# Nonlinear $k_\perp$-factorization: a new paradigm for hard processes in a nuclear environment

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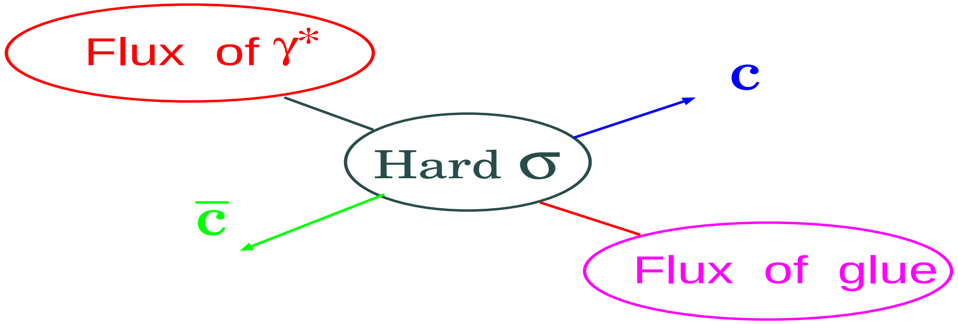
# pQCD factorization theorems

Example: open charm in ep --> c $\bar{c}$ X




⋆ Forward dijets: $x_\gamma = z_+ + z_- \approx 1$, jet-jet decorrelation momentum $\mathbf{\Delta} = \mathbf{p}_+ + \mathbf{p}_-$

$$\frac{d\sigma_N(\gamma^* \to c\bar{c})}{dz d^2\mathbf{p}_+ d^2\mathbf{\Delta}} = \frac{\alpha_S(\mathbf{p}^2)}{2(2\pi)^2} f(\mathbf{\Delta}) \left|\Psi(z,\mathbf{p}_+) - \Psi(z,\mathbf{p}_+ - \mathbf{\Delta})\right|^2 .$$

$$f(\boldsymbol{\kappa}) = \frac{4\pi}{N_c} \cdot \frac{1}{\kappa^4} \cdot \frac{\partial G_N(x,\boldsymbol{\kappa})}{\partial \log \kappa^2}$$

$$\sigma_0(x) = \int d^2\boldsymbol{\kappa} f(\boldsymbol{\kappa}) = \sigma(x,\mathbf{r})|_{r\to\infty}$$

⋆ A linear functional of the unintegrated glue.

⋆ Back to 1973-74: DIS at $x \ll 1$ in the Breit frame

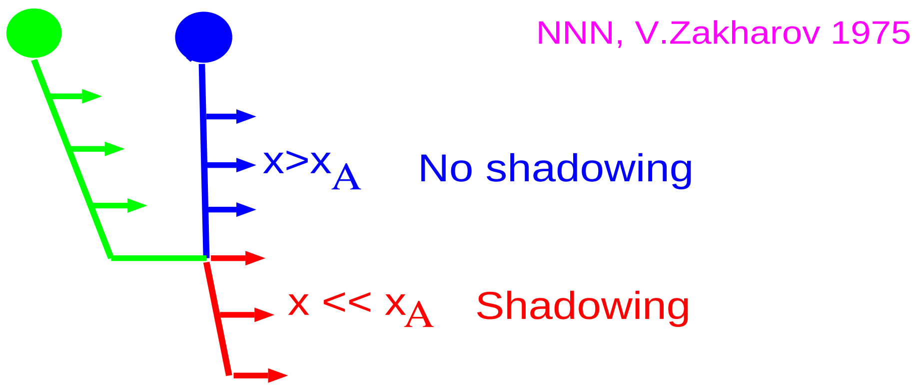
⋆ Lorentz-contracted ultrarelativistic nucleus:

$$R_A \to R_A \frac{m_N}{p_N} < \lambda = \frac{1}{k_z} = \frac{1}{x p_N}\,.$$

⋆ Spatial overlap of partons from many nucleons if

$$x \lesssim x_A = 1/R_A m_N$$

$\Longrightarrow$ FUSION & NUCLEAR SHADOWING.




⋆ Nuclear parton density (if it can be meaningfully defined!) is a nonlinear functional of the free nucleon parton density: the same sea is shared by many nucleons.

⋆ Major strategy of this talk: shadowing from unitarity for dipole amplitudes.

