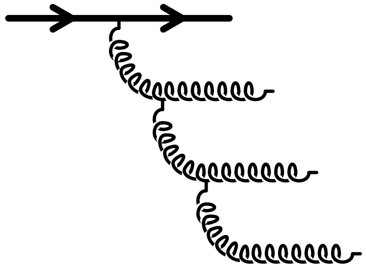
Particle procution from the CGC in pA collisions

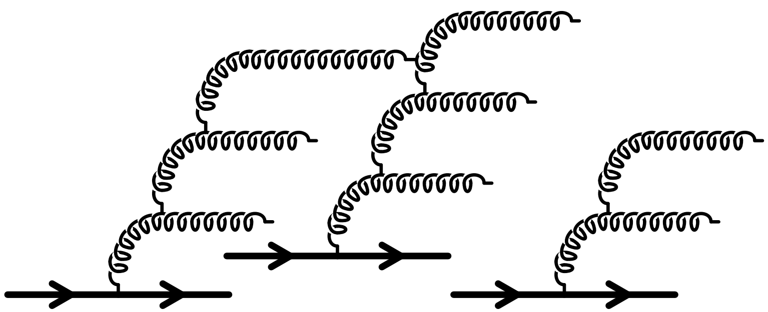


- Nucleus at high energy = dense system of small-x gluons

Introduction

Particle procution from the CGC in pA collisions

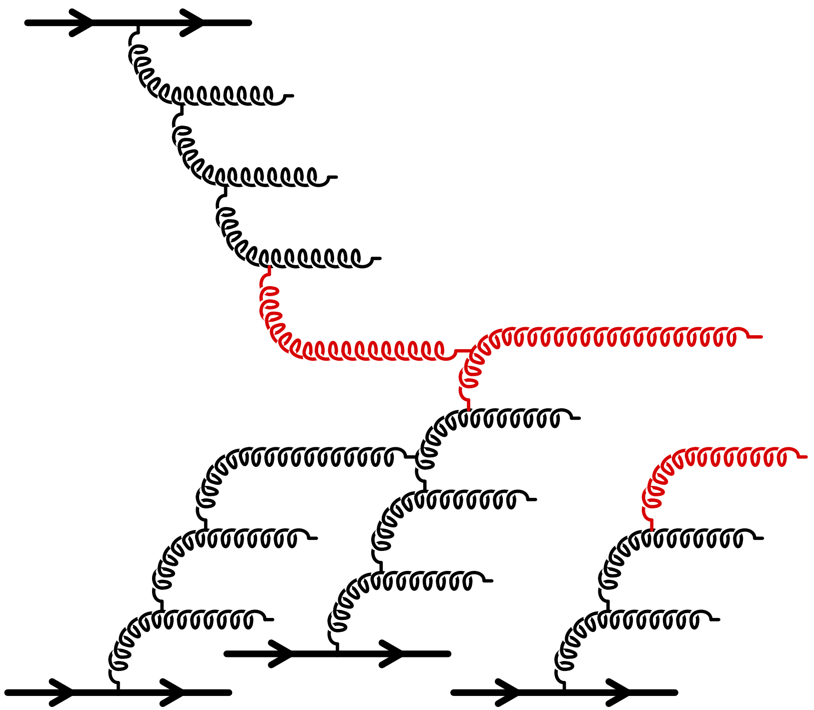




- Nucleus at high energy = dense system of small-x gluons
- Proton probes Nucleus

Introduction

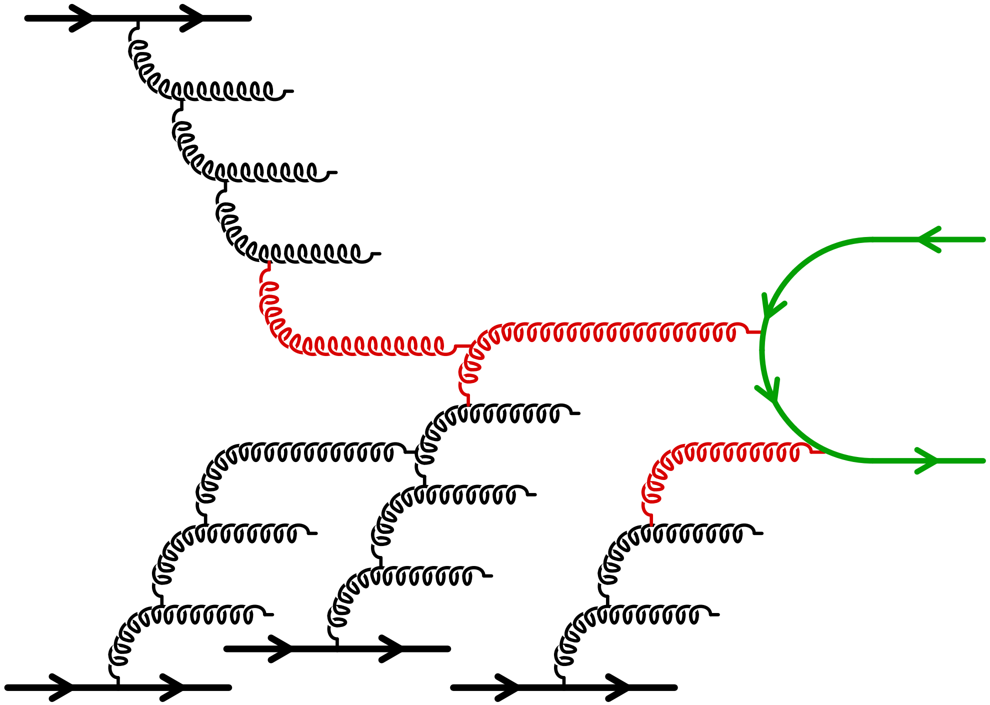
Particle procution from the CGC in pA collisions



- Nucleus at high energy = dense system of small-x gluons
- Proton probes Nucleus
  - gluon production

Introduction

Particle procution from the CGC in pA collisions



- Nucleus at high energy = dense system of small-x gluons
- Proton probes Nucleus
  - gluon production
  - quark production

Introduction

- Aim of this talk :
  - to provide a quantitative estimate for violation of $\boldsymbol{k}_{\perp}$ factoriztion in quark production in pA
- Framework :
  - McLerran-Venugopalan (Gaussian) model
  - Without x-evolution

Classical Color Field

- Small-x gluons are described as classical field generated by charge source $\rho_{p,A}$ on the light-front (McLerran Venugopalan 1994)

- Yang-Mills equations:

$$[D_\mu, F^{\mu\nu}] = J^\nu \quad , \qquad [D_\nu, J^\nu] = 0 \quad , \qquad \partial_\mu A^\mu = 0$$

