# Status of $\alpha_s$ Determinations

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- introduction
- previous world averages
- which measurements did they include?
- new results
- limiting factors
- perspectives / expectations

# Renormalization Group Equation

Renormalization Procedure introduces scale dependence of pert. cross section and $\alpha_s$
$\Rightarrow$ Observable R does not depend on choice of $\mu_r$:

Renormalization Group Equation: $$\mu_r^2 \frac{\partial R}{\partial \mu_r^2} + \mu_r^2 \frac{\partial \alpha_s}{\partial \mu_r^2} \frac{\partial R}{\partial \alpha_s} = 0$$

$$\frac{\partial a}{\partial \ln \mu_r^2} = \beta(a) = -\beta_0 a^2 - \beta_1 a^3 - \beta_2 a^4 - \beta_3 a^5 + \mathcal{O}(a^6) \qquad a(\mu_r) \equiv \alpha_s(\mu_r)/(4\pi)$$

computed up to NNNLL (=4-loops)

$$\begin{aligned}
\beta_0 &= 11 - \frac{2}{3} n_f \\
\beta_1 &= 102 - \frac{38}{3} n_f \\
\beta_2 &= \frac{2857}{2} - \frac{5033}{18} n_f + \frac{325}{54} n_f \\
\beta_3 &= \left(\frac{149753}{6} + 3564\zeta_3\right) - \left(\frac{1078361}{162} + \frac{6508}{27}\zeta_3\right) n_f + \left(\frac{50065}{162} + \frac{6472}{81}\zeta_3\right) n_f^2 + \frac{1093}{729} n_f^3
\end{aligned}$$

with: $\zeta_3 = 1.202056903\ldots$

# A Single Free Parameter: $\alpha_s(M_Z)$

$$a(\mu_r) = \frac{a(M_Z)}{1 + a(M_Z)\, L}$$

with

$$\begin{aligned} L \quad = \quad & \beta_0 \ln \frac{\mu_r^2}{M_z^2} + b_1 \ln \frac{a(M_Z)}{a(\mu_r)} + (b_2 - b_1^2)(a(M_Z) - a(\mu_r)) \\ & + \left( \frac{b_3}{2} - b_1 b_2 + \frac{b_1^3}{2} \right) (a^2(M_Z) - a^2(\mu_r)) \end{aligned}$$

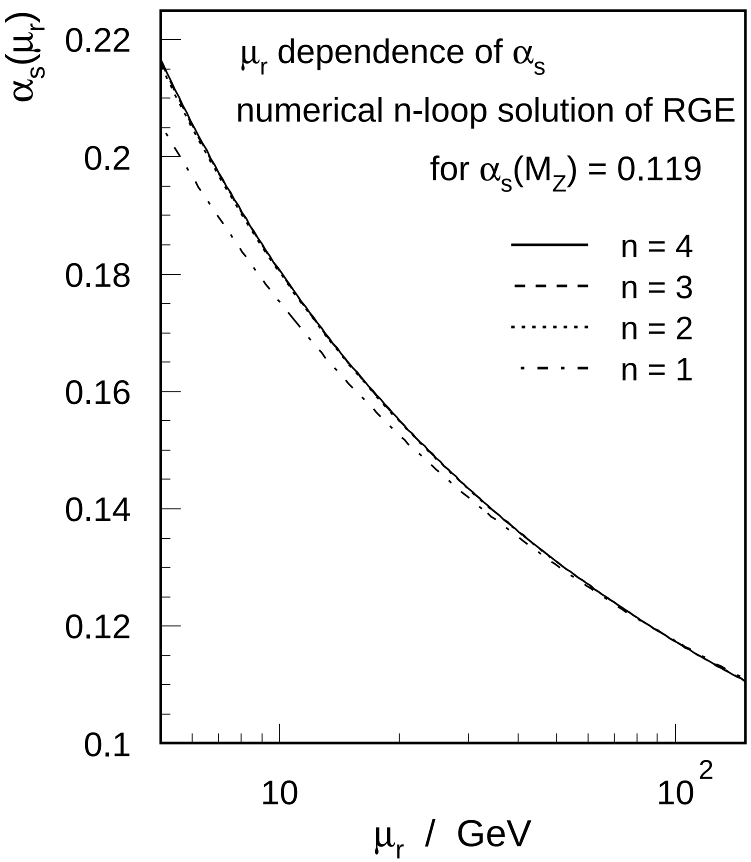
with: $b_N = \beta_N / \beta_0$ $\qquad$ $M_Z = 91.1876 \pm 0.0021$ (PDG 2005)

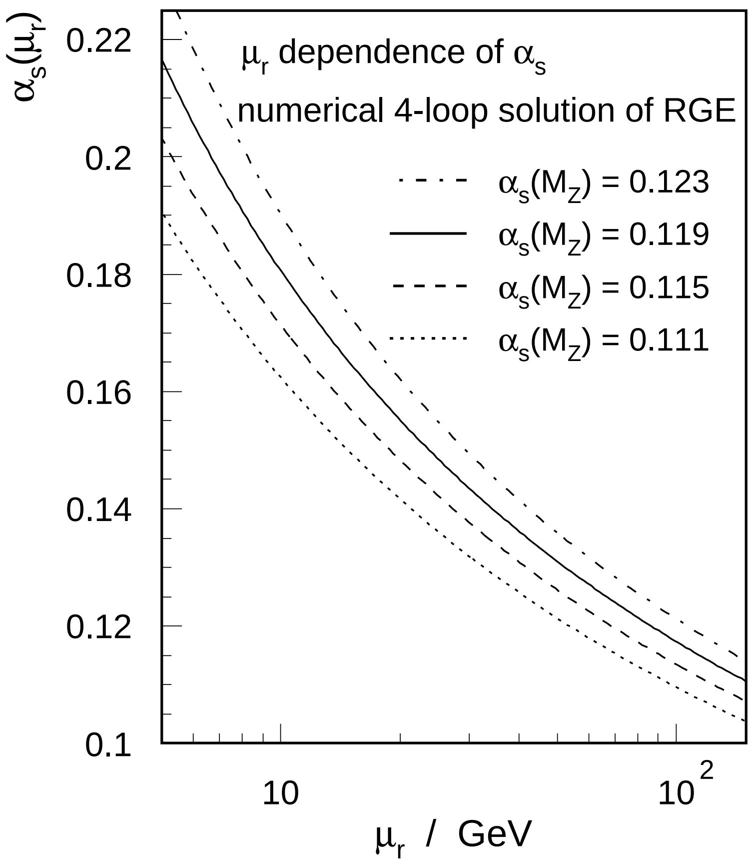
$\Longrightarrow$ important: no Lambda value is needed – much clearer to define in terms of $\alpha_s(M_Z)$

- approximate solution:
  solve iteratively and discard terms of $\mathcal{O}(1/\ln^N(\mu_r^2/M_Z^2))$

- exact solution using numerical methods
  (as used by PDF evolution program QCD-Pegasus by A. Vogt)

