Nonlinear QCD dynamics at high energy

Robi Peschanski[a]
(SPhT, Saclay)
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- Saturation and Non-Linear QCD Evolution:

Non-linear driving terms in QCD

- Mathematics:

Universality of Traveling Wave Solutions

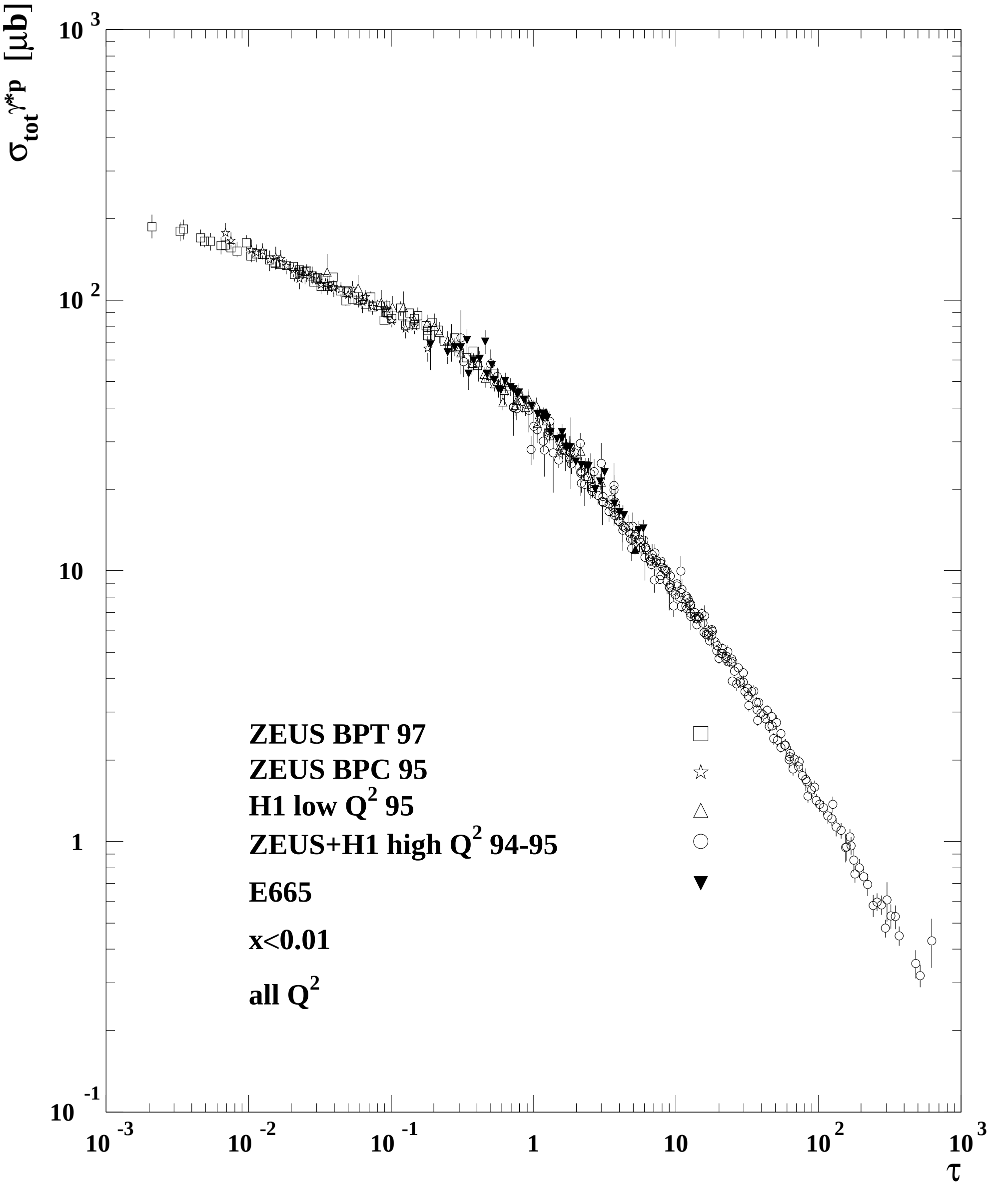
- Physics:

Geometric Scaling, Diffusion and Fluctuations

[a]hep-ph/0309177 /0310357 /0401215 (with S. Munier); hep-ph/0505237 and C.Marquet, R.P., G.Soyez, to appear.