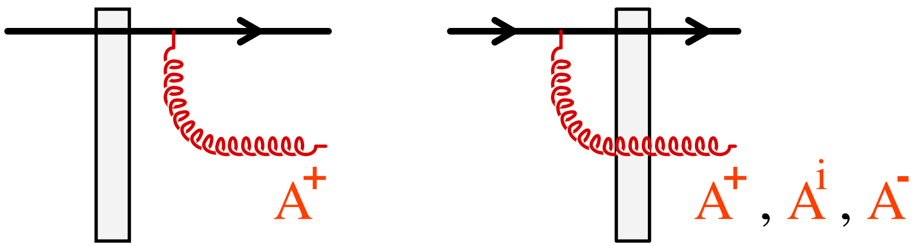
$$J^\nu\big|_{\mathrm{LO}} = \delta^{\nu+}\delta(x^-)\rho_p(\boldsymbol{x}_\perp) + \delta^{\nu-}\delta(x^+)\rho_A(\boldsymbol{x}_\perp)$$

- Gluon and Quark production amplitudes are known to first order in ρ_p and to all orders in ρ_A Kovchegov Muller 1998

  Blaizot Gelis Venugopalan 2004

Gluon production

- (Skechy) interpretation of solution:



- $\boldsymbol{k}_\perp$-factorization is valid for gluon production

$$\frac{d\sigma}{d^2\boldsymbol{q}dy_q} \sim \frac{\alpha_s N}{\pi^4 d_A \boldsymbol{q}_\perp^2} \int_{\boldsymbol{k}_\perp} \varphi_p^{g,g}(\boldsymbol{k}_\perp)\phi_A^{g,g}(\boldsymbol{q}_\perp - \boldsymbol{k}_\perp)$$

- $\phi_A^{g,g} = FT\langle U(\boldsymbol{x}_\perp)U^\dagger(0)\rangle$
  - obtained analytically in MV model
  - characterized by charge density $\mu_A^2 \sim Q_s^2$ upto log

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Quark cross section

Blaizot Gelis Venugopalan 2004

- Total amplitude (after some cancellation):

$$\mathcal{M}_F = g^2 \int_{\vec{k}_{1\perp},\vec{k}_\perp} \frac{\rho_{p,a}(\vec{k}_{1\perp})}{k_{1\perp}^2} \int_{\vec{x}_\perp,\vec{y}_\perp} e^{i\vec{k}_\perp\cdot\vec{x}_\perp} e^{i(\vec{p}_\perp+\vec{q}_\perp-\vec{k}_\perp-\vec{k}_{1\perp})\cdot\vec{y}_\perp}$$

$$\times \overline{u}(\vec{q}) \left\{ [\widetilde{U}(\vec{x}_\perp) t^a \widetilde{U}^\dagger(\vec{y}_\perp)] T_{q\bar{q}}(\vec{k}_\perp) + [t^b U_{ba}(\vec{x}_\perp)] \not{L} \right\} v(\vec{p})$$

with

$$T_{q\bar{q}}(\vec{k}_\perp) \equiv \frac{\gamma^+(\not{q}-\not{k}+m)\gamma^-(\not{q}-\not{k}-\not{k}_1+m)\gamma^+}{2p^+[(\vec{q}_\perp-\vec{k}_\perp)^2+m^2] + 2q^+[(\vec{q}_\perp-\vec{k}_\perp-\vec{k}_{1\perp})^2+m^2]}$$

and $\not{L}$ Lipatov's vertex

- (Skechy) interpretation of solution:

Quark cross section

- Pair production cross section:

$$\frac{d\sigma_{q\bar{q}}}{d^2\vec{\boldsymbol{p}}_\perp d^2\vec{\boldsymbol{q}}_\perp dy_p dy_q} = \frac{\alpha_s^2 N}{8\pi^4 d_A} \int_{\vec{\boldsymbol{k}}_{1\perp},\vec{\boldsymbol{k}}_{2\perp}} \frac{\delta(\vec{\boldsymbol{p}}_\perp + \vec{\boldsymbol{q}}_\perp - \vec{\boldsymbol{k}}_{1\perp} - \vec{\boldsymbol{k}}_{2\perp})}{\boldsymbol{k}_{1\perp}^2 \boldsymbol{k}_{2\perp}^2}$$

$$\times \Big\{ \int_{\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp} \mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)(\not{p}-m)T^*_{q\bar{q}}(\vec{\boldsymbol{k}}'_\perp)\Big]\phi_A^{q\bar{q},q\bar{q}}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp)$$

$$+ \int_{\vec{\boldsymbol{k}}_\perp} \mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)(\not{p}-m)\not{L}^* + \mathrm{h.c.}\Big]\phi_A^{q\bar{q},g}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp)$$

$$+\mathrm{tr}\Big[(\not{q}+m)\not{L}(\not{p}-m)\not{L}^*\Big]\phi_A^{g,g}(\vec{\boldsymbol{k}}_{2\perp})\Big\}\varphi_p(\vec{\boldsymbol{k}}_{1\perp})$$

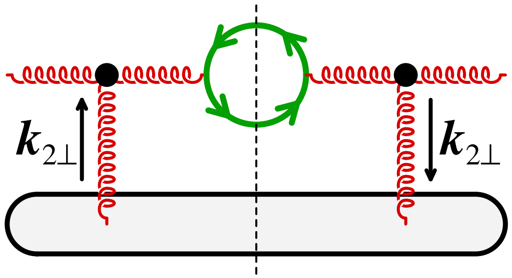
▷ $k_\perp$-factorization is violated for the nucleus:
one needs three different “distributions” to describe the nucleus

▷ $k_\perp$ factorization is recovered in the leading twist approximation
Gelis Venugopalan 2004

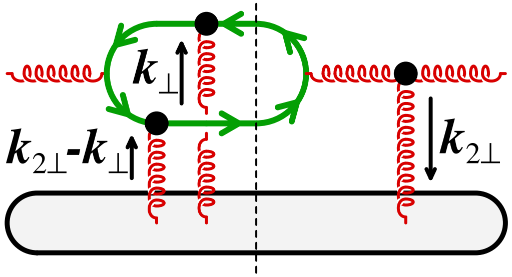
Quark cross section

- Interpretation:

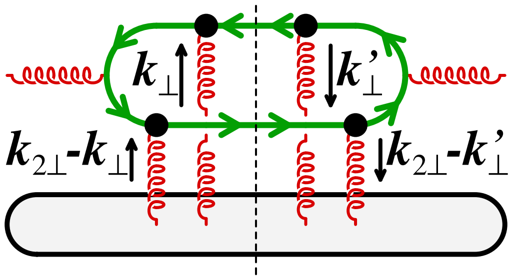
$$\phi_A^{g,g} \sim \mathrm{FT}\ \mathrm{tr}\langle U\ U^\dagger \rangle \quad \propto$$



$$\phi_A^{q\bar{q},g} \sim \mathrm{FT}\ \mathrm{tr}\langle \widetilde{U}\ t^a\ \widetilde{U}^\dagger\ t^b\ U_{ba} \rangle \quad \propto$$



$$\phi_A^{q\bar{q},q\bar{q}} \sim \mathrm{FT}\ \mathrm{tr}\langle \widetilde{U}\ t^a\ \widetilde{U}^\dagger \widetilde{U}\ t^a \widetilde{U} \rangle \quad \propto$$