# Running $\alpha_s$




- significant differences only between 1-loop and $n$-loop formulas ($n = 2, 3, 4$)
- $n = 2, 3, 4$-loop formulas agree within 0.4% for $\mu_r > m_b \quad (n_f = 5)$
- difference of $\pm 0.004$ @$\mu = M_Z$ evolves to $\pm 5\%$ @$10$ GeV and $\pm 6\%$ @$5$ GeV
- for $\mathcal{O}(\alpha_s^2)$ processes this means: $\pm 10\%$ @$10$ GeV and $\pm 12\%$ @$5$ GeV

# History of World Averages in Past Decade

$$\alpha_s(M_Z) = \ldots$$

| | | | |
|---|---|---|---|
| PDG 1996 | $0.118 \pm 0.003$ | | |
| PDG 1998 | $0.119 \pm 0.002$ | | |
| PDG 1999 | $0.1185 \pm 0.002$ | | |
| PDG 2000 | $0.1181 \pm 0.002$ | Bethke 2000 | $0.1184 \pm 0.0031$ |
| PDG 2002 | $0.1172 \pm 0.002$ | Bethke 2002 | $0.1183 \pm 0.0027$ |
| PDG 2004 | $0.1187 \pm 0.002$ | Bethke 2004 | $0.1182 \pm 0.0027$ |

$\Longrightarrow$ LEP EW WG uses currently $0.118 \pm 0.003$ – very reasonable!

# Overview

- go through different processes
- not exhaustive!
  special emphasis on jet production ... and selected new results
- discuss principles / issues
- results used in world average value
- emphasize new results, not included in previous world averages

not discussed here:

- Structure Functions (Scaling Violations and Sum Rules)
- Tau Decay
- Upsilon Decay
- Scaling Violations of Fragmentation Functions

# $e^+e^-$ Total Hadr. Cross Section & EW Precision Fits

rely on predictions (and validity) of standard model!!

- Combined from LEP-I and LEP-II electroweak precision measurements:
  $R = \Gamma_{\text{had}}/\Gamma_{\text{l}} = 20.767 \pm 0.025$
  $\rightarrow$ NNLO analysis:
  $\alpha_s(M_Z) = 0.1126 \pm 0.0038$ (exp.)$^{+0.0033}_{-0.0000}$ $(M_H)^{+0.0028}_{-0.0005}$ (QCD) (used by Bethke2004)

- global electroweak precision fits of $M_Z$, $M_H$, $M_t$, $\Delta\alpha^{(5)}_{\text{had}}$ in NNLO
  (all Z-pole data - plus direct $m_t$, $m_W$, $\Gamma_W$ determinations – i.e. all high $Q^2$ results)
  $\alpha_s(M_Z) = 0.1186 \pm 0.0027$ (exp.) (no dependence on Higgs mass)

  so far: theory uncertainty unknown $\rightarrow$ PDG2004 added difference to the value above:
  $\alpha_s(M_Z) = 0.1186 \pm 0.0027$ (exp.) $\pm\ 0.003$ (theor.)

- **new:** determination of theor. uncertainty by H. Stenzel (hep-ph/0501245)
  $\alpha_s(M_Z) = 0.1186 \pm 0.0027$ (exp.) $\pm\ 0.0013$ (theor.)
  (rare case: experimental effects dominate)

$\Longrightarrow$ significant improvement of already precise value – impact on future world average!

# *e+e- Jets and Hadronic Event Shapes*

study variables $V$, defined on the hadronic final state, which are IR and collinear safe and which are directly sensitive to higher order parton emissions, i.e. $V \propto \alpha_s$

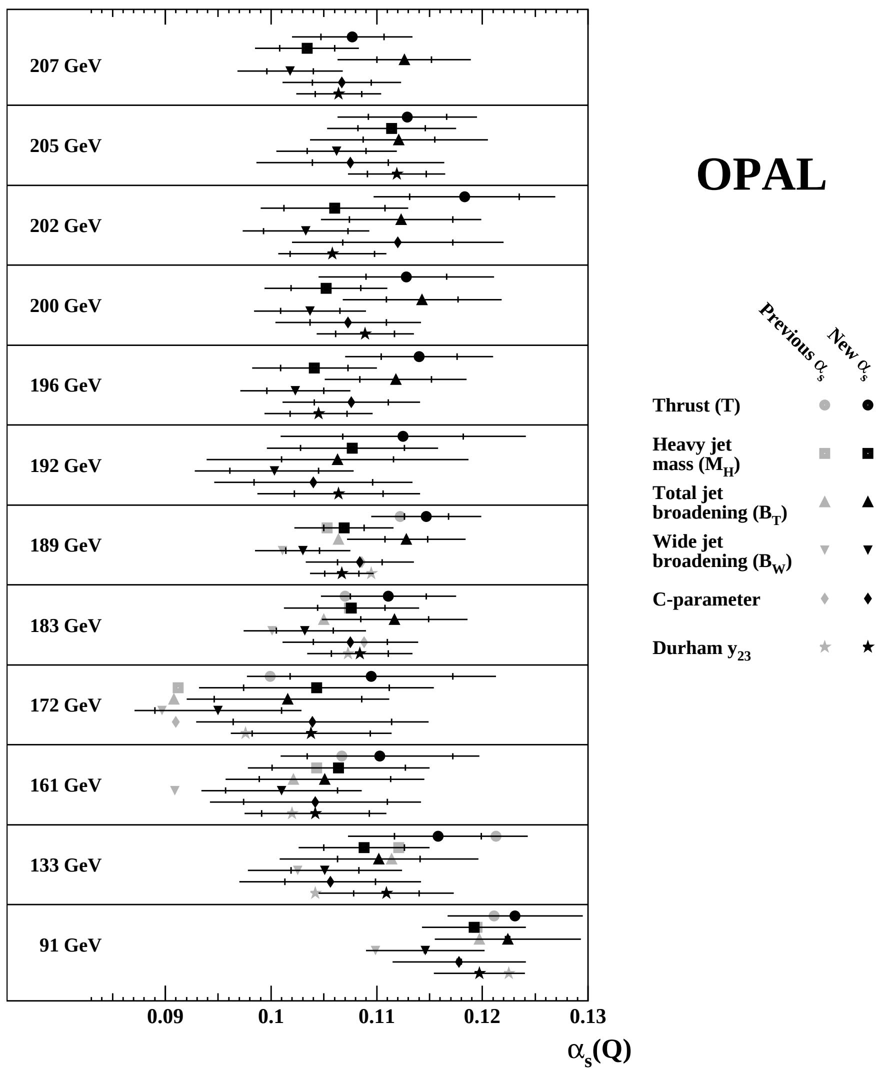
- event shape variables:
  variables, defined by weighted sum over all particles
  (1-Thrust), Heavy Jet Mass, C Parameter, 3-Jet Parameter
- jet rates:
  reconstruct collimated blocks of particle energies using jet algorithm:
  Durham, Cambridge, Cone, JADE
  study transition: for 2 jets $\longrightarrow$ 3 jets $\longrightarrow$ 4 jets

large No. of measurements from OPAL, DELPHI, ALEPH, L3 at different energies
theory NLO + NLLA – average value from 2004:

$$\alpha_s(M_Z) = 0.1202 \pm 0.0009 \text{ (exp.)} \pm 0.0009 \text{ (hadr)} \pm 0.0047 \text{ (theor.)}$$

