Yields and fluctuations in statistical models

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- Definition
- Equilibrium $(T = 170 MeV, \gamma_{q,s} = 1)$ vs
  explosive $(T = 140 MeV, \gamma_{q,s} > 1)$ vs
  Continuus emission+resonance modification vs
  More complicated models (dynamics)
- Yields and fluctuations in SHM
- Sensitive probes with yields and fluctuations
- Analysis of existing (130 and 200 GeV RHIC data)
- Do it yourself: SHARE

  http://www.physics.arizona.edu/∼torrieri/SHARE/share.html

The statistical model:

$$N = \int \mathcal{M} \prod_i \frac{d^3\vec{p}_i}{E_i} \delta_E \delta_Q$$

$\mathcal{M} \rightarrow constant$ (dynamics $\rightarrow$ phase space)

$$P_N = \frac{\Omega_N}{\sum_n \Omega_n} \qquad \Omega = \int \prod_i \frac{d^3\vec{p}_i}{E_i} \delta_E \delta_Q$$

Observables:

$$< N > \quad , \quad \omega = \frac{< N^2 > - < N >^2}{< N >}, \quad \mathrm{higher\ \ comulants}$$

calculable through partition function

Several ways of defining $\delta_{E,Q} \rightarrow$ Ensembles.

Ensembles , or how to deal with conservation laws

$\lim_{V\to\infty}^{N/V=const} < N >$ same in $\forall$ ensembles. not ω

Micro-canonical : EbyE conservation

$$\delta_E \delta_Q = \delta\left(\sum_i E_i - E_T\right)\delta\left(\sum_i Q_i - Q_T\right) \quad \omega_E = \omega_Q = 0$$

Canonical : Energy conserved on average

Appropriate for system in equilibrium with bath

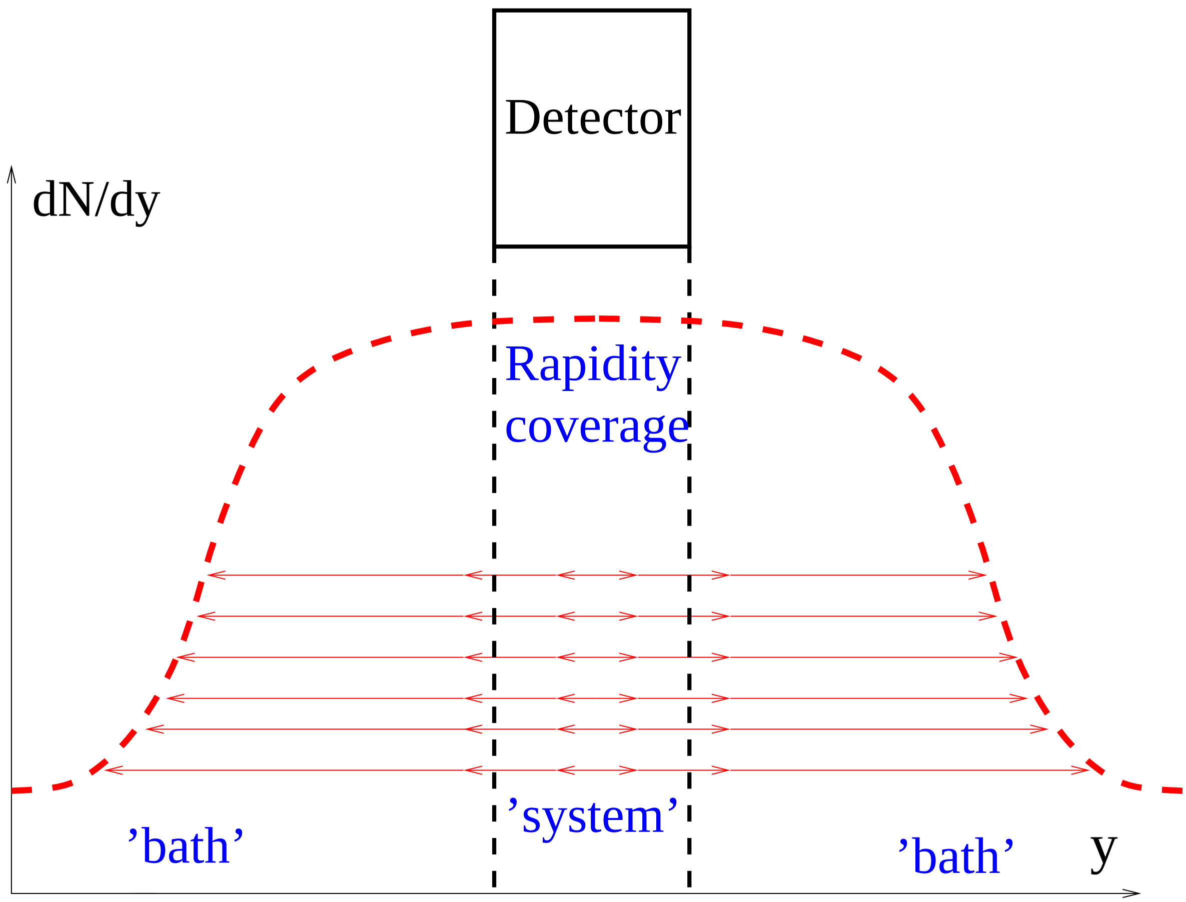
$$\delta_E \to \delta\left(E_T - < E >\right) \qquad \omega_E \sim 1$$

Grand Canonical : Charge conserved on average

$$\delta_Q \to \delta\left(Q_T - < Q >\right) \qquad \omega_E \sim \omega_Q \sim 1$$

Appropriate for detector sampling part of a fluid

Freeze−out from ideal fluid at mid−rapidity



Boost invariance: Rapidity ⇔configuration space

- Mid-rapidity ⇔system
- Peripheral regions ⇔bath

⇒ Grand Canonical ensemble needs to be used!

Cleymans, Redlich, PRC 60, 054908 (1999):

$$\left[\frac{dN}{dy}\right]_{b.i.} \sim < N >_{4\pi} \quad \left[\frac{d(\Delta N)^2}{dy}\right]_{b.i.} \sim (\Delta N)^2_{4\pi}$$

- All details of flow and freeze-out <u>integrate out</u>

- Up to Normalization, $< N >, \omega$ calculable from Grand Canonical T, λ_i

$$\left.\begin{array}{l} Ideal \quad hydro \\ Fast \quad freezeout \end{array}\right\} \begin{array}{l} Statistical \quad model \quad \underline{fits \quad well} \\ < N > \quad \underline{AND} \quad < N^2 > - < N >^2 \end{array}$$

<u>Clusters etc. $\Rightarrow$<u>deviation</u> from hydro.</u>

So lets see how the statistical model does!
But <u>which one</u>?