Geometric Scaling

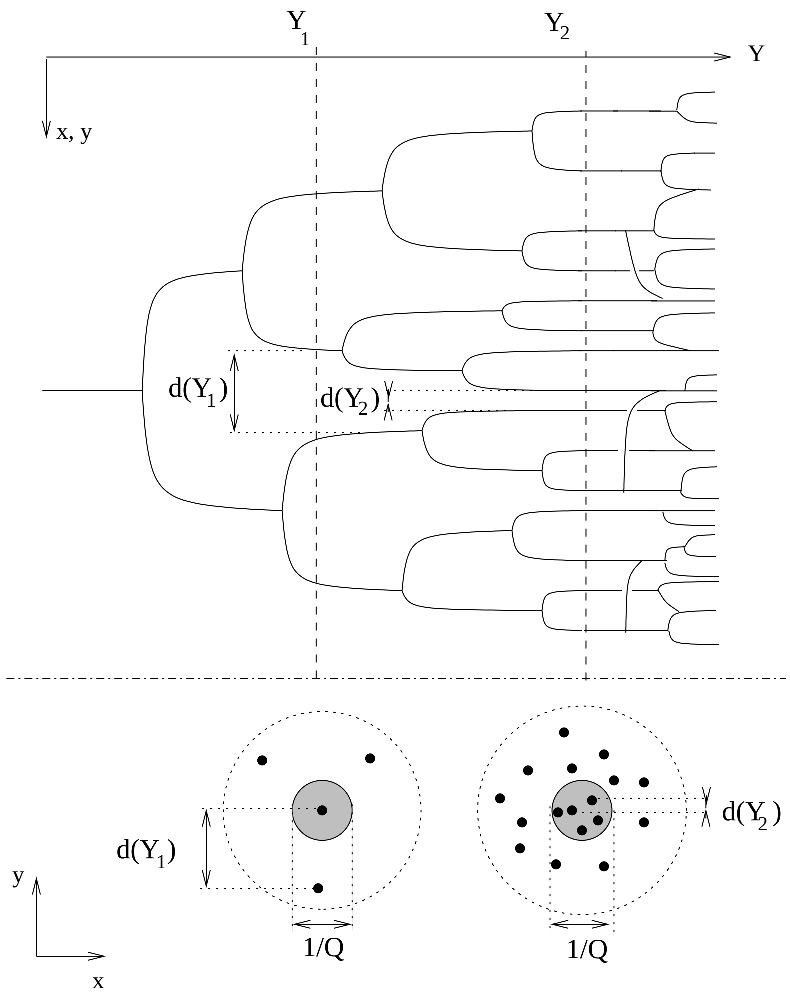
K.Golec-Biernat, J.Kwiecinski, A.Stasto (2000)



$$\tau = Q^2/Q_s(Y_{=\log 1/x_{Bj}})$$

Geometric Scaling from Non-Linear QCD ?

The Tree of Partons/Dipoles



$d(Y) \to 0 \Rightarrow$ QCD Driving Terms

$Y \sim Y_1$: Exponential growth: BFKL

$Y \sim Y_2$: Approach to Saturation: "mean field"

$Y > Y_2$: Beyond: Saturation, Fluctuations

The Balitskiĭ-Kovchegov (BK) Equation

- **The Dipole Tree Observed in DIS:**

$$\sigma^{\gamma^*}(Y,Q) = \int_0^\infty x_{01}^3 \, dx_{01} \, |\psi(x_{01}Q)|^2 \int k dk J_0(kx_{01}) \, \mathcal{N}(Y,k)$$

x_{01} : Dipôle Size

$\psi(x_{01}Q)$: $q\bar{q}$ Dipole Wave Function

$\mathcal{N}(Y,k)$: $\sim$ Unintegrated Gluon k-Distribution

- **The Non-Linear BK Equation for $\mathcal{N}$:**

$$\partial_Y \mathcal{N} = \bar{\alpha}\chi\left(-\partial_L\right)\mathcal{N} - \bar{\alpha}\,\mathcal{N}^2$$