$\Rightarrow$ Clearly limited by theoretical precision

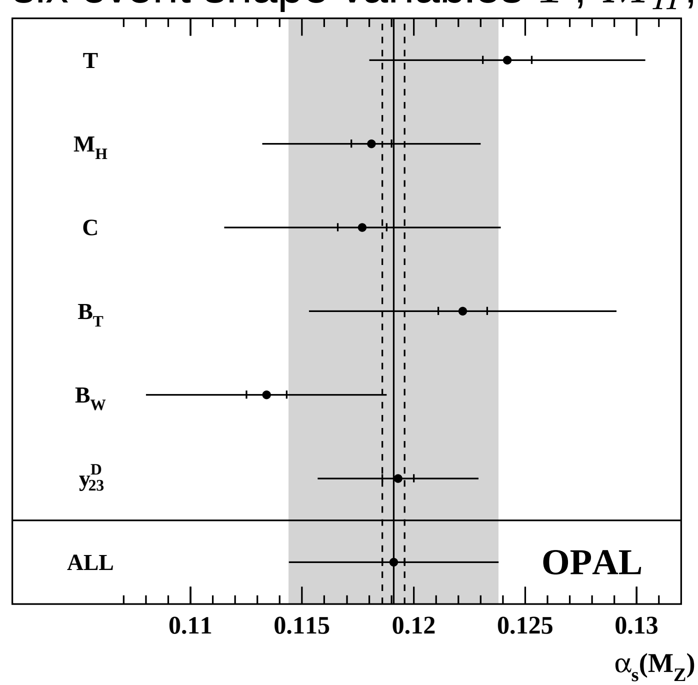
# e+e- Event Shape Distributions and Moments




**new result:** OPAL (hep-ex/0503051)

large range: $\sqrt{s} = 91$ GeV – $207$ GeV

six event shape variables $T$, $M_H$, $C$, $B_T$, $B_W$, $y_{23}^D$

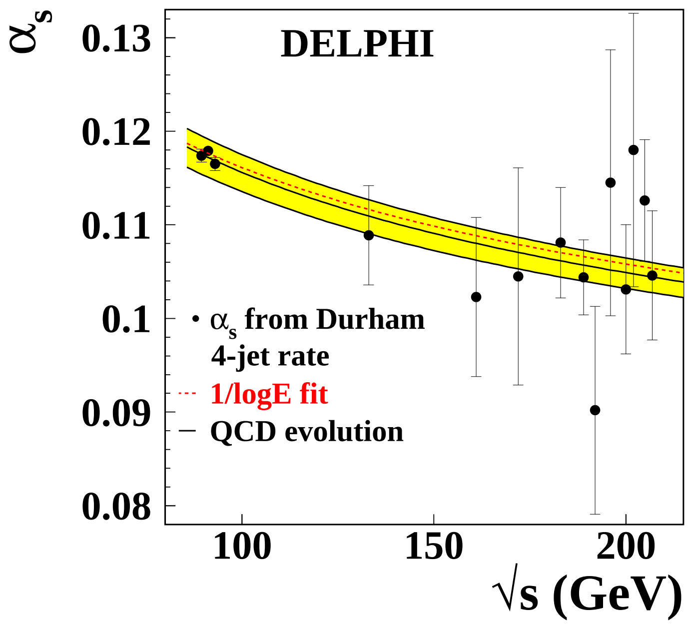


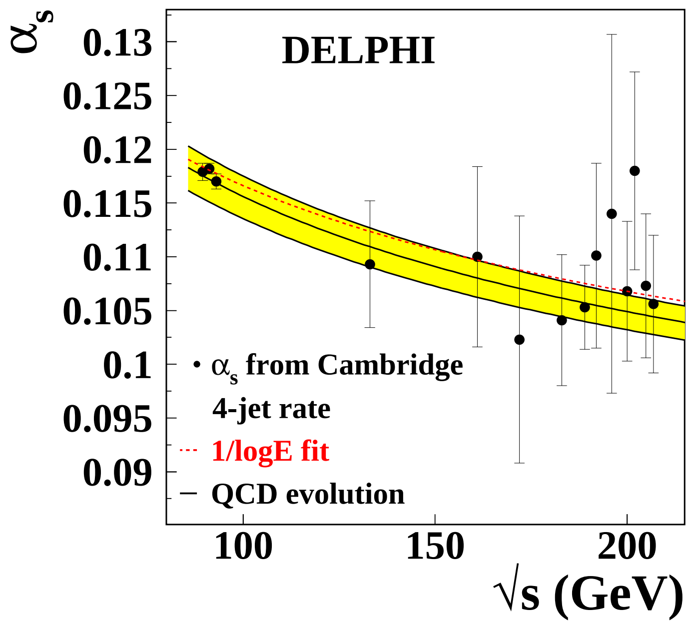

$\alpha_s(M_Z) = 0.1191 \pm 0.0011$ (exp) $\pm\, 0.0011$ (hadr) $\pm\, 0.0044$ (theor.)

$\Rightarrow$ good confirmation of existing results

# *e+e- Four-Jet Rate*

**new result:** DELPHI (hep-ex/0410071): ratio of four jet events in all events $\propto \alpha_s^2$
Durham, Cambridge and JADE jet algorithms – large range: $\sqrt{s} = 89$ GeV – $209$ GeV




smaller third-order contribution for Cambridge algorithm:
$\alpha_s(M_Z) = 0.1175 \pm 0.0010$ (exp) $\pm\, 0.0027$ (hadr) $\pm\, 0.0007$ (theor.)

questionable: "experimentally optimized" scales $\Rightarrow$ ren. scale has been fitted to data
$x_\mu = 0.015$ for Durham, $x_\mu = 0.042$ for Cambridge algo - why so different/small??

(I personally don't trust the quoted theory uncertainty)