Grand canonical statistical hadronization

All particles described in terms of T and $\lambda_{q,s,I3}$.
Detailed balance: $\lambda_{\overline{q}} = \lambda_q^{-1}$Integral can be done in rest-frame wrt flow using Bessel function K_2

$$N_i = V' \sum_{n=1}^{\infty} (\mp 1)^{n+1} \frac{\lambda_i^n}{n} m_i^2 T K_2 \left(\frac{n m_i}{T} \right)$$

$$\Delta N_i^2 = V' \sum_{n=1}^{\infty} (\mp 1)^{n+1} \frac{\lambda_i^n}{n} \left(\begin{array}{c} 2+n-1 \\ n \end{array} \right) m_i^2 T K_2 \left(\frac{n m_i}{T} \right)$$

$$V' = V(2J_i + 1) \frac{4\pi}{(2\pi)^3}$$

Chemical potentials for conserved quantities

$$\lambda_i = \lambda_u^{u-\overline{u}} \lambda_d^{u-\overline{u}} \lambda_s^{s-\overline{s}}$$

Non-equilibrium through phase space occupancies

$$\lambda_i \rightarrow \lambda_i^{\mathrm{eq}} \gamma_u^{u+\overline{u}} \gamma_d^{u+\overline{u}} \gamma_s^{s+\overline{s}} \qquad \gamma^{\mathrm{eq}} = 1$$

Resonance feed-down

$$N_i = N_i^{direct} + \sum_j b_{j\to i} N_j$$

$$\Delta N_i^2 = \Delta N_i^2 + \sum_j \left[b_{j\to i} \left(1 - b_{j\to i}\right) N_j + b_{j\to i}^2 \Delta N_j^2) \right]$$

Widths

Decay $M \to m_1, m_2, ...$ width Γ_i, total width Γ_T
relative angular momentum l, threshold mass M_{th}

$$N(M,\lambda) \to \frac{\int_{M_{Th}}^{\infty} \rho(m) N(m,T,\lambda) dm}{\int_{M_{Th}}^{\infty} \rho(m) dm}$$

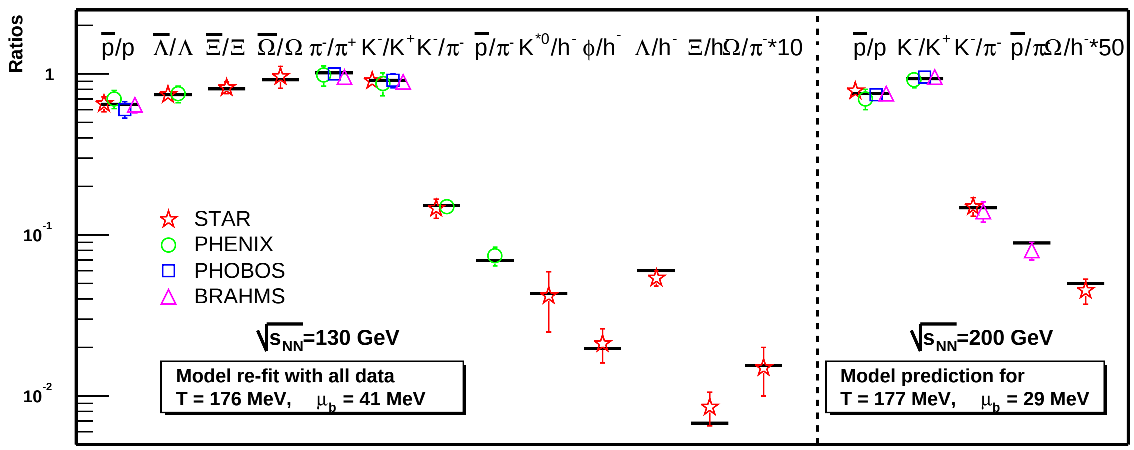
Where

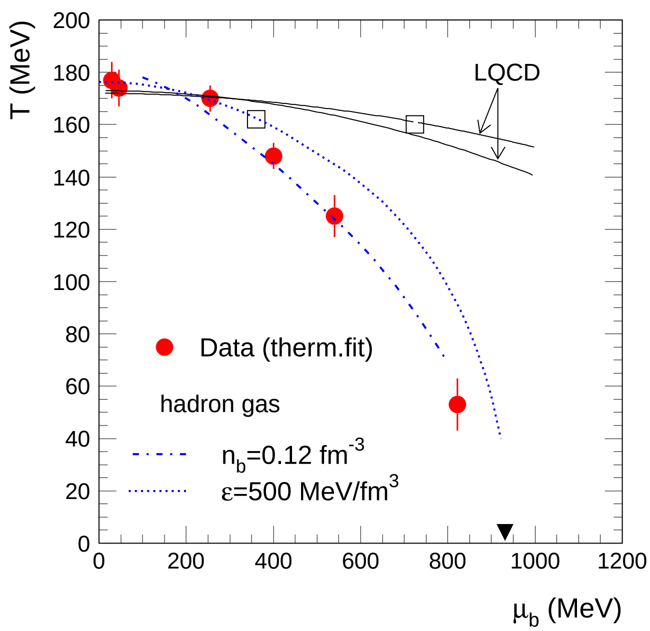
$$\rho(m) = \frac{\Gamma_T \Gamma(m)}{(M-m)^2 + \Gamma^2(m)/4}$$

$$\Gamma(m) = \Gamma_i \left[1 - \left(\frac{M_{Th}}{m} \right) \right]^{l/2}$$

Model I:Equilibrium statistical mechanics

(Braun-Munzinger,Magestro,Florkowski,Broniowski,Redlich,...)