- BFKL kernel

$$\chi\left(-\partial_L\right) = 2\psi(1) - \psi\left(-\partial_L\right) - \psi\left(1+\partial_L\right) \;\; ; \;\; L \equiv \log\frac{k^2}{\Lambda^2}$$

- QCD Coupling (fixed $\rightarrow$ running)

$$\bar{\alpha} \;= cste. \; or \; \bar{\alpha} \;= \frac{1}{bL}$$

- No Fluctuations: $\equiv$ “Mean Field”

Mathematical Problem

1^{st} step: $\longrightarrow$ **Non-Linear Diffusion**

- **Diffusive Approximation of BK** ($\bar{\alpha} = cst.$)

$$\bar{\chi}\left(-\partial_L\right) \sim \chi\left(\tfrac{1}{2}\right) + \tfrac{D}{2} \times \left(\partial_L + \tfrac{1}{2}\right)^2$$

- **Equation BK $\Rightarrow$ F-KPP**

S.Munier, R.P., 2003,2004

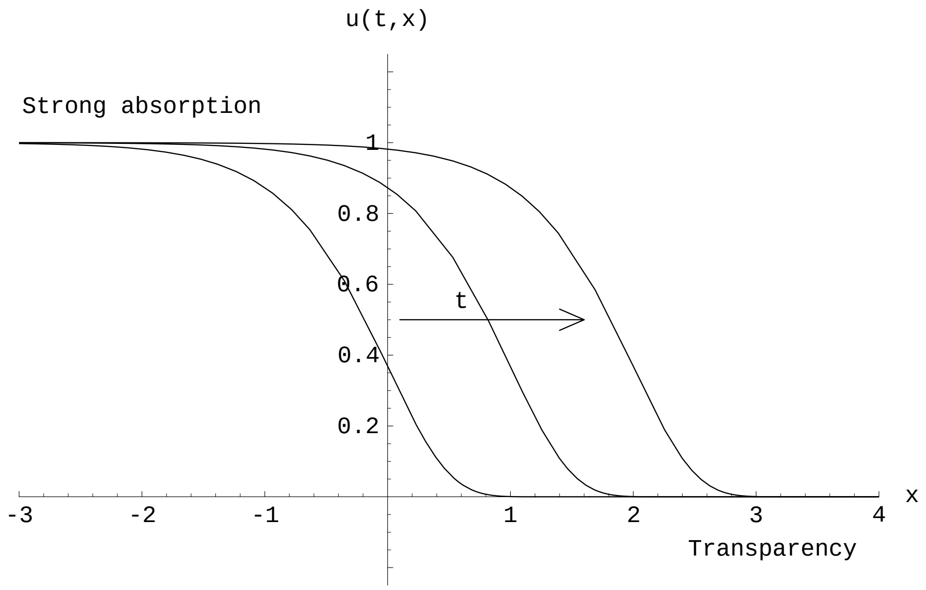
$$\boxed{\partial_t u(t,x) = \partial_x^2 u(t,x) + u(t,x)(1 - u(t,x))}$$

- **"Dictionnary"**

$$\begin{aligned} Time &= t \propto Y \\ Space &= x \propto L + \frac{\bar{\alpha} D}{2} Y \\ Wave\ Front &= u(t,x) \ \propto\ \mathcal{N}(Y,k) \end{aligned}$$

"Universality" of Saturation

Bramson (1983)