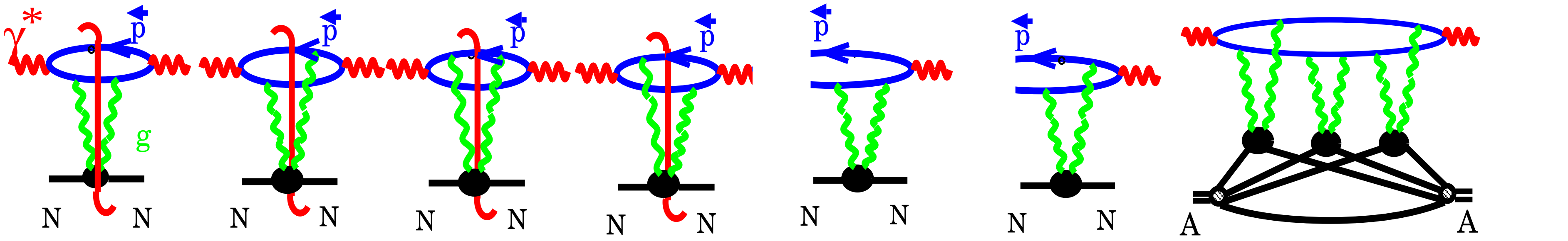
# Coherent diffractive and truly inelastic DIS

⋆ Color dipole is coherent over whole nucleus for $x \lesssim x_A$:

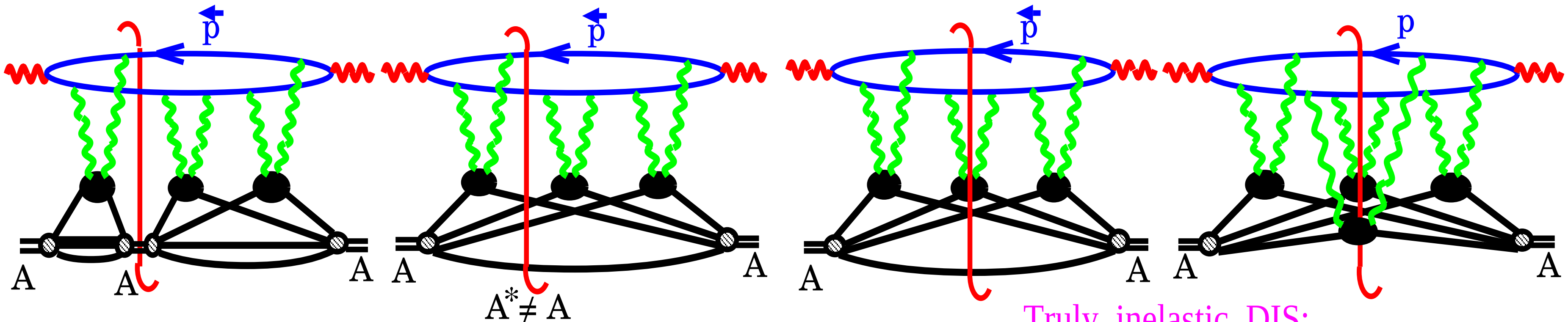
$\Longrightarrow$ Glauber–Gribov formalism ( NNN, Zakharov (91)):

$$\sigma_A(\mathbf{r}) = 2\int d^2\mathbf{b}\Gamma_A(\mathbf{b},\mathbf{r}) = 2\int d^2\mathbf{b}[1-\exp(-\frac{1}{2}\sigma(\mathbf{r})T(\mathbf{b}))]$$

⋆ The unitarity content of DIS



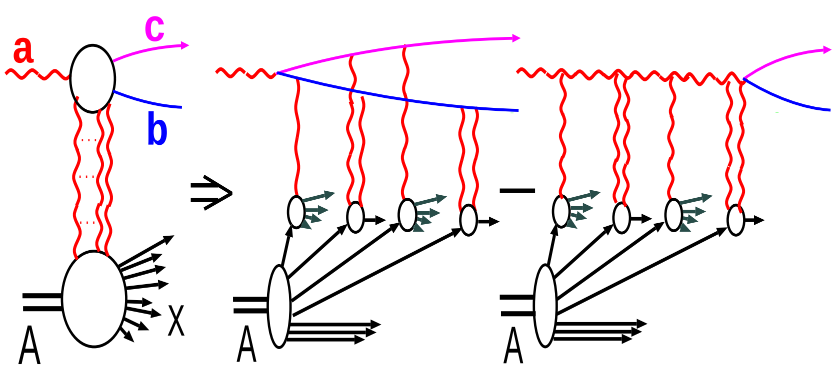

Unitarity cuts for DIS off free nucleons Amplitude of diffractive DIS γ*A Compton amplitude