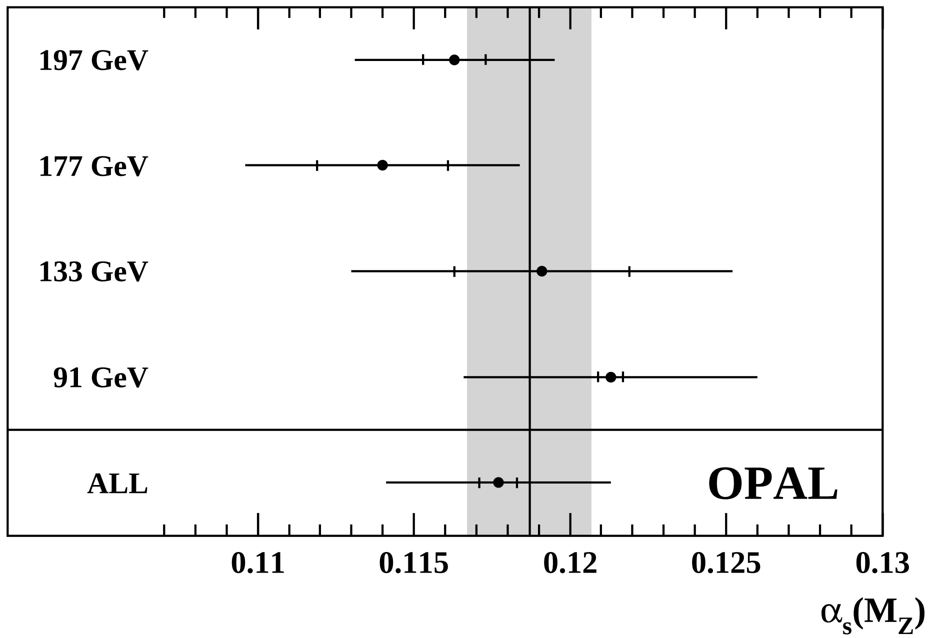
# *e+e- Jet Rates*

**most recent result:** OPAL (hep-ex/0507047): differential two jet rate and average jet rate

large range: $\sqrt{s} = 91$ GeV – $209$ GeV

using four jet algorithms:
Durham, Cambridge, Cone and JADE

theory: NLO $(\mathcal{O}(\alpha_s^2))$
with matched NLLA predictions




$$\alpha_s(M_Z) = 0.1177 \pm 0.0013 \text{ (exp)} \pm 0.0010 \text{ (hadr)} \pm 0.0032 \text{ (theor.)}$$

$\Rightarrow$ error is slightly larger than the one from a previous publication: $\sqrt{s} = 35$ GeV – $189$ GeV

# $\alpha_s$ *in Hadron-Induced Reactions (DIS/pp)*

- Problem: need PDF knowledge $\longrightarrow$ correlation $\alpha_s$ and PDFs (especially the gluon!)

$$\chi^2(\alpha_s) = \frac{(D - T(\alpha_s))^2}{\sigma^2} \qquad \longrightarrow \qquad \frac{(D - T(\alpha_s, \text{PDF}))^2}{\sigma^2}$$

- In addition: PDF knowledge is already coupled to $\alpha_s$ knowledge

- one approximation:
  use best PDF fit result and ignore correlation
  later: use different PDFs – demonstrate small dependence (if you're lucky!)
- different approximation:
  accept that PDFs fit results depend on $\alpha_s$ – choose PDF sets for different values of $\alpha_s(M_Z)$

$$\chi^2(\alpha_s) = \frac{(D - T(\alpha_s, \text{PDF}(\alpha_s)))^2}{\sigma^2}$$

- problem: these PDFs are not all standing on the same footing!!!!
  PDFs for different $\alpha_s$ values had different $\chi^2$ values in PDF fit

$$\chi^2(\alpha_s) = \frac{(D - T(\alpha_s, \text{PDF}(\alpha_s)))^2}{\sigma^2} + \Delta\chi^2(\text{PDF}(\alpha_s))$$

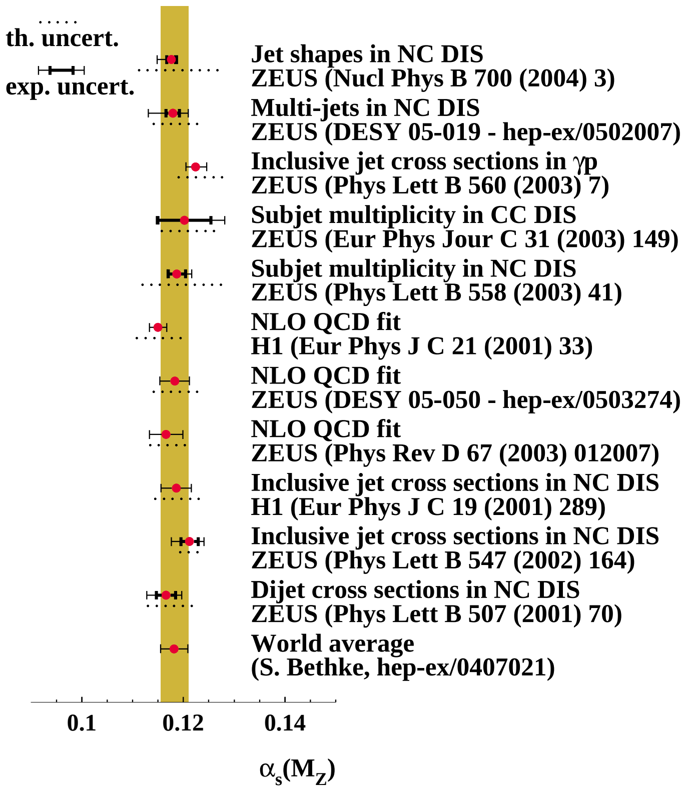
...but usually ignored — real problem: no modern PDFs available for flexible $\alpha_s(M_Z)$

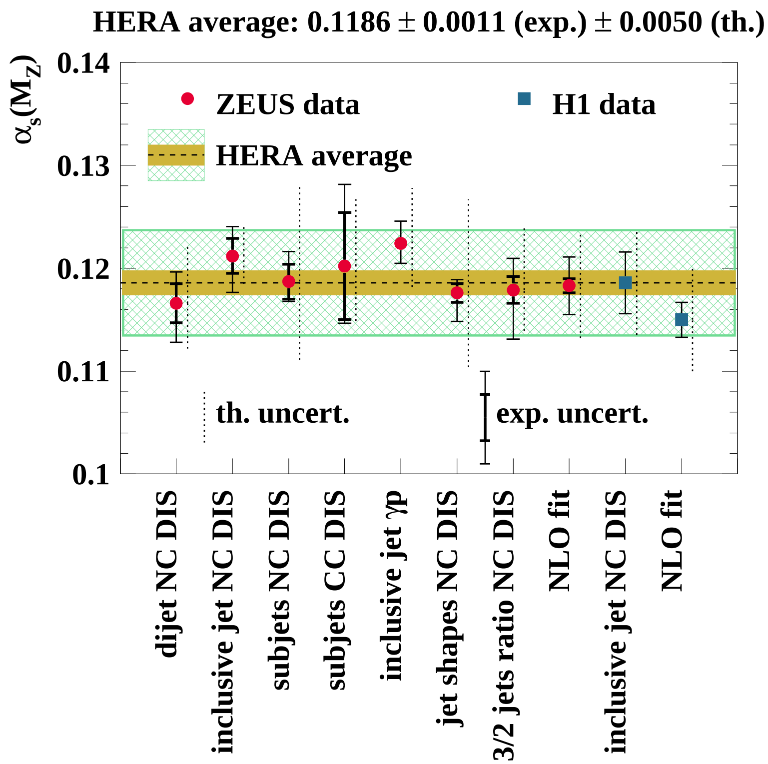
$\Rightarrow$ only one real solution: should aim for combined fits of $\alpha_s$ and the PDFs