- Traveling wave $\longrightarrow$ Geometric Scaling

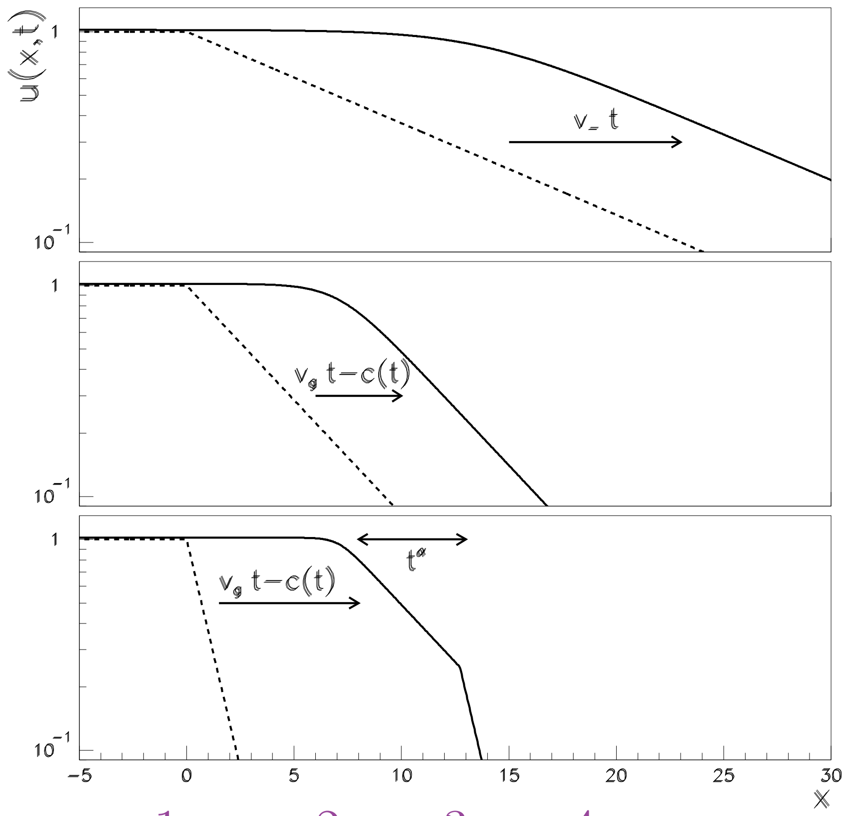
$$u(t,x) \xrightarrow[t\,\to\infty]{} w(x - m_{\bar\gamma}(t)) \Rightarrow \boxed{\mathcal{N}(Y,k) = \mathcal{N}\left(\frac{k}{Q_s(Y)}\right)}$$

- I: "Universality" of Saturation Scale

$$\boxed{\log Q_s^2(Y) = \bar\alpha \frac{\chi(\bar\gamma)}{\bar\gamma} Y - \frac{3}{2\bar\gamma}\log Y - \frac{3}{(\bar\gamma)^2}\sqrt{\frac{2\pi}{\bar\alpha\chi''(\bar\gamma)}}\frac{1}{\sqrt{Y}} + \mathcal{O}(1/Y)}$$

The Wave Front Structure

Derrida, Van Saarlos: “Pulled vs. Pushed fronts”

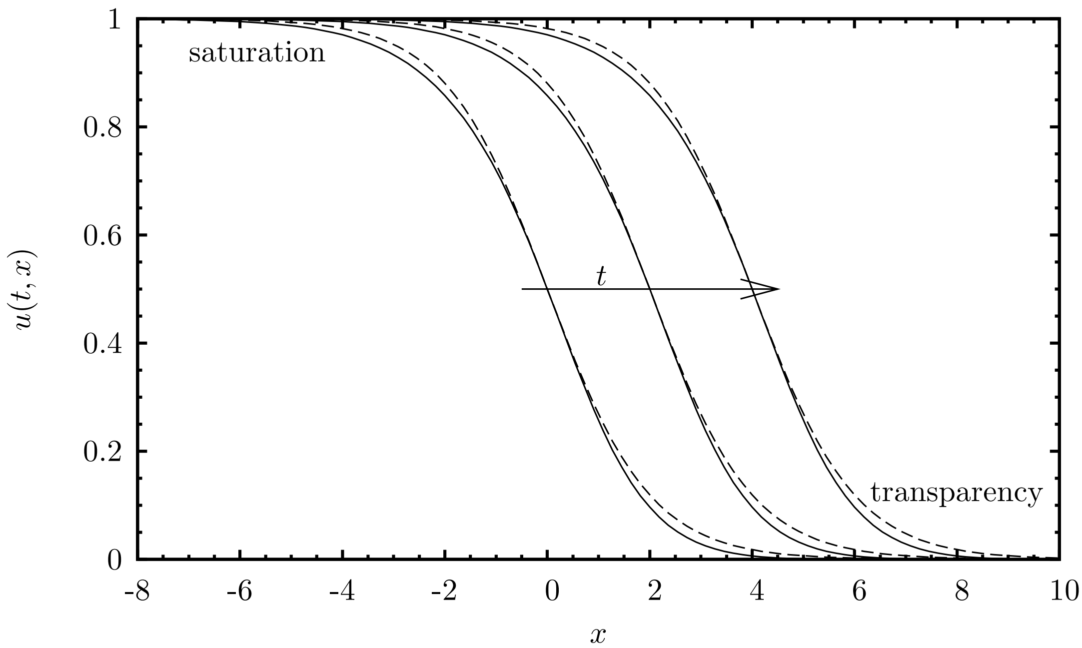


−1 − −2 − −3 − −4−

Supercritical “Pulled” Fronts: 4 Regions

- -1- “Absorptive”: Deep Saturation
- -2- “Interior”: Geometrical Scaling
- -3- “Leading Edge”: Transition to Saturation
- -4- “Conserved Velocity”: Transparency limit