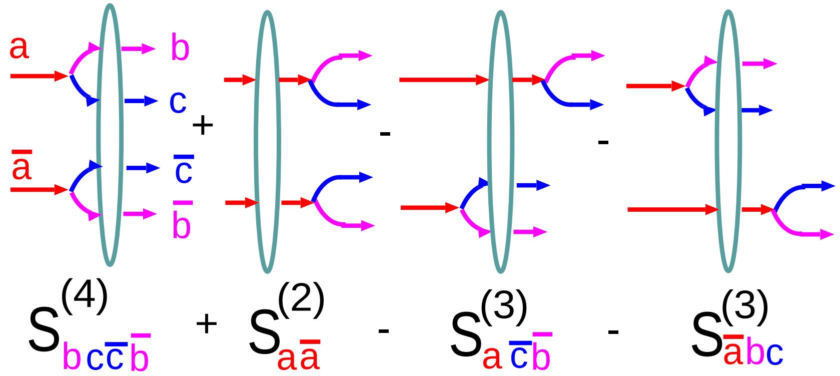



Coherent diffractive cut Incoherent diffractive cut Truly inelastic DIS: Single color excitation Multiple color excitation

# Production processes as excitation of beam Fock states

Zakharov (87), NNN, Zakharov, Piller (95), NNN, Schäfer, Zakharov, Zoller (03)




⋆ Interactions with the nucleus after and before the virtual decay interfere destructively.

⋆ Apply closure over the nucleon & nucleus excitations

⋆ Partial cross sections with color-excitation of $\nu$ nucleons ($\nu$ cut pomerons in the Abramovsky-Gribov-Kancheli language)

⋆ Requires evaluation of specific intermediate states in $S^{(n)}$:
well developed technique is available ( NNN, Schafer, Zakharov (05))

# Non-Abelian coupled-channel evolution and master formula for dijets

NNN, Zakharov, Piller (95), NNN, Schäfer, Zakharov, Zoller (03)

$$\frac{d\sigma(a^* \to bc)}{dz_b d^2\mathbf{p}_b d^2\mathbf{p}_c} = \frac{1}{(2\pi)^4} \int d^2\mathbf{b}_b d^2\mathbf{b}_c d^2\mathbf{b}'_b d^2\mathbf{b}'_c \exp[-i\mathbf{p}_b(\mathbf{b}_b - \mathbf{b}'_b) - i\mathbf{p}_c(\mathbf{b}_c - \mathbf{b}'_c)]$$
$$\times \Psi(z_b, \mathbf{b}_b - \mathbf{b}_c)\Psi^*(z_b, \mathbf{b}'_b - \mathbf{b}'_c)$$
$$\times \{S^{(4)}_{\bar{b}\bar{c}cb}(\mathbf{b}'_b, \mathbf{b}'_c, \mathbf{b}_b, \mathbf{b}_c) + S^{(2)}_{\bar{a}a}(\mathbf{b}', \mathbf{b}) - S^{(3)}_{\bar{b}\bar{c}a}(\mathbf{b}, \mathbf{b}'_b, \mathbf{b}'_c) - S^{(3)}_{\bar{a}bc}(\mathbf{b}', \mathbf{b}_b, \mathbf{b}_c)\} \, .$$

⋆ The dilute-gas nucleus: $S_i(\mathbf{b}_c{}', \mathbf{b}_b{}', \mathbf{b}_c, \mathbf{b}_b) = \exp\{-\frac{1}{2}\Sigma_i(\mathbf{b}_c{}', \mathbf{b}_b{}', \mathbf{b}_c, \mathbf{b}_b)T(\mathbf{b})\}$