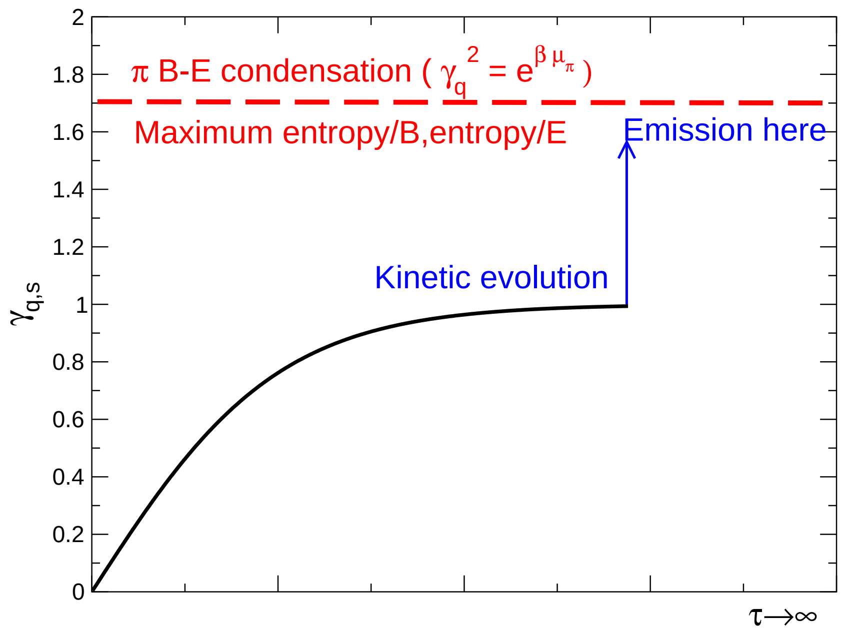




$$\frac{dT}{d\sqrt{s}} > 0 \qquad \frac{d\mu_B}{d\sqrt{s}} < 0 \qquad \frac{E}{V} \sim 1GeV$$

Resonances not well described (Rescattering?)

Alternatively... Supercooling+oversaturation
(chemical Non-equilibrium but statistical emission)



inaccessible by kinetic evolution
(if put "in a box", this system would heat)
but accessible in a fast phase transition
from a high entropy phase $\gamma_q > 1, \gamma_s/\gamma_q > 1$.

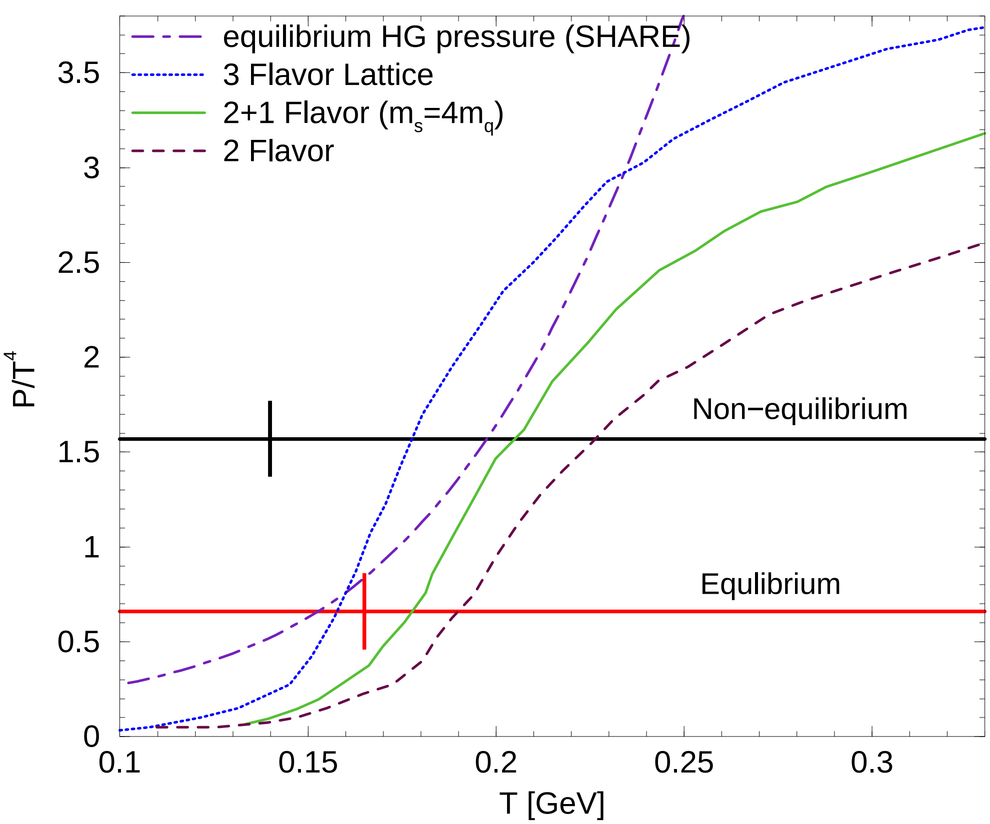
- Small $\lambda_{eq} \rightarrow \gamma$ dominates $\Leftrightarrow$ Statistical coalescence!

- Enthropy survives coalescence by $\gamma > 1$

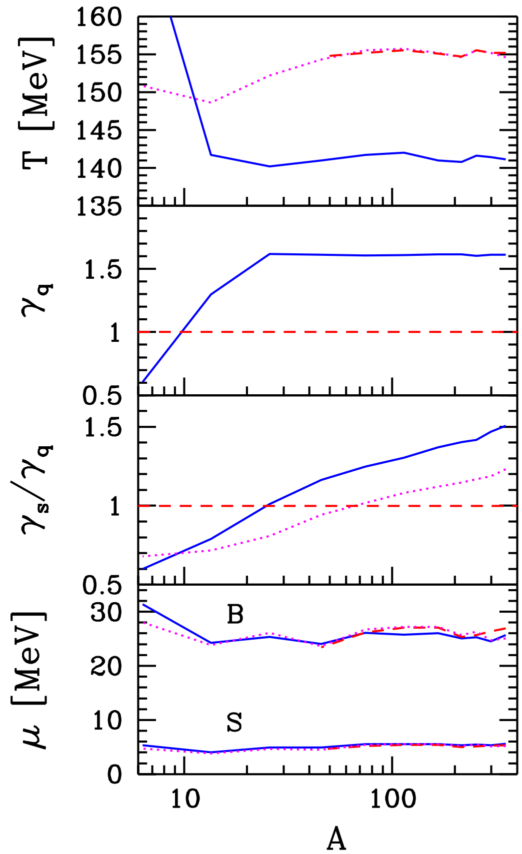
J. Rafelski, J Letessier, PRL 85:4695-4698,2000:
Explosive hadronization from supercooled QGP