“Universality” of the Front



- - - $\mathcal{O}(1)$; —— $\mathcal{O}(1/c^2)$

- Result up to $\mathcal{O}(1/c^2)$

$$U(s) = \frac{1}{1+e^s} - \frac{1}{c^2}\,\frac{e^s}{(1+e^s)^2}\,\log\frac{(1+e^s)^2}{4e^s}$$

- Geometric Scaling

$$\mathcal{N} \propto \frac{1}{1+\left[\frac{k^2}{Q_s^2(Y)}\right]^{\mu_1}} - \frac{1}{c^2}\,\frac{\left[\frac{k^2}{Q_s^2(Y)}\right]^{\mu_1}}{\left(1+\left[\frac{k^2}{Q_s^2(Y)}\right]^{\mu_1}\right)^2}\,\log\frac{\left(1+\left[\frac{k^2}{Q_s^2(Y)}\right]^{\mu_1}\right)^2}{4\left[\frac{k^2}{Q_s^2(Y)}\right]^{\mu_1}}$$

Running Coupling

- **Running Coupling**

$$bL\,\partial_Y \mathcal{N}(L,Y) = \chi\,(-\partial_L)\,\mathcal{N}(L,Y) - \mathcal{N}^2(L,Y)$$

- **Scaling Solution**

$$\mathcal{N}(L,Y) \equiv \mathcal{N}\left\{L\ \varphi(\frac{Y}{L^2})\right\}$$

- **New scaling variable**

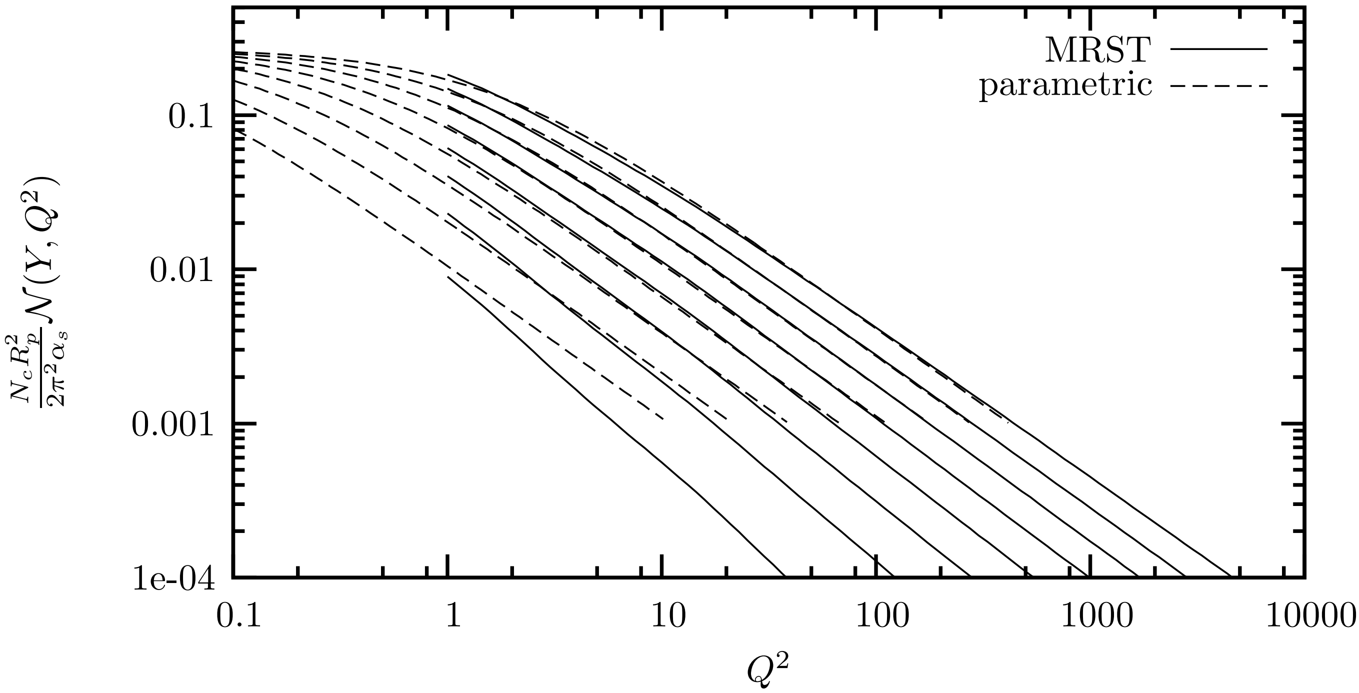
$$s \propto L - \kappa\sqrt{L^2 + \frac{v_g^2}{\kappa^2}Y} \sim L - v_g\sqrt{Y}$$