- DIS: $\gamma^* \to q\bar{q}$ : $\Longrightarrow \underbrace{1}_{1} + \underbrace{8}_{N_c^2}$
- Open charm: $g \to c\bar{c}$ : $\Longrightarrow \underbrace{1}_{1\ (N_c\ suppressed)} + \underbrace{8}_{N_c^2}$
- Forward dijets: $q \to qg$ : $\Longrightarrow \underbrace{3}_{N_c} + \underbrace{6 + 15}_{N_c \times N_c^2}$
- Central dijets: $g \to gg$ : $\Longrightarrow \underbrace{1}_{1\ (N_c\ suppressed)} + \underbrace{8_A + 8_S}_{N_c^2} + \underbrace{10 + \overline{10} + 27 + R_7}_{N_c^2 \times N_c^2}$

⋆ Universality classes depending on color excitation

- $T(\mathbf{b}) = \int dz n_A(\mathbf{b}, z)$: new large dimensional scale.

$$\Gamma_A(\mathbf{b}, \mathbf{r}) = 1 - \exp[-\frac{1}{2}\sigma(\mathbf{r})T(\mathbf{b})] = \int d^2\boldsymbol{\kappa}\phi(\mathbf{b}, \boldsymbol{\kappa})\{1 - \exp[i\boldsymbol{\kappa}\mathbf{r}]\}$$

- Nuclear glue per unit area in the impact parameter space

$$\phi(\mathbf{b}, \boldsymbol{\kappa}) = \sum_{j=1}^{\infty} w_j(\mathbf{b}) \frac{f^{(j)}(\boldsymbol{\kappa})}{\sigma_0^j}$$

$$\Phi(\mathbf{b}, \boldsymbol{\kappa}) = \phi(\mathbf{b}, \boldsymbol{\kappa}) + w_0(\mathbf{b})\delta(\boldsymbol{\kappa})$$

- Probability to find $j$ overlapping nucleons

$$w_j(\mathbf{b}) = \frac{\nu_A^j(\mathbf{b})}{j!} \exp\left[-\nu_A(\mathbf{b})\right], \quad \nu_A(\mathbf{b}) = \frac{1}{2}\sigma_0 T(\mathbf{b})$$

- Collective glue of $j$ overlapping nucleons:

$$f^{(j)}(\boldsymbol{\kappa}) = \int \prod_i^j d^2\boldsymbol{\kappa}_i f(\boldsymbol{\kappa}_i)\delta(\boldsymbol{\kappa} - \sum_i^j \boldsymbol{\kappa}_i), \quad f^{(0)}(\boldsymbol{\kappa}) \equiv \delta(\boldsymbol{\kappa})$$

- Antishadowing of hard, $\boldsymbol{\kappa}^2 \gtrsim Q_A^2$, glue per bound nucleon ( NNN,Schäfer, Schwiete '00):

$$
\begin{aligned}
f_A(\mathbf{b}, x, \boldsymbol{\kappa}) &= \frac{\phi(\mathbf{b}, \boldsymbol{\kappa})}{\nu_A(\mathbf{b})} \\
&= f(x, \boldsymbol{\kappa}) \left[ 1 + \frac{\gamma^2}{2} \cdot \frac{\alpha_S(\boldsymbol{\kappa}^2) G(x, \boldsymbol{\kappa}^2)}{\alpha_S(Q_A^2) G(x, Q_A^2)} \cdot \frac{Q_A^2(\mathbf{b})}{\boldsymbol{\kappa}^2} \right] .
\end{aligned}
$$

- $\gamma =$ exponent of the large–$\boldsymbol{\kappa}^2$ tail $f(\boldsymbol{\kappa}) \sim \alpha_S(\boldsymbol{\kappa}^2)/(\boldsymbol{\kappa}^2)^\gamma$ .
- Antishadowing $\Longrightarrow$ the Cronin effect.
- Plateau for softer collective glue

$$
\phi(\mathbf{b}, \boldsymbol{\kappa}) \approx \frac{1}{\pi} \frac{Q_A^2(\mathbf{b})}{(\boldsymbol{\kappa}^2 + Q_A^2(\mathbf{b}))^2} ,
$$

- Width of the plateau ( saturation & higher twist scale, independent of auxiliary soft $\sigma_0(x)$)

$$
Q_A^2(\mathbf{b}, x) \approx \frac{4\pi^2}{N_c} \alpha_S(Q_A^2) G(x, Q_A^2) T(\mathbf{b}) .
$$