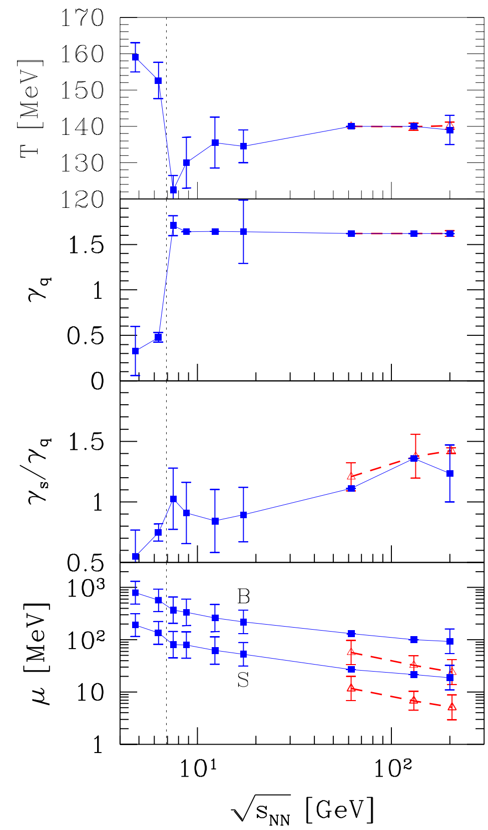
$$P_{vacuum} = P_{QGP} \qquad S_{HG} = S_{QGP} \qquad V \sim \frac{2}{3} V_{equilibrium}$$

$$T = 140 MeV, \qquad \gamma_q \sim 0.9 e^{m_\pi/2T} \sim 1.6$$

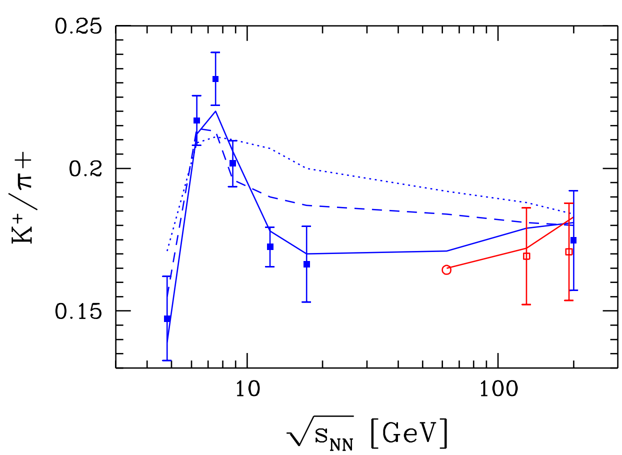


$\gamma_q > 1, T \rightarrow \sim 140 MeV$ @ critical $\sqrt{s}, N_{part}$



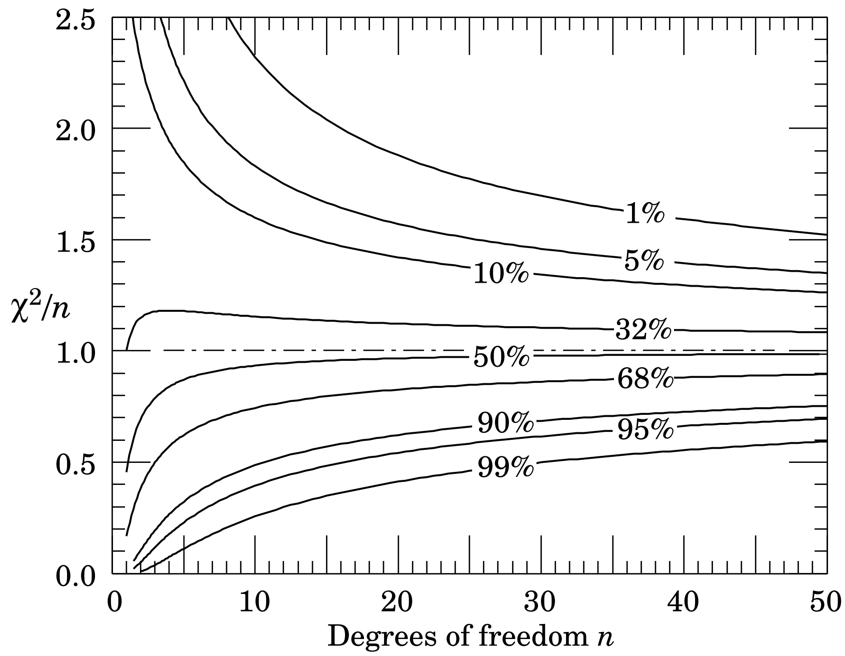


"Horn" explained by $\sqrt{s}$ dependance of T, γ_q



Smoking gun 4 deconfinement?? Perhaps...

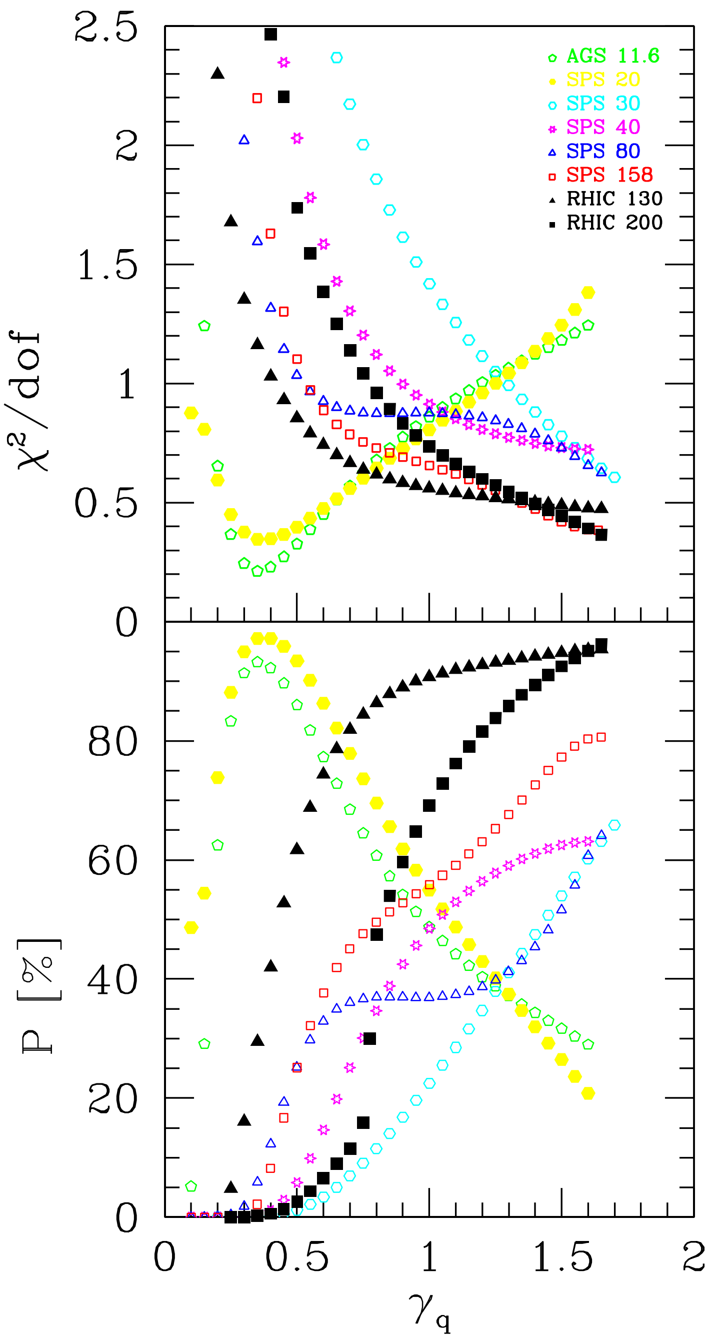
Equilibrium and non-equilibrium models have a different number of parameters.
Comparison standard: Statistical significance



- Statistical significance, the probability of getting χ^2 with n DoF given that "your model is true", is a quantitative measure of your fit's goodness

- With few DoF, "nice" looking graphs can have a very small statistical significance.