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Quark cross section

- Properties of ϕ_A's:
  - Sum rule

$$\int_{\vec{k}_\perp, \vec{k}'_\perp} \phi_A^{q\bar{q},q\bar{q}}(\vec{k}_{2\perp}|\vec{k}_\perp, \vec{k}'_\perp) = \int_{\vec{k}_\perp} \phi_A^{q\bar{q},g}(\vec{k}_{2\perp}|\vec{k}_\perp) = \phi_A^{g,g}(\vec{k}_{2\perp})$$

$\triangleright$ $k_\perp$-factorization is recovered if one can neglect $\boldsymbol{k}_\perp$ in $T_{q\bar{q}}(\boldsymbol{k}_\perp)$

  - In large N limit, 4-, 3-pt fns become a product of 2-pt fns

$$\phi_A^{q\bar{q},q\bar{q}}(\vec{k}_{2\perp}|\vec{k}_\perp, \vec{k}'_\perp) \xrightarrow{\text{large } N} \# C(\boldsymbol{k}_\perp) C(\boldsymbol{k}_{2\perp} - \boldsymbol{k}_\perp)(2\pi)^2 \delta(\vec{k}_\perp - \vec{k}'_\perp)$$

where $C(\boldsymbol{k}_\perp) = FT \langle \tilde{U}(\boldsymbol{x}_\perp) \tilde{U}^\dagger(0) \rangle$ expresses multiple scattering of a quark in the nucleus

- Large N approximation is rather good:
  we present the large N results (mainly)

Quark cross section

- Single quark cross section:

$$\frac{d\sigma_q}{d^2\vec{\boldsymbol{q}}_\perp dy_q} = \frac{\alpha_s^2 N}{8\pi^4 d_A} \int dy_p \int_{\vec{\boldsymbol{k}}_{1\perp},\vec{\boldsymbol{k}}_{2\perp}} \frac{1}{\boldsymbol{k}_{1\perp}^2 \boldsymbol{k}_{2\perp}^2}$$

$$\times\Big\{\mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)(\not{p}-m)T^*_{q\bar{q}}(\vec{\boldsymbol{k}}'_\perp)\Big]\frac{C_F}{N}\phi_A^{q,q}(\vec{\boldsymbol{k}}_{2\perp})$$

$$+\int_{\vec{\boldsymbol{k}}_\perp}\mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)(\not{p}-m)\not{L}^* + \mathrm{h.c.}\Big]\phi_A^{q\bar{q},g}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp)$$

$$+\mathrm{tr}\Big[(\not{q}+m)\not{L}(\not{p}-m)\not{L}^*\Big]\phi_A^{g,g}(\vec{\boldsymbol{k}}_{2\perp})\Big\}\varphi_p(\vec{\boldsymbol{k}}_{1\perp})$$

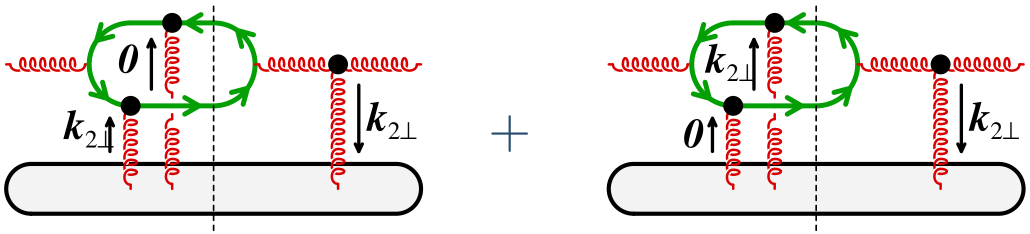
- A simplification : 4-point fn does not appear
- $k_\perp$-factorization is still violated for the nucleus

Numerical evaluation of violation

HF, Gelis, Venugopalan 2005

- $\boldsymbol{k}_\perp$-factorization is recovered if

$$\phi_A^{q\bar{q},g}(\boldsymbol{k}_{2\perp}|\boldsymbol{k}) = \frac{1}{2}(2\pi)^2 \left[\delta(\boldsymbol{k}_\perp) + \delta(\boldsymbol{k}_\perp - \boldsymbol{k}_{2\perp})\right] \phi_A^{g,g}(\boldsymbol{k}_{2\perp})$$

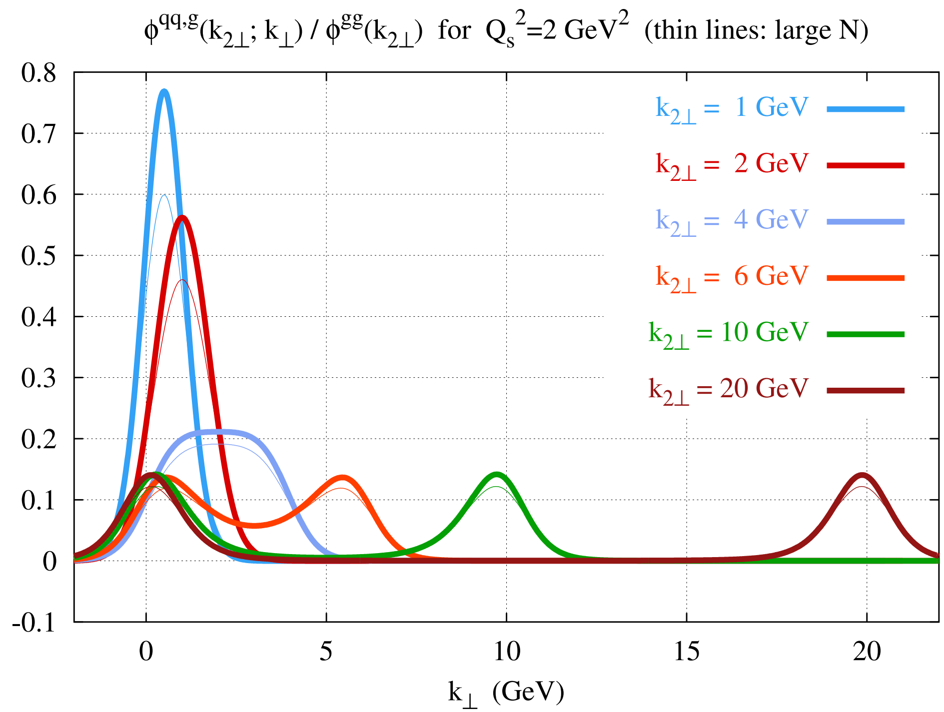
$=$ 

- This means either of Q or $\bar{Q}$ exchanges all the momentum

- Is the ratio $\phi_A^{q\bar{q},g}/\phi_A^{g,g}$ close to two δ-fns?