# Exceptional case of single-quark spectrum in DIS:

## Abelianization of evolution

$$\frac{d\sigma_{in}}{d^2\mathbf{b}d^2\mathbf{p}_+dz} = \frac{1}{(2\pi)^2} \times \Big\{\int d^2\boldsymbol{\kappa}\phi(\boldsymbol{\kappa})\,|\langle\gamma^*|z,\mathbf{p}\rangle - \langle\gamma^*|z,\mathbf{p}-\boldsymbol{\kappa}\rangle|^2 - \underbrace{\Big|\int d^2\boldsymbol{\kappa}\phi(\boldsymbol{\kappa})(\langle\gamma^*|z,\mathbf{p}\rangle - \langle\gamma^*|z,\mathbf{p}-\boldsymbol{\kappa}\rangle)\Big|^2}_{Nonlinear}\Big\}$$

Coherent diffraction = 50 per cent of total DIS for heavy nucleus (NNN, Zakharov, Zoller '94).

$$\frac{d\sigma_D}{d^2\mathbf{b}d^2\mathbf{p}dz} = \frac{1}{(2\pi)^2} \times \underbrace{\Big|\int d^2\boldsymbol{\kappa}\phi(\boldsymbol{\kappa})(\langle\gamma^*|z,\mathbf{p}\rangle - \langle\gamma^*|z,\mathbf{p}-\boldsymbol{\kappa}\rangle)\Big|^2}_{Nonlinear}.$$

$$d[\sigma_D + \sigma_{in}]/d^2\mathbf{b}d^2\mathbf{p}dz = d\sigma_A/d^2\mathbf{b}d^2\mathbf{p}dz$$
$$= \frac{1}{(2\pi)^2}\int d^2\boldsymbol{\kappa}\phi(\boldsymbol{\kappa})\,|\langle\gamma^*|z,\mathbf{p}\rangle - \langle\gamma^*|z,\mathbf{p}-\boldsymbol{\kappa}\rangle|^2$$

⋆ Exceptional case of linear $k_\perp$-factorization: FSI and ISI are fully reabsorbed into collective nuclear glue!

⋆ Doesn't hold for the two-particle and all other single-particle spectra

# Dijets: Universality class of coherent diffraction

⋆ Coherent distortion of dipole WF in slice $[0, \beta]$ of the nucleus:

$$\Psi(\beta; z, \mathbf{p}) = \int d^2\boldsymbol{\kappa}\Phi(\beta; \mathbf{b}, x, \boldsymbol{\kappa})\Psi(z, \mathbf{p} + \boldsymbol{\kappa})$$

$$\exp\left[-\frac{1}{2}\beta\sigma(x, \mathbf{r})T(\mathbf{b})\right] = \int d^2\boldsymbol{\kappa}\Phi(\beta; \mathbf{b}, x, \boldsymbol{\kappa})\exp(i\boldsymbol{\kappa}\mathbf{r})$$

⋆ Diffractive DIS:

$$\frac{(2\pi)^2 d\sigma_A(\gamma^* \to Q\overline{Q})}{d^2\mathbf{b}dzd^2\mathbf{p}d^2\boldsymbol{\Delta}} = \delta^{(2)}(\boldsymbol{\Delta})|\Psi(1; z_g, \mathbf{p}) - \Psi(z_g, \mathbf{p})|^2\,,$$

⋆ Exactly back-to-back dijets

⋆ $q \to qg$: net color charge of the incident parton

$$\frac{(2\pi)^2 d\sigma_A(q^* \to qg)}{d^2\mathbf{b}dzd^2\mathbf{p}_g d^2\boldsymbol{\Delta}} = \delta^{(2)}(\boldsymbol{\Delta})S_{abs}(2\nu_A(\mathbf{b}))|\Psi(1; z_g, \mathbf{p}_g) - \Psi(z_g, \mathbf{p}_g)|^2\,.$$

⋆ Intranuclear attenuation of the incident quark wave:

$$S_{abs}(2\nu_A(\mathbf{b})) = \exp[-2\nu_A(\mathbf{b})]$$

## Dijets: Universality class of dijet in higher color representation from partons in lower representation: $q \to qg|_{6+15}$