- It is said that you can fit an elephant with enough parameters. Maybe so, but if you are honest, you won't get a good statistical significance.

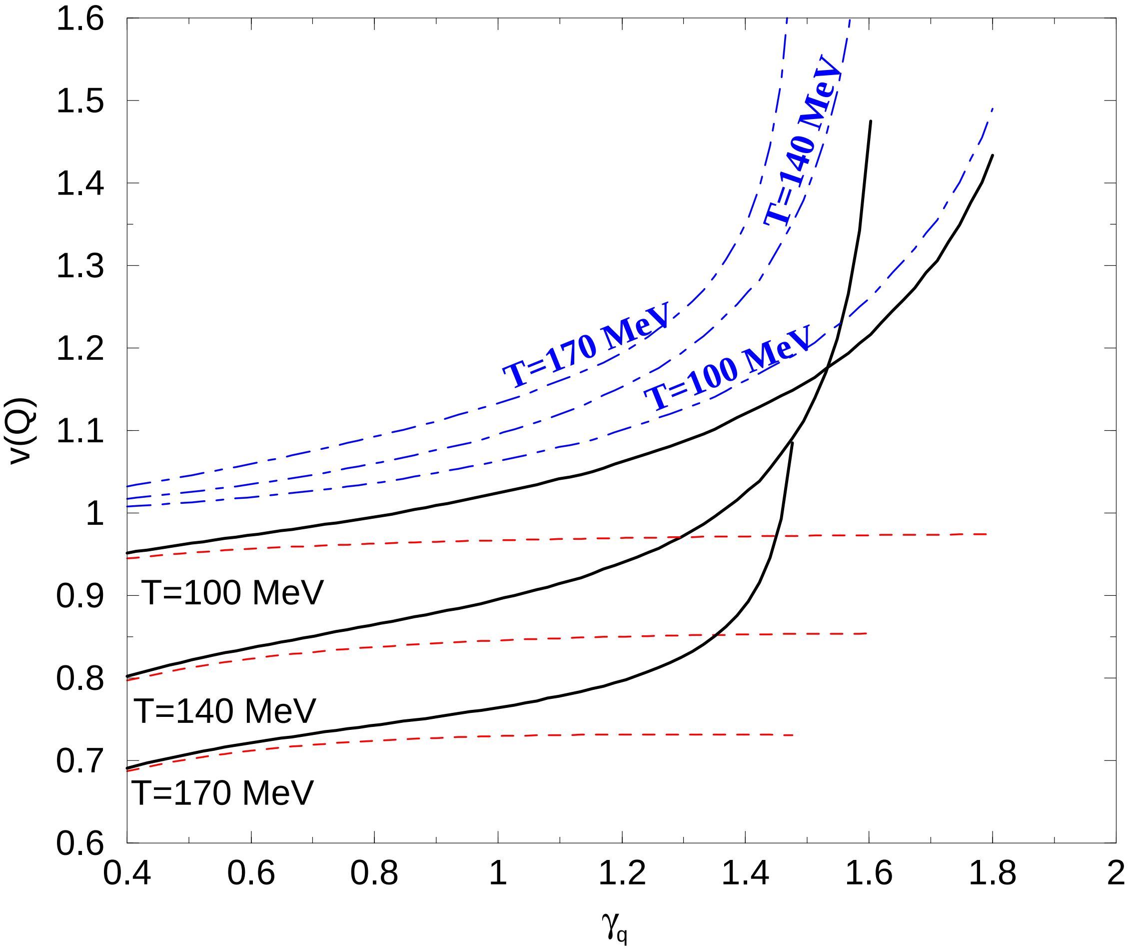


Maximum for SPS and RHIC is at $\gamma_q > 1$, but equilibrium not ruled out!

Need further data capable of determining γ_q.

fluctuations!

Yields <u>and</u> fluctuations: Non-equilibrium



T increase $\Rightarrow$ π Fluctuations <u>decrease</u> because of enhanced resonance production

over-saturation ($\gamma_q > 1$) $\Rightarrow$ π Fluctuations <u>increase</u> because of BE corrections

$\gamma_q > 1$, <u>unlike</u> resonances (detectable,rescattered,in-medium modified,...) affects fluctuations rather than correlations

Yields and fluctuations: Reinteraction (or not)

Consider $Y^* \longrightarrow Y\pi$

$\sigma_{Y/\pi}$ probes correlation of Y and π from Y^* at chemical freeze-out.
(further rescattering/regeneration does not change the correlation.

Y^*/Y **yield** probes Y^*at thermal freeze-out (after all rescattering.

So...

- If can fit stable particles and resonances and fluctuations in same fit $\rightarrow$ no reinteraction
- If Stable particles$+$ Fluctuations fit gives wrong value for resonances $\rightarrow$ magnitude of reinteraction

Suitable:

Yields Independent of γ_s, λ_s,volume

- Λ/K^-, better
  - Λ corrected for $\Xi, \Omega \rightarrow \Lambda$
  - K^- corrected for $\phi \rightarrow K^- K^+$
- Ξ/ϕ

Fluctuations $v(Q)$,σ_{π^+/π^-}

$$v(Q) = \frac{\langle Q^2 \rangle - \langle Q \rangle^2}{N_{ch}}$$

$$\sigma_{N_1/N_2} = \frac{\omega_{N_1}}{\langle N_1 \rangle} + \frac{\omega_{N_2}}{\langle N_2 \rangle} - 2\frac{\langle N_1 N_2 \rangle - \langle N_1 \rangle \langle N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle}$$

σ_{π^+/π^-}can also be used to probe ρ, f_0 mass modificationdue to $\pi^+ - \pi^-$ correlations

undetectable or rescattered resonances also contribute to fluctuations!!!