# DIS Jets

In the last years: large number of jet measurements from HERA

**new:** HERA summary by C. Glasman (hep-ex/0506035):



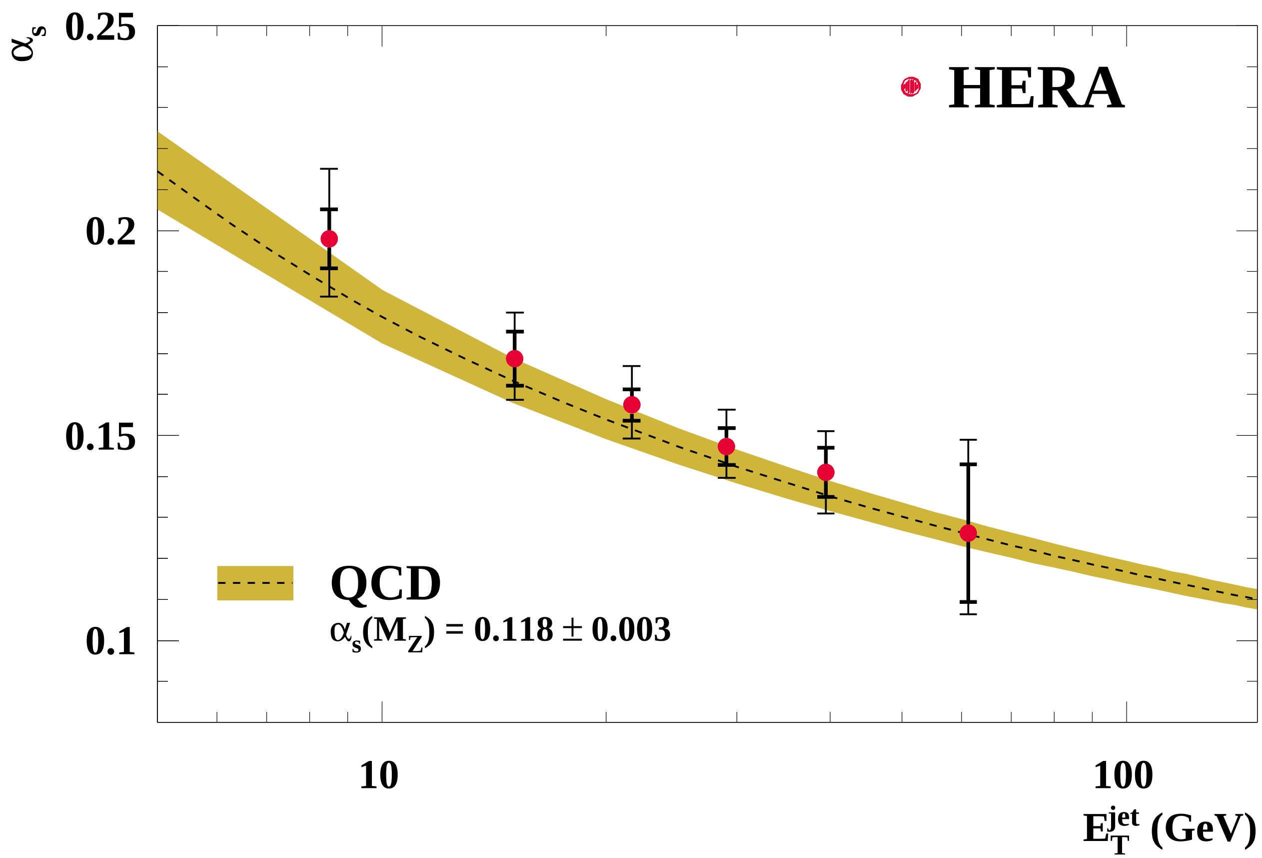




$$\alpha_s(M_Z) = 0.1186 \pm 0.0011\,\text{(exp)} \pm 0.0050\,\text{(theor.)}$$

⇒ includes Jets and Structure Function results – it might be preferable to separate these

# *DIS Jets*

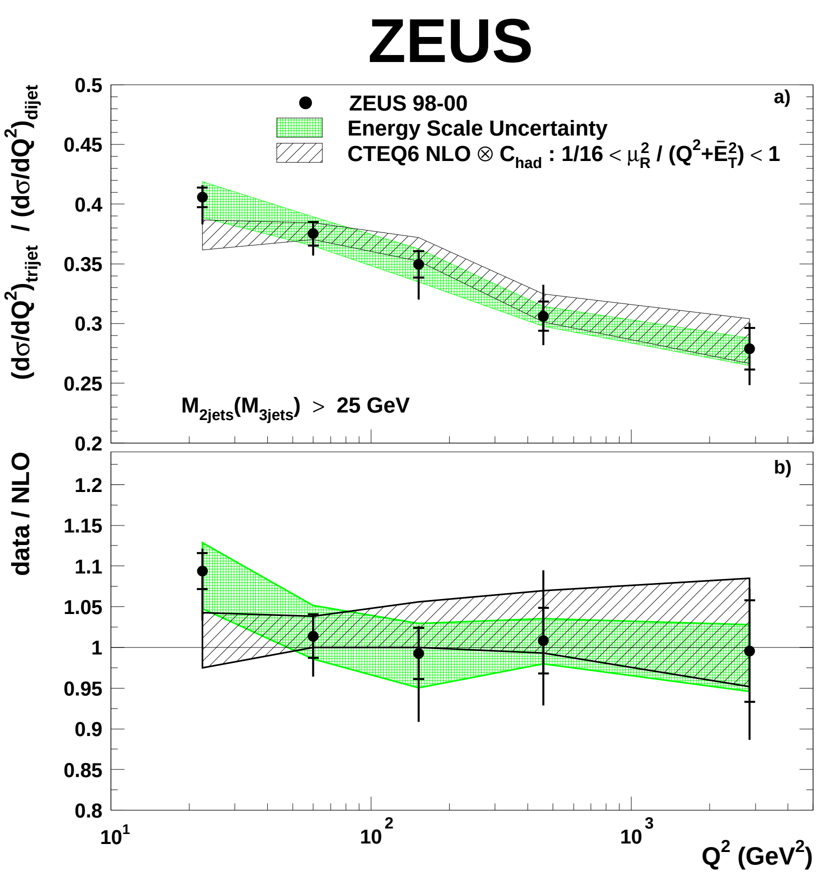
the energy dependence of $\alpha_s$ — for HERA jet results:

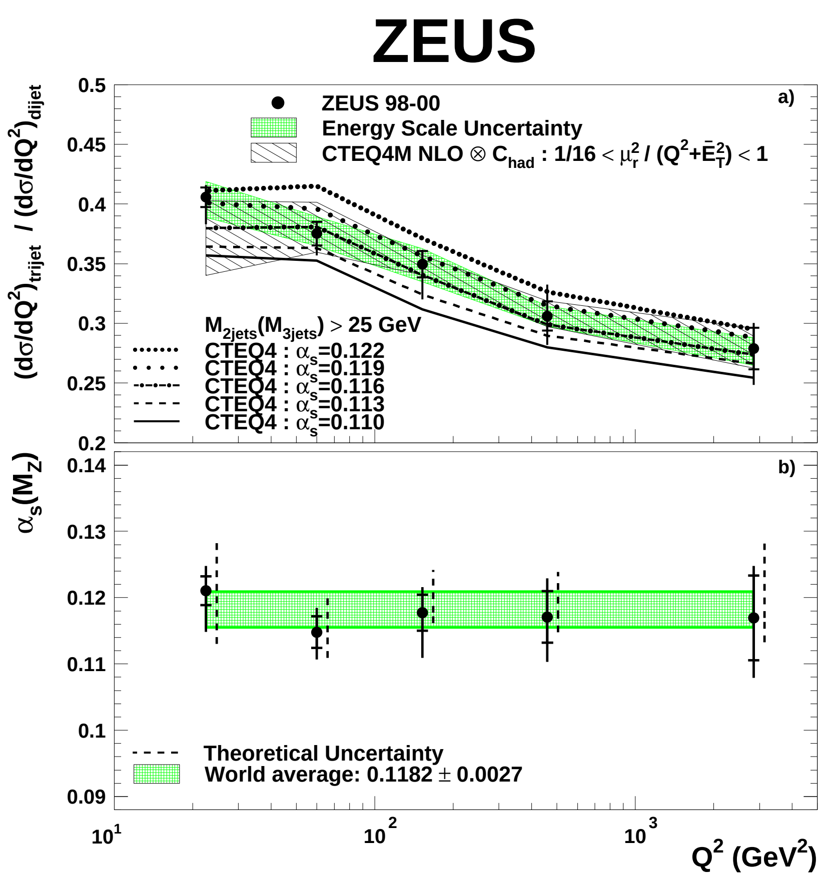



$\Rightarrow$ nice demonstration of the running of $\alpha_s(\mu_r)$

# DIS jets – Ratio 3-Jet/2-Jet Production

**new:** ZEUS (hep-ex0502007) $\sigma_{3-\text{jet}}/\sigma_{2-\text{jet}} \propto \alpha_s$ measurement (left) / $\alpha_s$ (right)



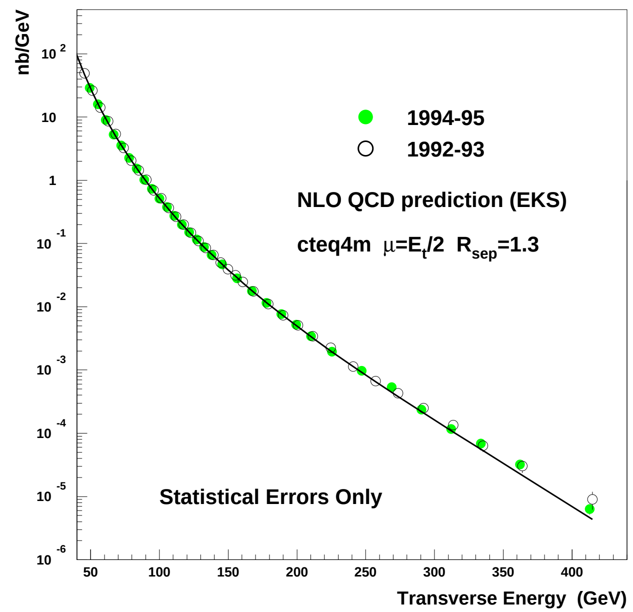




$$\alpha_s(M_Z) = 0.1179 \pm 0.0013\,\text{(stat)}^{+0.0028}_{-0.0046}\,\text{(exp.)}^{+0.0046}_{-0.0064}\,\text{(theor)}$$

$\Rightarrow$ jet ratio: new approach — important to demonstrate consistency

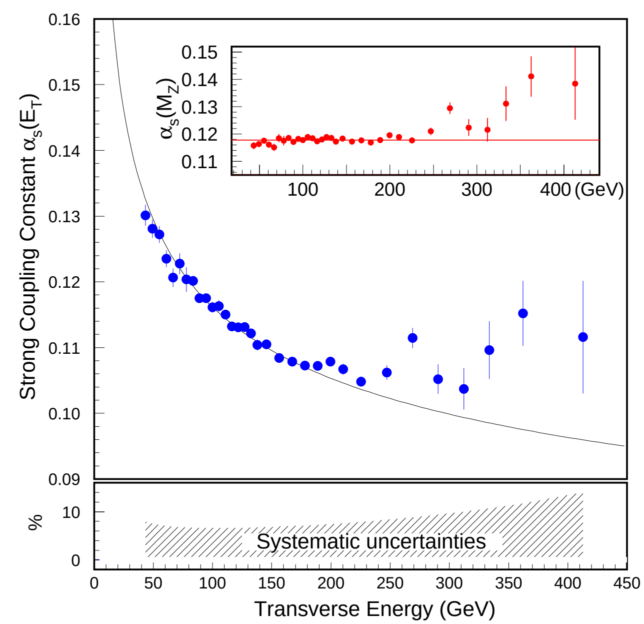
# *Hadron-Hadron $\longrightarrow$ Jets*

- CDF Collaboration: inclusive jet cross section
  Tevatron Run I / at $\sqrt{s} = 1800$ GeV
  in $40 < E_{T,\,\text{jet}} < 450$ GeV
  (CDF, Phys.Rev. D64 (2001) 032001)




- each data point $\sigma_{\text{jet}}(E_T) \propto \alpha_s^2(\mu = E_T) \Rightarrow \alpha_s(E_T)$
  combined fit: $40 < E_{T,\,\text{jet}} < 250$ GeV $\Rightarrow \alpha_s(M_Z)$
  exclude high $E_T$ data – not consistent (old PDFs?)
  (but no statistical power if included)

- input PDFs: CTEQ4 and CTEQ4A series
  ... but exclude the PDFs with $\chi^2/\text{ndf} > 5$




- $\alpha_s(M_Z) = 0.1178^{+0.0081}_{-0.0095}$(exp) ${}^{+0.007}_{-0.005}$(theory)
  $\pm 0.006$ (PDF)

  (CDF, Phys.Rev.Lett.88:042001,2002)
  $\longrightarrow$ used by PDG and Bethke

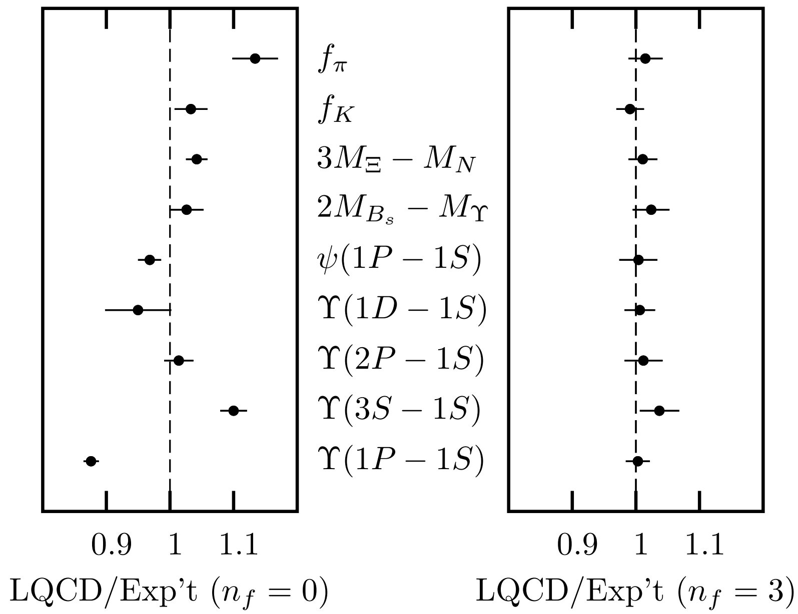
$\Rightarrow$ rare case: dominated by large exp. uncertainties
... would benefit from combining with DØ jets!