$$
\begin{aligned}
\left.\frac{d\sigma(q^* \to qg)}{d^2\mathbf{b}dzd^2\mathbf{\Delta}d^2\mathbf{p}}\right|_{6+15} &= \frac{1}{(2\pi)^2}T(\mathbf{b})\int_0^1 d\beta \\
&\times \int d^2\boldsymbol{\kappa}d^2\boldsymbol{\kappa}_1d^2\boldsymbol{\kappa}_2d^2\boldsymbol{\kappa}_3\delta(\boldsymbol{\kappa}+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2+\boldsymbol{\kappa}_3-\mathbf{\Delta}) \\
&\times \underbrace{\Phi(\beta;\mathbf{b},\boldsymbol{\kappa}_3)}_{Quark\quad ISI}\underbrace{f(\boldsymbol{\kappa})\left|\Psi(\beta;z,\mathbf{p}-\boldsymbol{\kappa}_1)-\Psi(\beta;z,\mathbf{p}-\boldsymbol{\kappa}_1-\boldsymbol{\kappa})\right|^2}_{Hard\quad Excitation} \\
&\times \underbrace{\Phi(1-\beta;\mathbf{b},\boldsymbol{\kappa}_1)}_{Quark\quad FSI}\underbrace{\Phi(\frac{C_A}{C_F}(1-\beta);\mathbf{b},\boldsymbol{\kappa}_2)}_{Gluon\quad FSI}
\end{aligned}
$$

⋆ $\gamma^* \to q\bar{q}|_8$: the same as $q \to qg|_{6+15}$ modified for vanishing ISI

⋆ $g \to gg|_{10+\overline{10}+27+R_7}$: the same as $q \to qg$ subject to two modifications:

(i) Quark FSI/ISI $\Longrightarrow$ Gluon FSI/ISI

(ii) $C_A/C_F$: collective glue is different, defined for octet-octet dipoless!

## Dijets: Universality class of dijets in the same lower color representation as the beam parton: $q \longrightarrow qg|_3$

$$\left.\frac{d\sigma(q^*A \to qg)}{d^2\mathbf{b}dzd^2\mathbf{\Delta}d^2\mathbf{p}}\right|_3 = \frac{1}{(2\pi)^2}\phi(\mathbf{b},\mathbf{\Delta})\left|\Psi(1;z,\mathbf{p}-\mathbf{\Delta}) - \Psi(z,\mathbf{p}-z\mathbf{\Delta})\right|^2$$

$\star$ $\Psi(z,\mathbf{p}-z\mathbf{\Delta})$ = probability amplitude for the $qg$ state in physical quark - driving term of quark jet fragmentation

$\star$ Color triplet dijets: fragments of the multiply-scattered quark

$\star$ Coherent nuclear-distorted $\Psi(1;z,\mathbf{p}-\mathbf{\Delta})$:

$$|\underbrace{\Psi(z,\mathbf{p}-\mathbf{\Delta})}_{in-vacuum} - \Psi(z,\mathbf{p}-z\mathbf{\Delta})|^2 \Longrightarrow |\underbrace{\Psi(1;z,\mathbf{p}-\mathbf{\Delta})}_{in-nucleus\quad distorted} - \Psi(z,\mathbf{p}-z\mathbf{\Delta})|^2$$

Interpretation: nuclear modification of the fragmentation function

$\star$ More universality classes: $g \longrightarrow q\bar{q}|_8$, $g \longrightarrow gg|_{8_A+8_S}$, $g \longrightarrow gg|_{8_S}$

$\star$ Different collective nuclear glue - density matrix in the space of color representations, not a single function.

## Fixed multiplicity of color-excited nucleons: Unitarity cuts and AGK rules

⋆ Multiple-scattering theory for final states with $j$ color-excited nucleons $= j$ cut pomerons in the AGK language

⋆ Hadronic activity in the nucleus hemisphere

$$\eta_{Lab} \lesssim \log(R_A m_\perp)$$

⋆ Manifest unitarity at the level of fully differential cross sections:

$$\sum_j \sigma_j = \sigma_{inclusive}$$

⋆ Unitarity rules depend on the universality class

⋆ Example of AGK rule: coherent diffractive mechanism:

$$d\sigma_j = \delta_{j0} d\sigma_D$$

⋆ Coherent diffraction $=$ 50% of total DIS (NNN, Zakharov, Zoller (95))