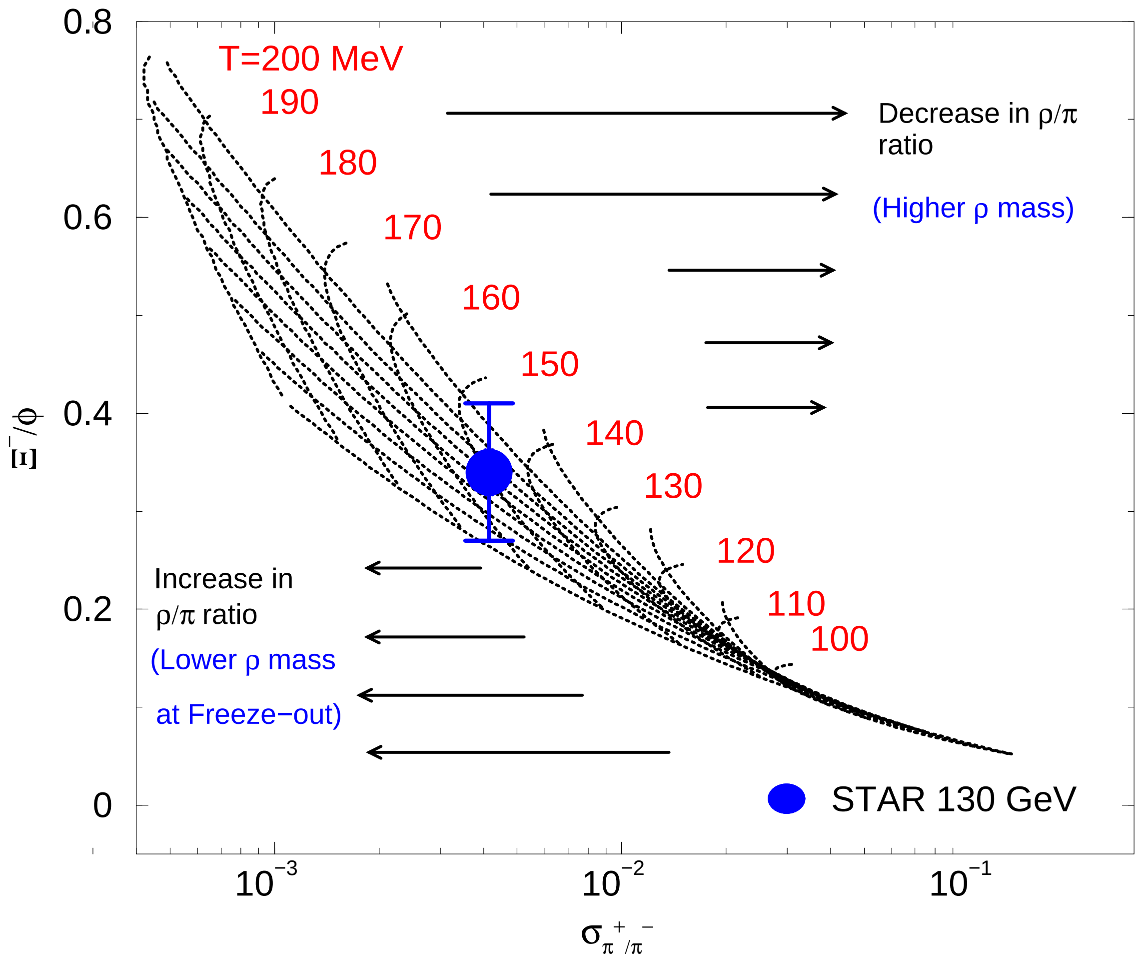
Volume fluctuations are not well understood, and show up in all $< N^2 > - < N >^2$. Avoid them choosing observables such as

- $(\Delta Q)^2$. $\frac{<Q>}{V}$small, so is $\Delta V \frac{<Q>}{V}$ (Jeon, Koch)

- Fluctuations of ratios(Jeon, Koch)
  Volume fluctuations irrelevant to 1st order

- For most other data-points can fit ΔV, $(\Delta N)^2 = V(\Delta\rho)^2 + \left[\Delta V < N >\right]^2$