(limit: sensitivity only at large scales $E_T$)

# *Lattice QCD (1.)*

results from `hep-lat/0404004`, C. Davies et al. – and **new**: `hep-lat/0503005`, Q. Mason et al.

- 5 LQCD parameters:
  - bare quark masses $m_u = m_d, m_s, m_c, m_b$
  - bare QCD coupling
- tune bare quark masses to reproduce **exp. results**:
  $m_\pi^2, 2m_K^2 - m_\pi^2, m_{D_s}, m_\Upsilon$
  (high sensitivity / small intercorrelation)
- fix bare coupling: determine lattice spacing $a$
  from $\Upsilon$ mass splitting: $\Upsilon' - \Upsilon$
  new: show agreement with ten other physical quantities (pion, kaon leptonic decay constants, $B_s$ mass, $\Omega$ baryon mass)

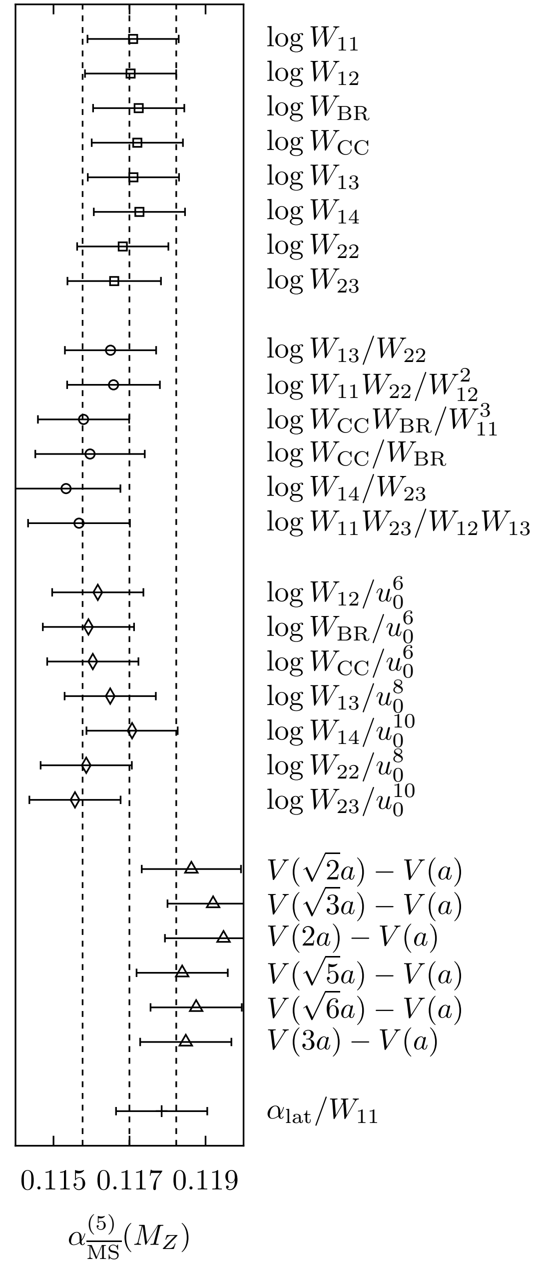



- new: first analyses to include vacuum polarization from all three light-quarks
  older results: "quenched approxim." $n_f = 0$ or $n_f = 2$ had to be extrapolated to $n_f = 3$
  $\rightarrow$ major source of uncertainty – more consistent results in new analysis

- plot: ratios Lattice QCD over Experiment (after tuning, as described above) for $n_f = 0, 3$
  $\Rightarrow$ consistent only for $n_f = 3$

# Lattice QCD (2.)

- use accurately tuned QCD simulation to compute non-perturbative results for a variety of short-distance quantities
- compare results with NNLO pQCD predictions
  $\Rightarrow$ extract $\alpha_s$
- 28 quantities:
  6 vacuum expectation values of Wilson Loop Operators
  $\Rightarrow$ ... large estimated NNNLO coeffi cients
  $\rightarrow$ 6 Creutz ratios
  $\rightarrow$ 7 "tadpole improved" Wilson Loops (divided by $u_0^{2(n+m)}$)
  $\Rightarrow$ the"improved" quantities have smaller HO corrections
  but also reduced sensitivity
  $\rightarrow$ 6x static quark potential V(r) (perturbative at short distances)
  $\rightarrow$ tadpole-improved bare lattice coupling $\alpha_{\mathrm{lat}}/W_{11}$
- extract 3-flavor coupling $\alpha_V$ from each of the 28 quantities
  $\rightarrow$ convert from "V-scheme" to $\overline{\mathrm{MS}}$-scheme
  $\rightarrow$ add $c$ and $b$ quark vacuum polarization (perturbatively)
  $\Rightarrow$ evolve to the Z-mass
- weighted average: $\alpha_s(M_Z) = 0.1170 \pm 0.0012$



# Lattice QCD (3.)

- significant progress – first time:
  - include vacuum polarization from all three light-quark flavors
  - include third order terms in perturbation theory
  - systematically estimate fourth and higher order terms
- Three dominant sources of uncertainty
  - uncertainty in inverse lattice spacing
  - residual uncertainties in perturbative coefficients (from numerical calculation)
  - uncertainty from perturbative coefficients from higher orders
- PDG 2004 and Bethke 2004 used previous result from `hep-lat/0404004`, C. Davies et al.

$$\alpha_s(M_Z) = 0.121 \pm 0.003$$

- Significant improvements in recent analysis `hep-lat/0503005`, Q. Mason et al.