- **Same Universal Parametric Wave**

$$U(s) = \frac{1}{1+e^s} - \frac{1}{c^2}\ \frac{e^s}{(1+e^s)^2}\ \log\frac{(1+e^s)^2}{4e^s}$$

Back to Scaling

C.Marquet,R.P.,G.Soyez, preliminary

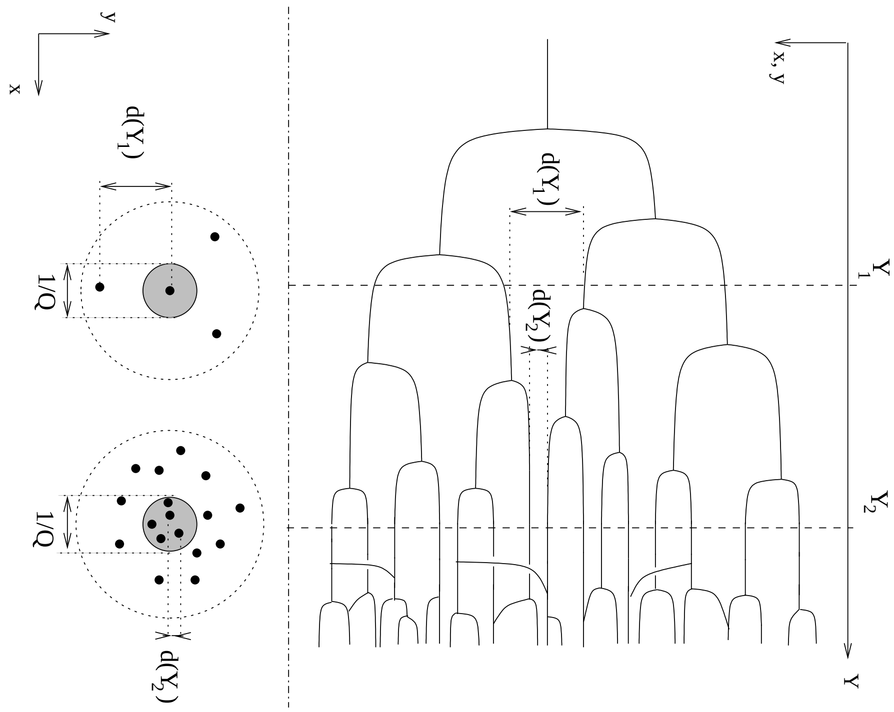


Unintegrated Gluon $Y : 3 \rightarrow 10$

Conclusions

- BK Equation and Beyond
  Nonlinear Traveling wave solutions
- Physics Properties:
  Geometric Scaling from Basics of QCD
- Results:
  "Universality" $\Rightarrow$ Scale + Front
- High Energy Frontier in QCD:
  New Relation with Mathematics (non-linear Eqs.) and Physics (Disordered systems, Polymer diffusion and Spin glass phase transitions)

Questions and Prospects :



- Phenomenology

  Build a full (Q^2, Y) solution

- Fluctuations and Stochasticity

  Theoretical Extension needed

- Many-Body Correlations

  Relation with Advanced Statistical Physics

- $\rightarrow$ Strong Coupling

  Relation with String Theory?