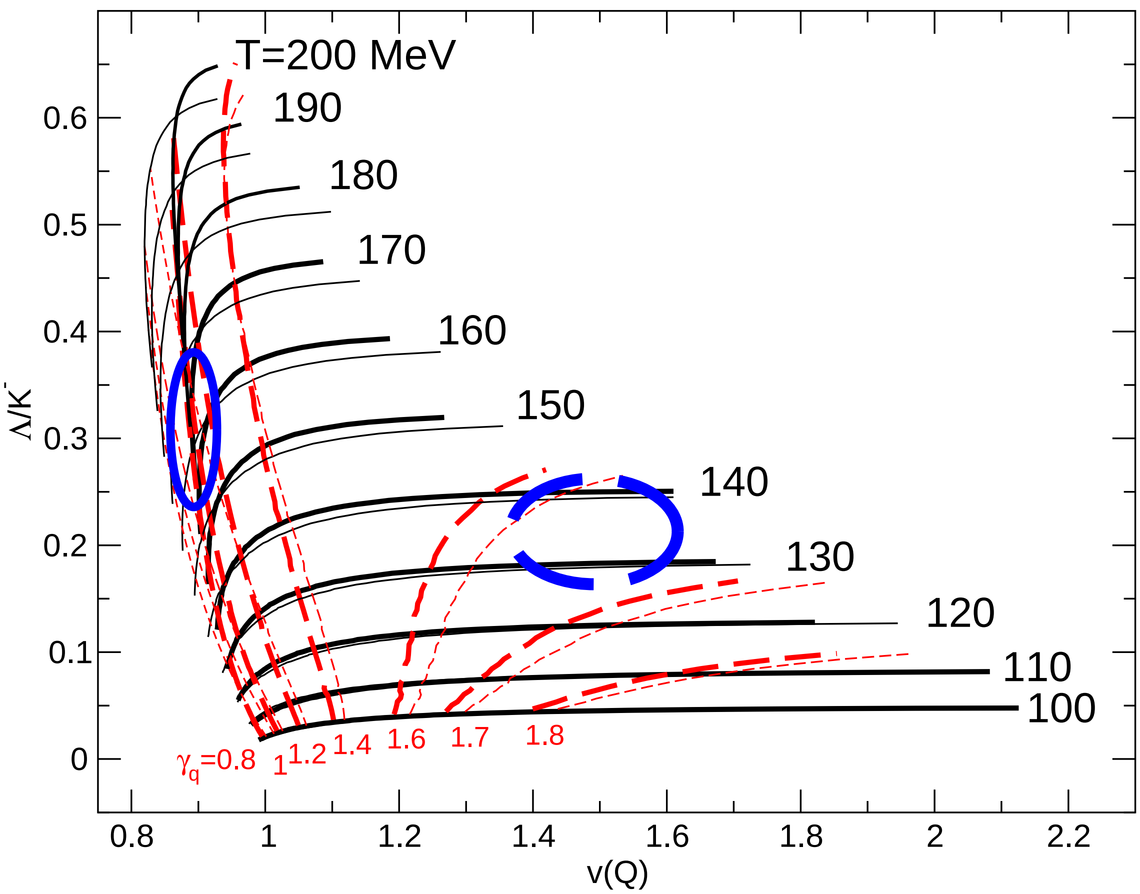
σ_{π^+/π^-} vs Ξ^-/ϕ



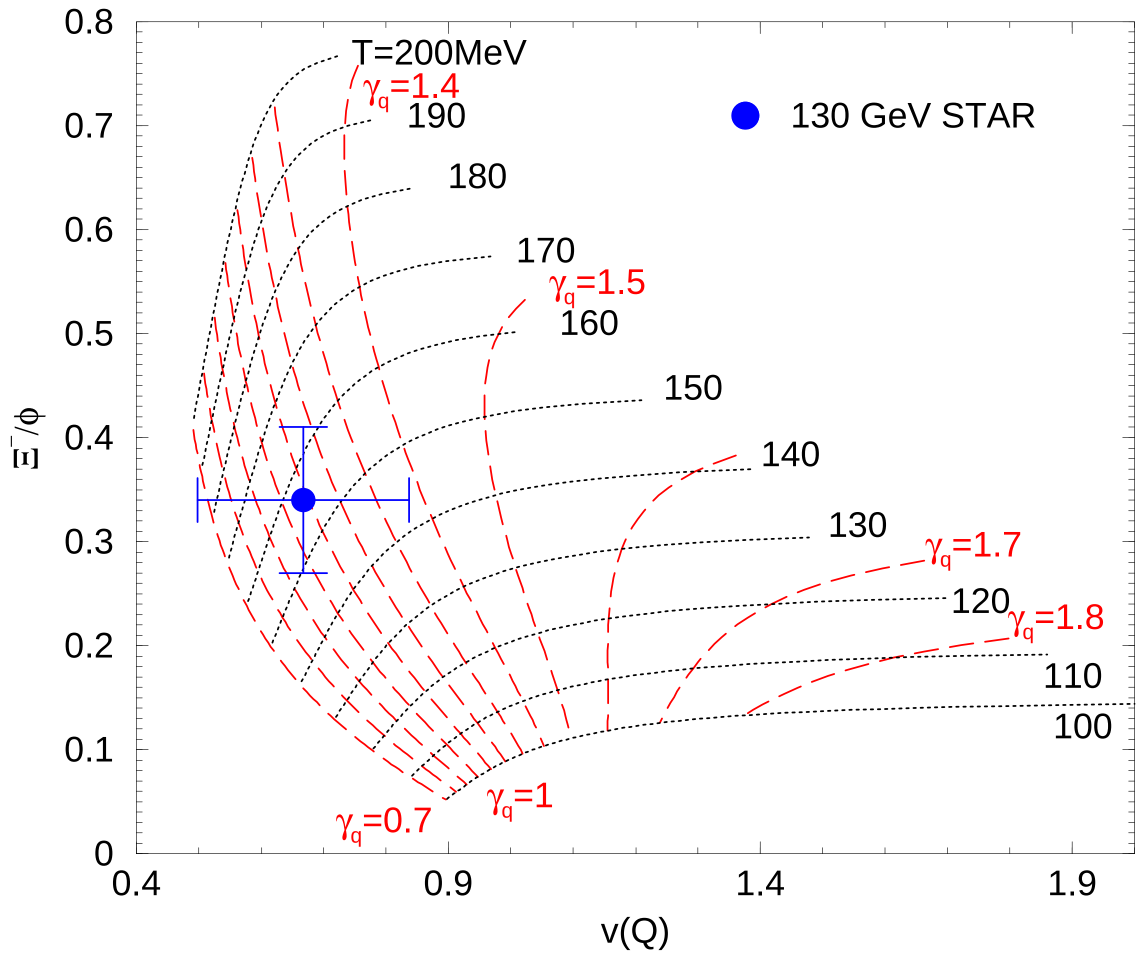
- σ_{π^+/π^-} probes T independently of γ_q
- Allowed region narrow $\rightarrow$ mass modification probe