3-point function

- Exact 3-point function in the MV model:

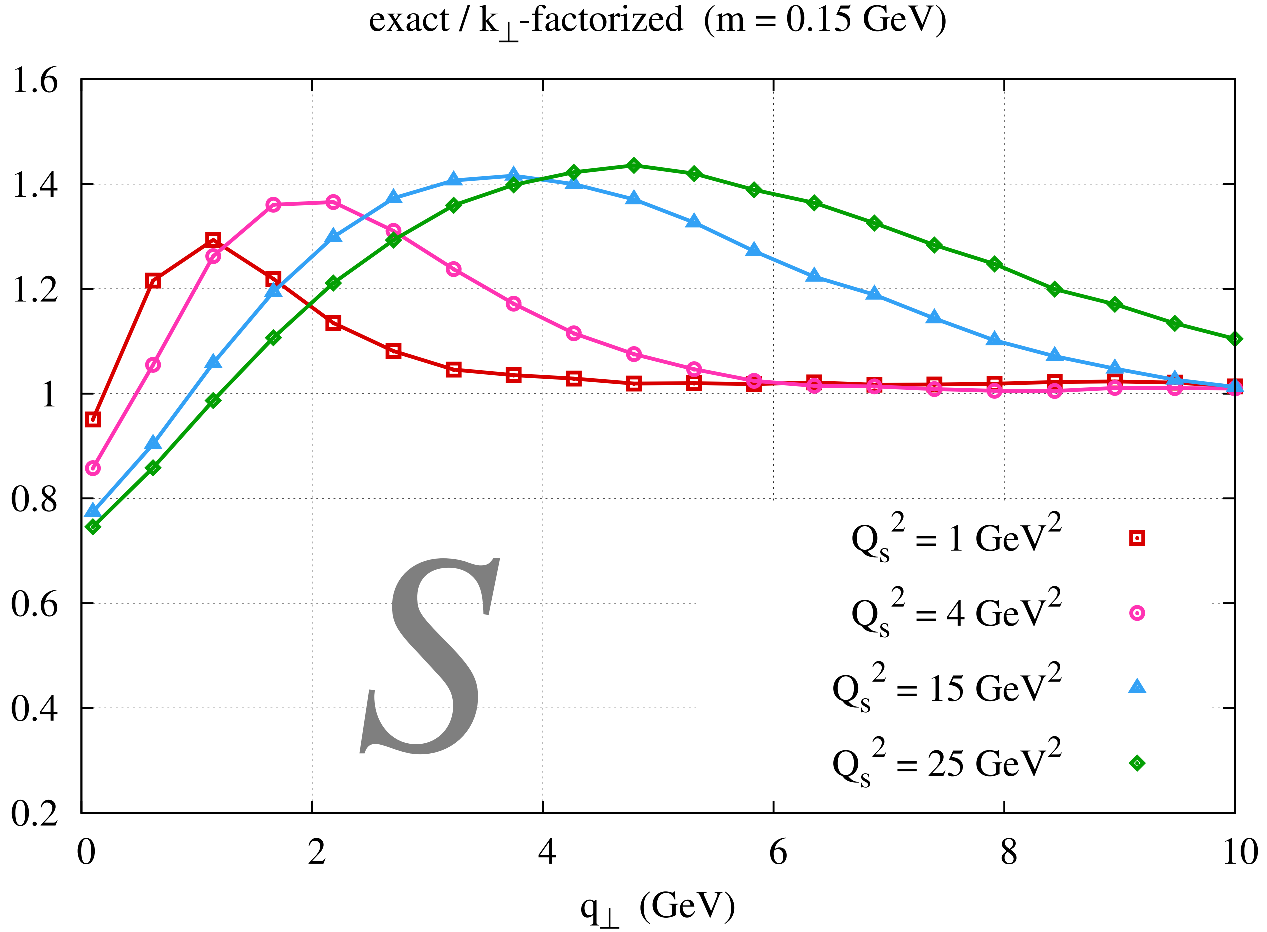


- For $\boldsymbol{k}_{2\perp} \gg Q_s$ two peaks with width $\sim Q_s$ separated by $\boldsymbol{k}_{2\perp}$
- For $\boldsymbol{k}_{2\perp} \lesssim Q_s$ two peaks merge into one

- $\boldsymbol{k}_\perp$-factorization holds if $\sqrt{s} \gg m_{p_\perp} \gg Q_s$
  - typical $\boldsymbol{k}_{2\perp}$ is large since $\boldsymbol{p}_\perp + \boldsymbol{q}_\perp = \boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}$
  - multiple scatteing $\sim Q_s$ cannot see pair structure $\sim m_{p_\perp}$

Single quark production

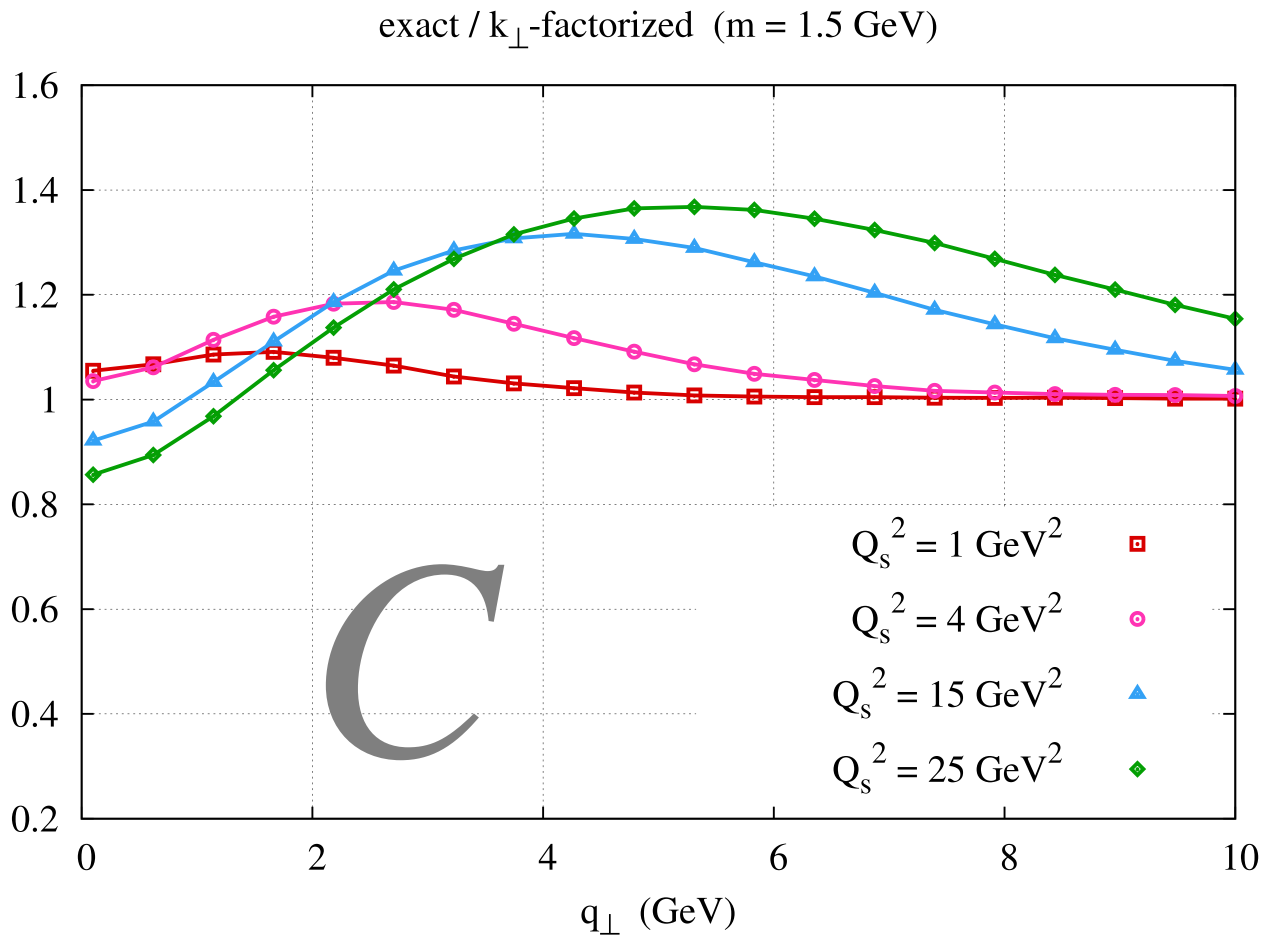
- Numerical estimate of 'exact / $k_\perp$-fact' for a light quark with several values of Q_s in the MV model