⋆ Persists in all processes, albeit suppressed by nuclear attenuation

# Example: AGK rules for $q \to qg|_3$ (j cut pomerons)

$$\left.\frac{d\sigma_j(q^*A \to qg)}{d^2\mathbf{b}dzd^2\boldsymbol{\Delta}d^2\mathbf{p}}\right|_3 = \frac{1}{(2\pi)^2} w_j(\nu_A(\mathbf{b})) \frac{f^{(j)}(\boldsymbol{\Delta})}{\sigma_0^j} \left|\Psi(1; z, \mathbf{p} - \boldsymbol{\Delta}) - \Psi(z, \mathbf{p} - z\boldsymbol{\Delta})\right|^2$$

⋆ Quark-nucleon quasielastic scattering $qN \to q'N^*$ ($N^*$ in the color-excited state) :

$$\frac{d\sigma_{qN}}{d^2\boldsymbol{\kappa}} = \frac{1}{2} f(\boldsymbol{\kappa})$$

⋆ $j$-fold quasielastic scattering $\Longrightarrow$ $j$ cut pomerons :

$$\frac{d\sigma^{(j)}}{d^2\boldsymbol{\kappa}} = \frac{1}{2} \frac{f^{(j)}(\boldsymbol{\kappa})}{\sigma_0^{j-1}}$$

⋆ Multiple uncut (elastic) pomeron exchanges: the unitarity sum rule

$$\sum_{j=0} w_j(\nu_A(\mathbf{b})) = 1$$

⋆ Multiple uncut pomeron exchanges in $\Psi(1; z, \mathbf{p} - \boldsymbol{\Delta})$

⋆ Hard fragmentation of scattered quark independent of $j$

# Example: AGK rules for $q \to qg|_{6+15}$ (j=m+1+k + n cut pomerons)

$$\left.\frac{d\sigma_j(q^* \to qg)}{d^2\mathbf{b}dzd^2\boldsymbol{\Delta}d^2\mathbf{p}}\right|_{6+15} = \frac{1}{(2\pi)^2}T(\mathbf{b})\int_0^1 d\beta$$

$$\times \int d^2\boldsymbol{\kappa}d^2\boldsymbol{\kappa}_1d^2\boldsymbol{\kappa}_2d^2\boldsymbol{\kappa}_3\delta(\boldsymbol{\kappa}+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2+\boldsymbol{\kappa}_3-\boldsymbol{\Delta})\sum_{n,k,m}\delta(j-m-1-k-n)$$

$$\times \underbrace{w_m(\beta\nu_A(\mathbf{b}))\frac{f^{(m)}(\boldsymbol{\kappa}_3)}{\sigma_0^m}}_{Quark\quad ISI}\underbrace{f(\boldsymbol{\kappa})\left|\Psi(\beta;z,\mathbf{p}-\boldsymbol{\kappa}_1)-\Psi(\beta;z,\mathbf{p}-\boldsymbol{\kappa}_1-\boldsymbol{\kappa})\right|^2}_{Hard\quad excitation}$$

$$\times \underbrace{w_k(\frac{C_A}{C_F}(1-\beta)\nu_A(\mathbf{b}))\frac{f^{(k)}(\boldsymbol{\kappa}_1)}{\sigma_0^k}}_{Gluon\quad FSI}\times\underbrace{w_n((1-\beta)\nu_A(\mathbf{b}))\frac{f^{(n)}(\boldsymbol{\kappa}_1)}{\sigma_0^n}}_{Quark\quad FSI}$$

- ⋆ $1$ cut pomeron for hard excitation at the depth $\beta$
- ⋆ $m$ cut pomeron exchanges between incident quark and nucleons in the slice $[0,\beta]$
- ⋆ $k$ cut pomeron exchanges between final-state gluon and nucleons in the slice $[\beta,1]$
- ⋆ $n$ cut pomeron exchanges between final-state quark and nucleons in the slice $[\beta,1]$

- ⋆ DIS: the same minus ISI

# Summary and further applications:

- Nonlinear $k_\perp$-factorization: explicit quadratures in terms of the collective glue defined by coherent diffraction
- Expansion of nuclear unintegrated glue in terms of *collective glue of overlapping nucleons*.
- *A non-abelian* intranuclear evolution of color dipoles.
- Non-trivial interplay of coherent and incoherent nuclear effects
- Universality classes for nonlinear $k_\perp$-factorization.
- Explicit quadratures are available for single-jet and dijet spectra from all pQCD subprocesses
- Excitation of nuclei: universality class-dependent AGK rules
- Application of AGK rules: energy loss for color-excitation of nucleons $\Longrightarrow$ quenching of forward jets