$$\alpha_s(M_Z) = 0.1170 \pm 0.0012$$

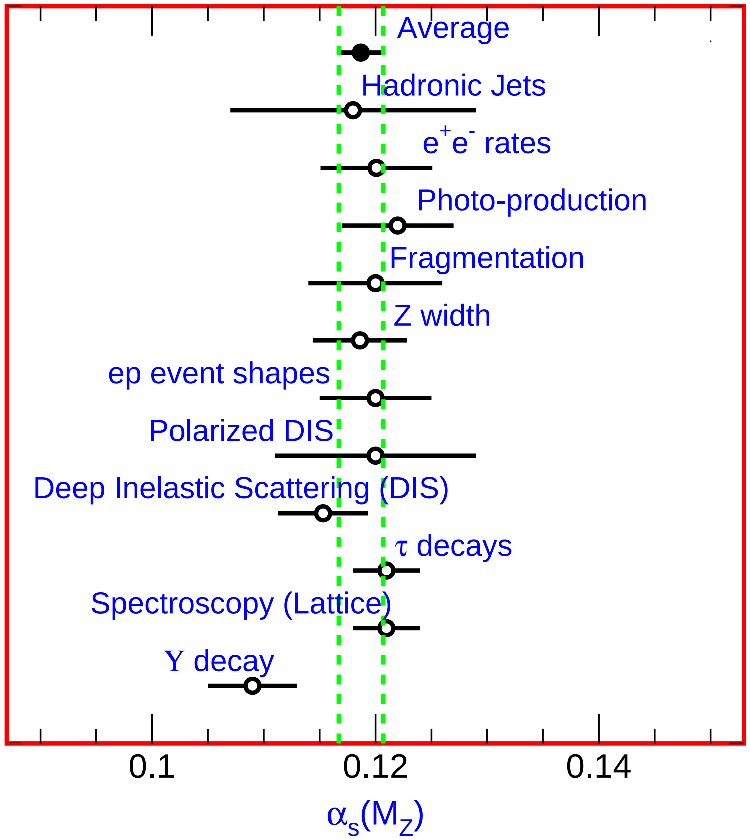
$\Longrightarrow$ Only result in the world with precision better than 2% (and it's much better!)

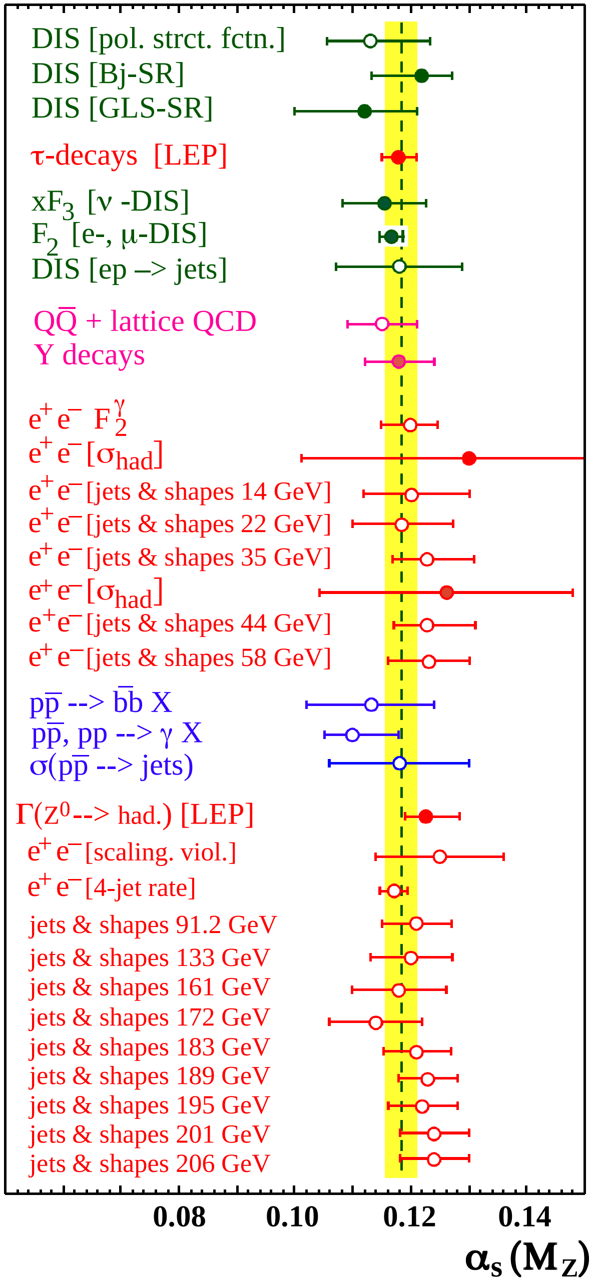
... so revolutionary, that we would like to see some independent confirmation!

# Overview of $\alpha_s(M_Z)$ results – 2004

Bethke 2004 (right)

PDG 2004 (below)




- large difference in the number of input values
- ... also large differences in choices

# How the World Averages are Obtained

- PDG and Bethke both use error-weighted average for central value
- average not dominated by single measurement several results with compatible small uncertainties $\tau$ decay, lattice, DIS, $\upsilon$ decay, $Z$ width
- errors are dominated by theory!!!!!! (not gaussian - meaning unclear)
  have: correlations between similar observables (e+e- jet rates and event shapes)
  maybe: also between similar observables from different processes (jets e+e-, DIS, pp)

- PDG: quote result with arbitrarily increased error $0.0013 \to 0.002$

- Bethke: more detailed procedure:
  weighted average gives similar error as PDG - with much smaller $\chi^2/\mathtt{ndf}$
- error weighted average and an “optimized” correlation error are calculated from the error covariance matrix, assuming overall correlation factor between the total errors of all measurements $\to$ adjust factor to get overall $\chi^2/ndf = 1$
- simple unweighted RMS of the mean values of all measurements
- assume rectangular shaped errors (instead of gaussian probability distributions)
- all three methods have different advantages and different problems
  important: demonstrate consistency / no strong dependence
- also: restrict the average to most significant subsets of data
- final result: include only determinations from NNLO theory

# Previous World Average Values

$$\alpha_s(M_Z) = ...$$

PDG 2004 $0.1187 \pm 0.002$

Bethke 2004 $0.1182 \pm 0.0027$

(these numbers from 2004 do not include most recent results)

$\Longrightarrow$ I am not trying to come up with a new world average value, including the most recent results $\rightarrow$ ... let's wait for S. Bethke's update

... the new values have a tendency to slightly lower the value — but no significant change

$\Rightarrow$ $\alpha_s(M_Z) = 0.118$ is still a reasonable value

# Summary and Conclusions

- The present talk focused on a single number: $\alpha_s(M_Z)$
  $\Rightarrow$ Very convenient to address consistency of a large set of measurements
  $\Rightarrow$ much more important: theory describes all kinds of differential distributions

- The large variety of experimental analyses proves that QCD is a perfectly adequate to describe all high energy phenomena at present colliders

- for all $\alpha_s$ results: almost all experimental errors are dominated by systematics

- almost all $\alpha_s$ results are limited by theoretical uncertainties
  $\Rightarrow$ scale dependence of the NLO / NLO+NLLA calculations

- a good modern result: $\alpha_s(M_Z) = 0.117 - 0.119 \quad \pm 0.003\,(\text{exp}) \quad \pm 0.004\,(\text{teor})$

- be aware: true theory uncertainty can be larger than scale dependence
  (there can be different contributions from higher orders)

- no further progress without significant theoretical improvements!

# Perspectives

- Where do we stand today?
  most important question: Can we trust the Result from Lattice QCD?
  if yes: GREAT!! we have reached 1% precision

- Where are we going?
  at LHC: $\alpha_s$ determinations at typical scales of large $\mu = p_T$ will not be very sensitive
  on the other hand: $\alpha_s$ uncertainty is not that critical for predictions at large $p_T$

- What do we need?
  Progress in Theory!!

- What we don't need:
  Any further $\alpha_s$ determinations with experimental or theoretical uncertainties above $\pm 0.005$