- Please don't confuse this with the Cronin peak

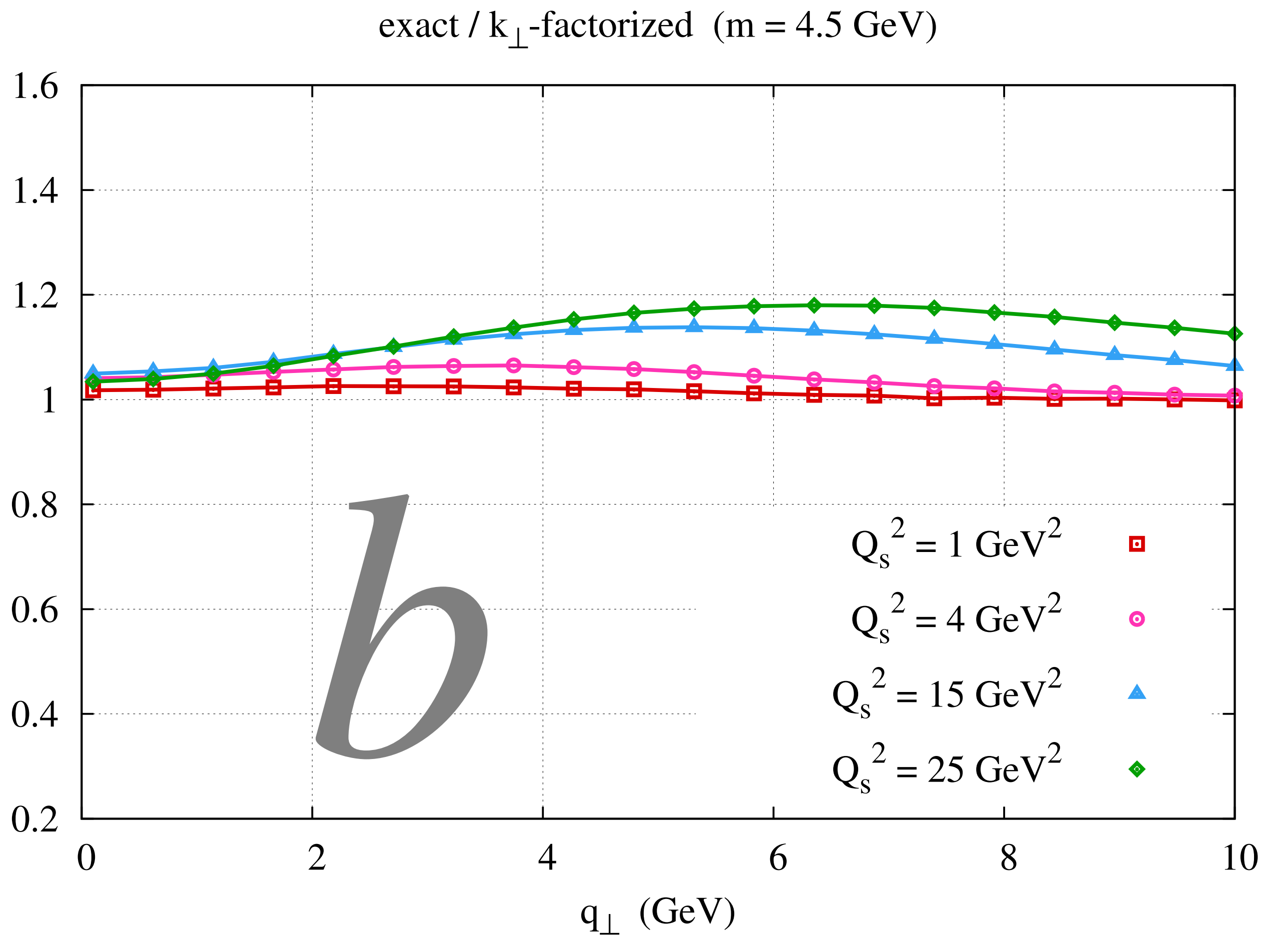
Single quark production

- charm quark



Single quark production

- bottom quark



Single quark production

- Asymptoticaly small-x regime — another extreme:

Charge distribution becomes a non-local gaussian

Iancu Itakura McLerran 2003

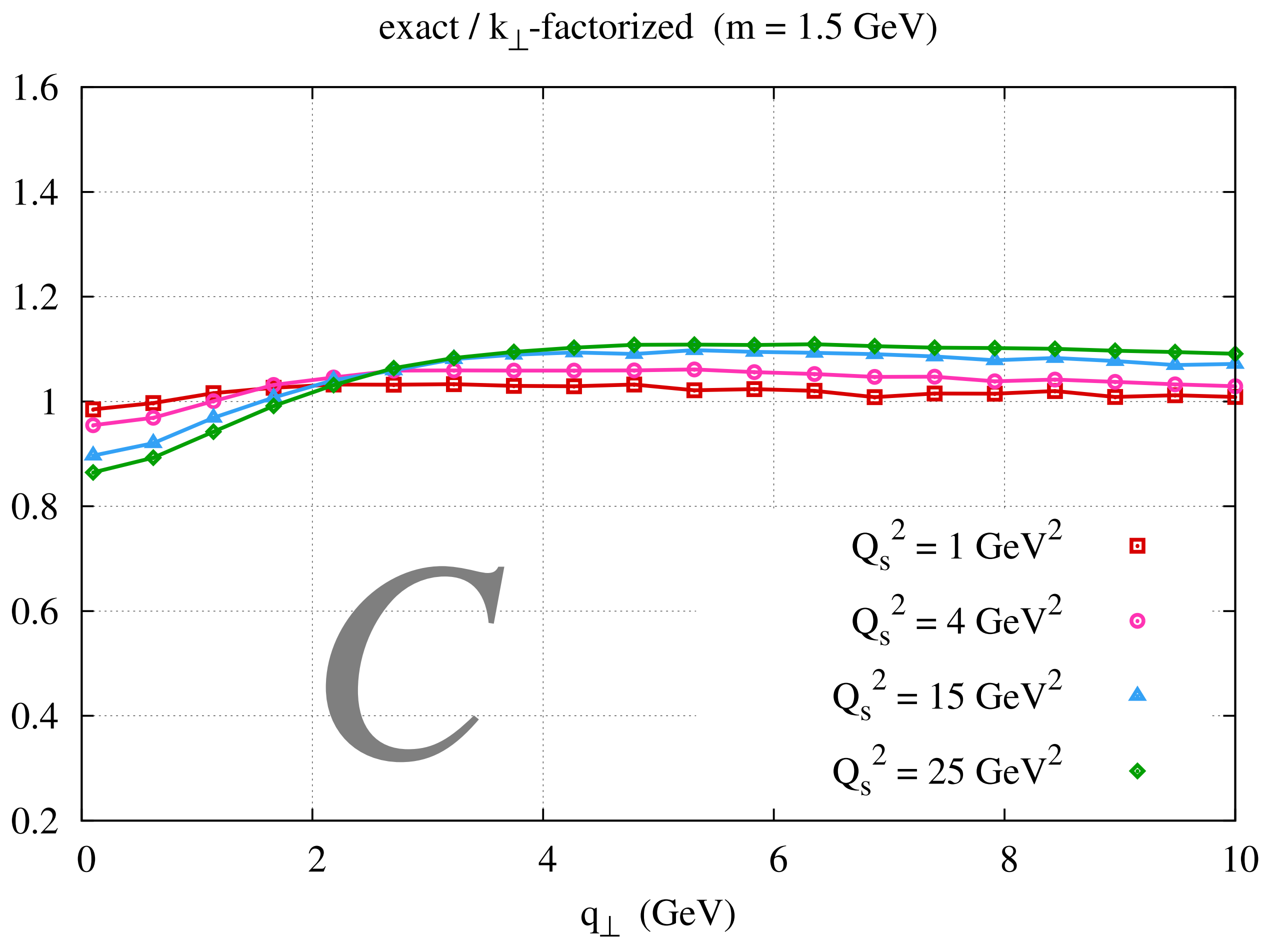
$$W_x[\rho_A] = \exp\left[-\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} \frac{\rho_{A,a}(\boldsymbol{x}_\perp)\rho_{A,a}(\boldsymbol{y}_\perp)}{2\mu_A^2(x,\boldsymbol{x}_\perp-\boldsymbol{y}_\perp)}\right]$$

with

$$\mu_A^2(x,\boldsymbol{k}_\perp) = \frac{2}{\gamma c}\boldsymbol{k}_\perp^2 \ln\left(1+\left(\frac{Q_s^2(x)}{k_\perp^2}\right)\right)$$

Single quark production

- charm quark with ϕ_A's of non-local gaussian



Single quark production

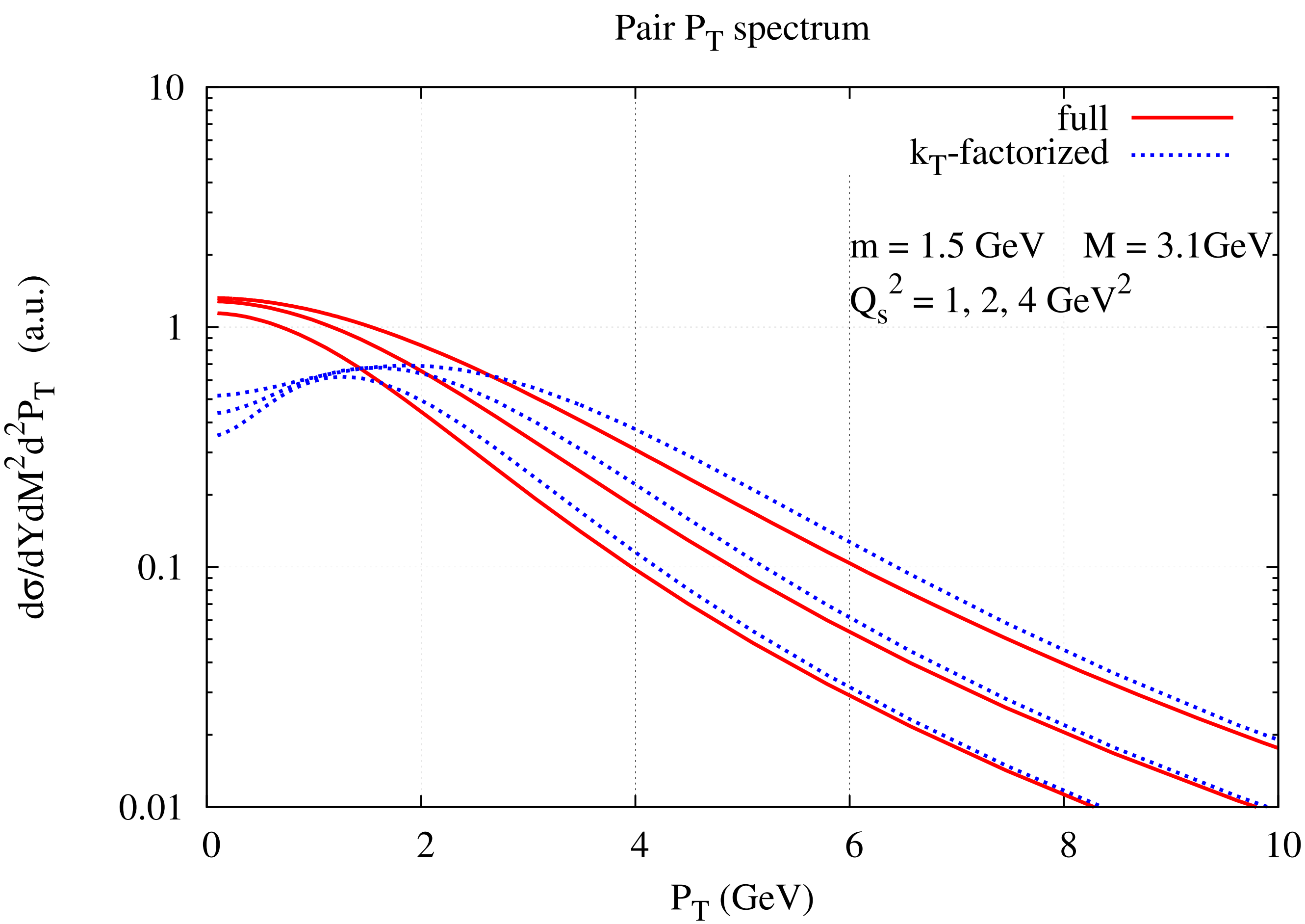
General trends :

- Violation is more significant for larger Q_s and smaller m
- Violation has a maximum around $q_\perp \sim Q_s$ and is recovered as $q_\perp \to \infty$
- If $Q_s \lesssim m, q_\perp$, the violation enhances the cross section
- If $m, q_\perp \ll Q_s$, the violation reduces the cross section : quark is likely to be produced with a transverse mass $\gtrsim Q_s$

Pair production

HF Gelis Venugopalan, in progress

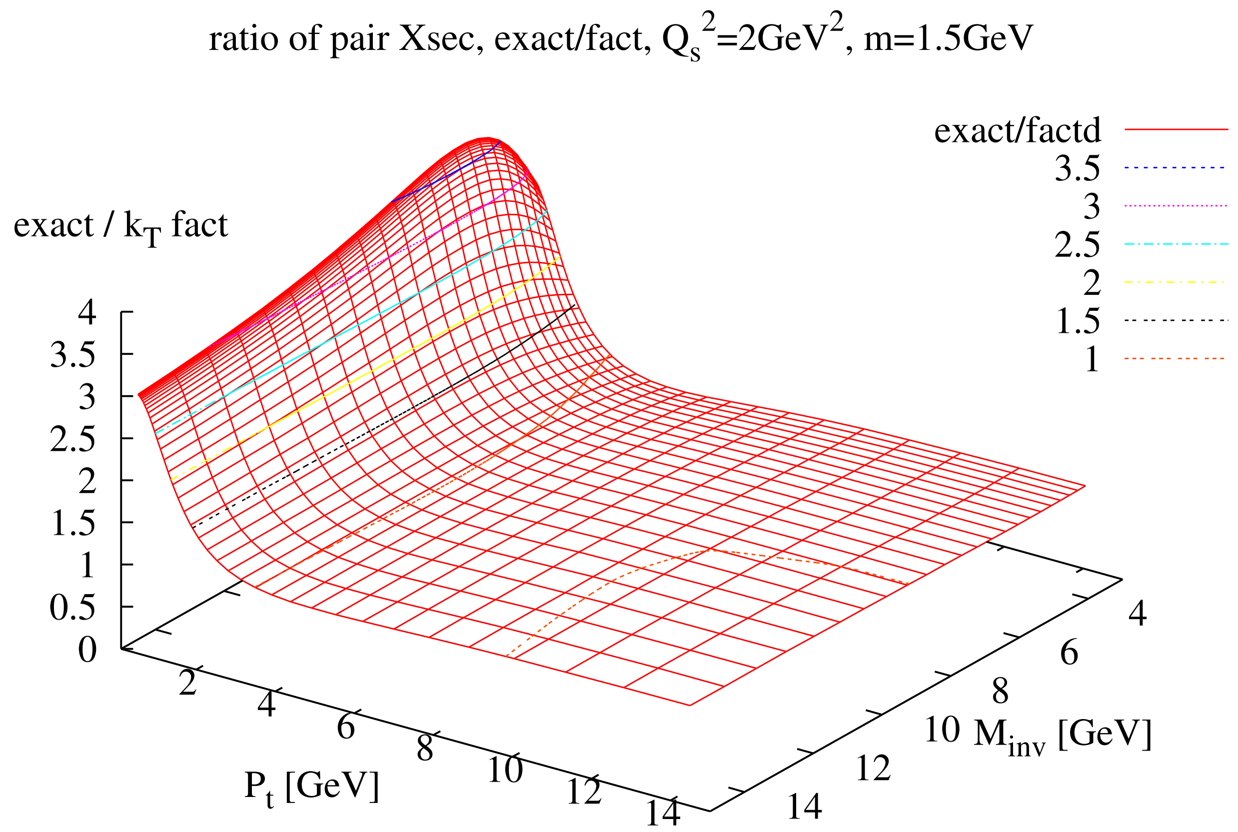
- Pair spectrum for total mom $P_\perp$ with a fixed invariant mass M



- Bump around Q_s in $k_\perp$-factorization approach $\Leftarrow \phi_A^{g,g}(\boldsymbol{k}_{2\perp})$
- Full calculation smears out the bump

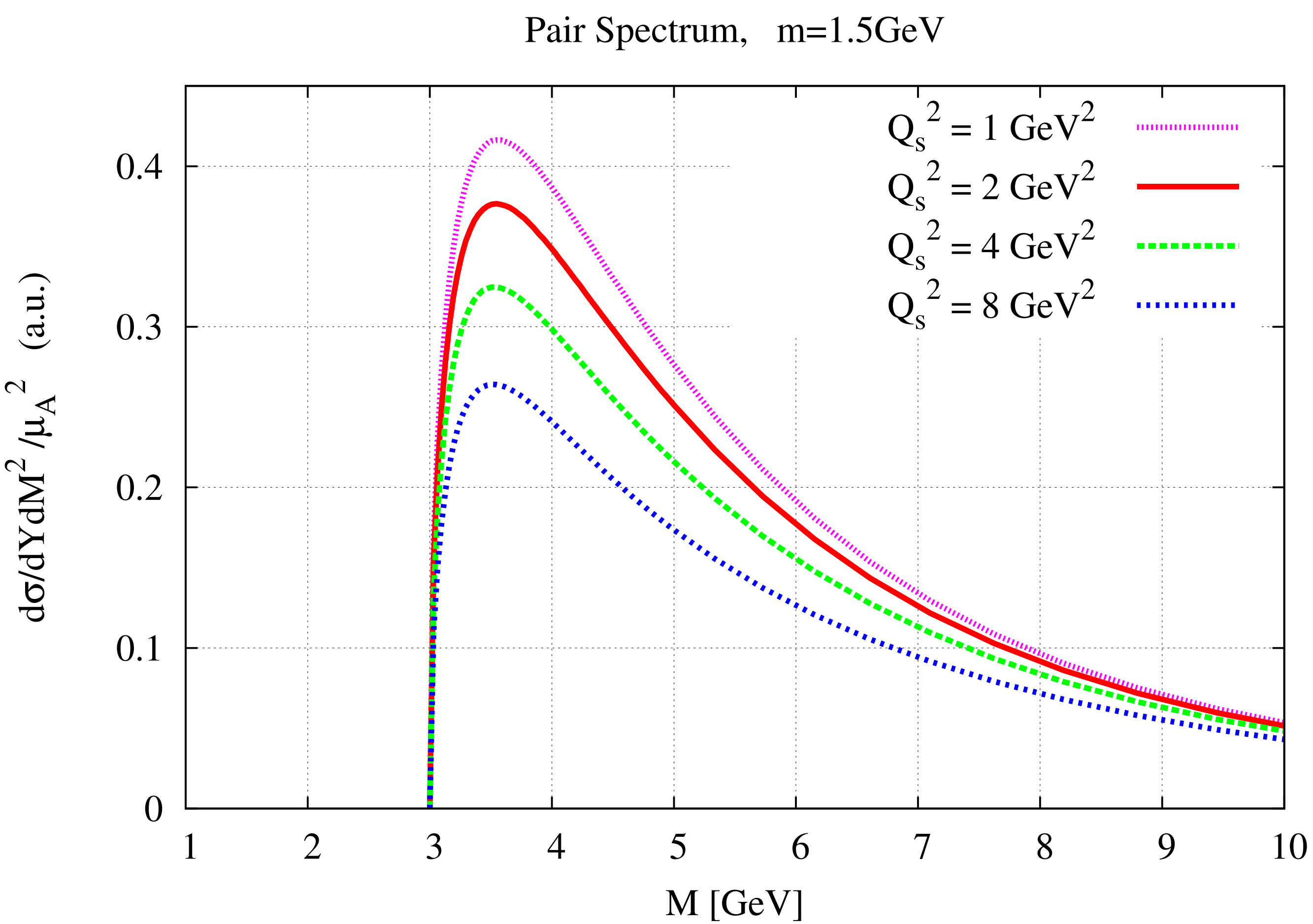
Pair production

- Ratio of 'exact / $k_\perp$-fact' for pair production



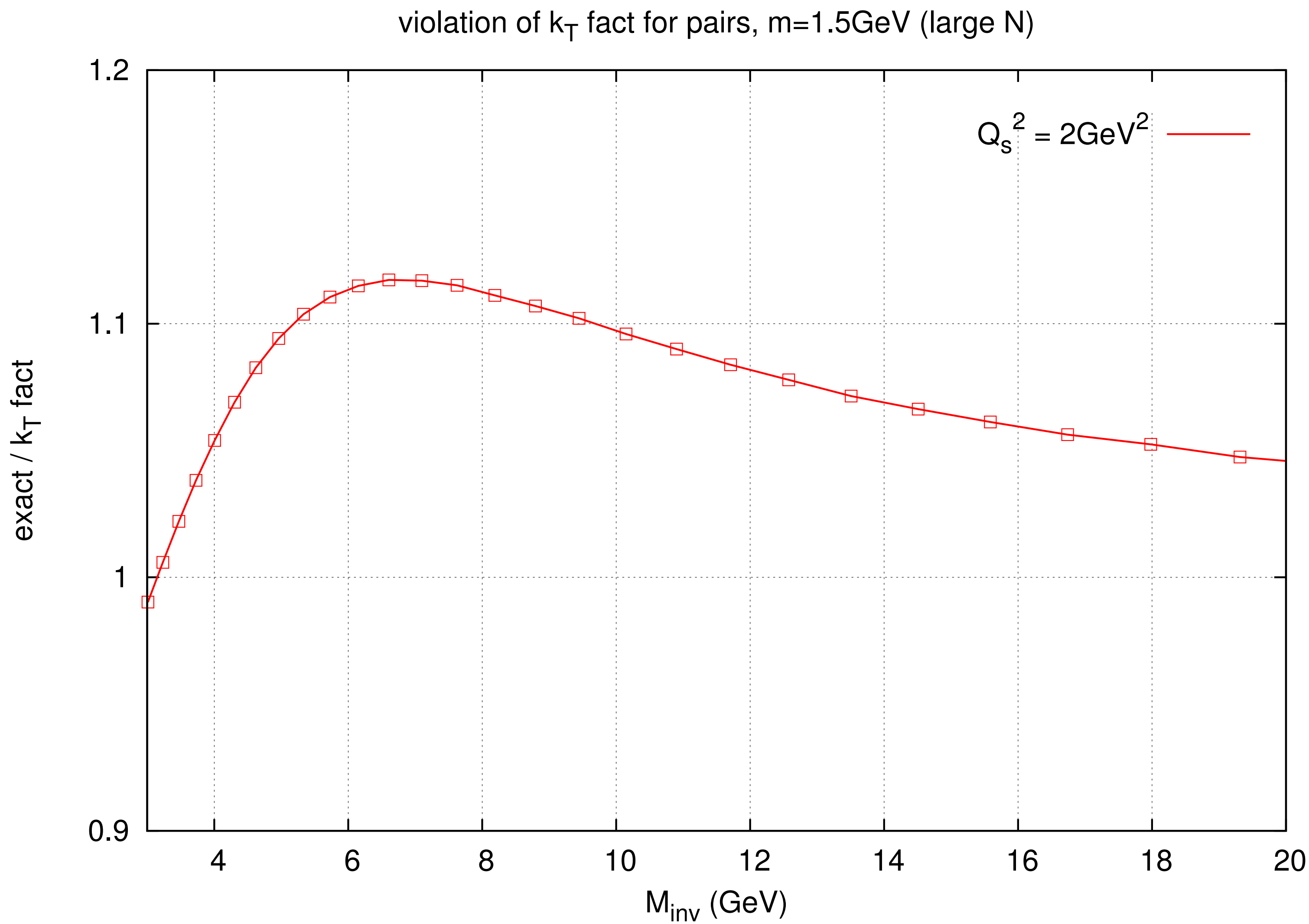
- $k_\perp$ factorization is recovered at large $P_\perp$
- Violation of $k_\perp$ enhances small-$P_\perp$ cross section :
  $\Rightarrow$ Opposite to the single quark case

Pair production



- invariant mass spectrum of the pair
- binary scaling holds at large M
- suppression at smaller M

Pair production



- Ratio of the invariant mass spectrum of full & $k_\perp$-fact results
- Size and trend are similar to single quark case

Conclusions

- Violation of $k_\perp$-factorization in pA collisions :
  - one needs 2-, 3-, 4-pnt fns to describe the nuclues side
  - quantitative estimate is shown in the MV model and in the saturated limit
- Single quark production :
  - violation can be significant when $Q_s \gtrsim m_{p\perp}$
    - small effect for charm quark with realistic Q_s at RHIC
    - significant with large Q_s at LHC and at very forward rapidities
- Pair production :
  - violation of $k_\perp$ factoriztion significantly depends on the pair momentum $P_\perp$
  - Bump in $P_\perp$ is seen in $k_\perp$ factorization approximation, but smeared out in full result due to multiple scattering
- Outlook :
  - more realistic $\phi_{p,A}$'s for phenomenology
  - x-evolution and rapidity dependence