130 GeV within band! Good agreement with non-equilibrium No freeze-out ρ, σ modification needed

v(Q) vs Λ/K^-



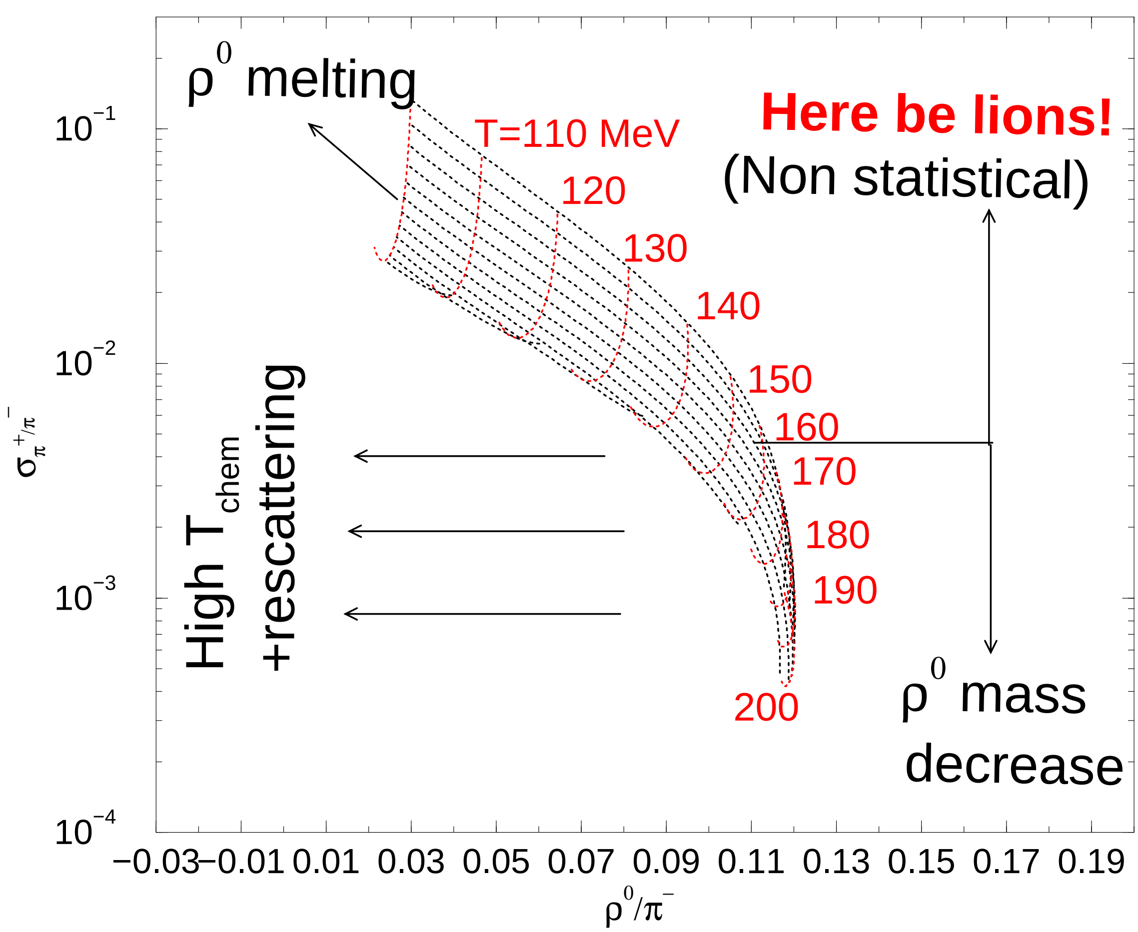
v(Q)vs Ξ^-/ϕ



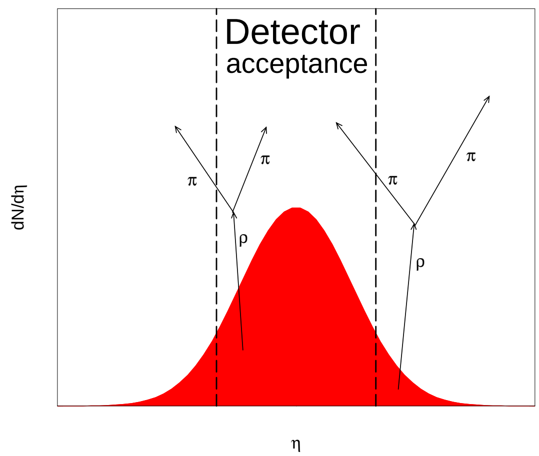
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.4
0.9
1.4
1.9
v(Q)
Ξ⁻/φ
T=200MeV
γq=1.4
190
180
170
γq=1.5
160
150
140
130
γq=1.7
120
γq=1.8
110
100
γq=1
γq=0.7
130 GeV STAR

σ_{π^+/π^-}vs ρ^0/π^-

Probes (lack of?) reinteraction <u>and</u> mass modification separately



These diagrams are made with static fluctuations susceptible to detector response effects



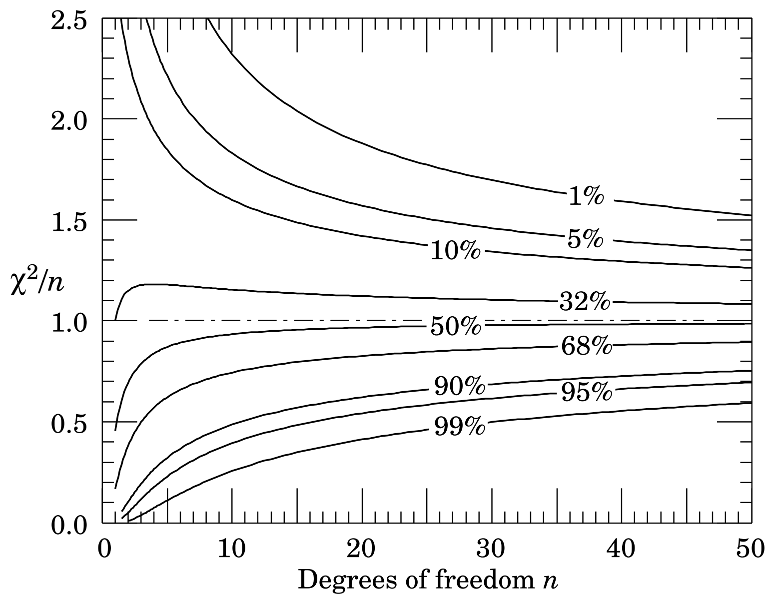
use dynamical fluctuations $\sigma_{dyn} = \sigma - \sigma_{stat}$ Where $\sigma_{stat} \sim \frac{1}{<N_1>} + \frac{1}{<N_2>}$ obtained by mixed event technique

σ_{dyn} robust against detector acceptance but needs more parameters ("volume") to be described $\Rightarrow$ no diagrams. Can use it in fit, including one/more yields at same centrality as σ_{dyn} .

For large acceptance, can hope fitting both σ_{dyn} and $\sigma, \omega, v(Q)$

Fit exp. yields, ratios, ω, σ, $< s >= 0, \frac{Q}{B} = 0.4$ for

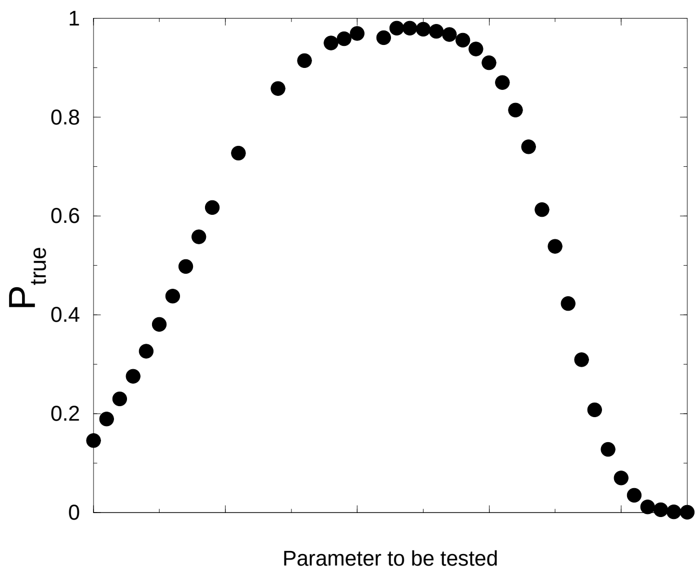
- Equilibrium parameters T, λ_{q,s,I_3}
- Non-equilibrium parameters γ_{q,s,I_3}
- System "volume" dV/dy

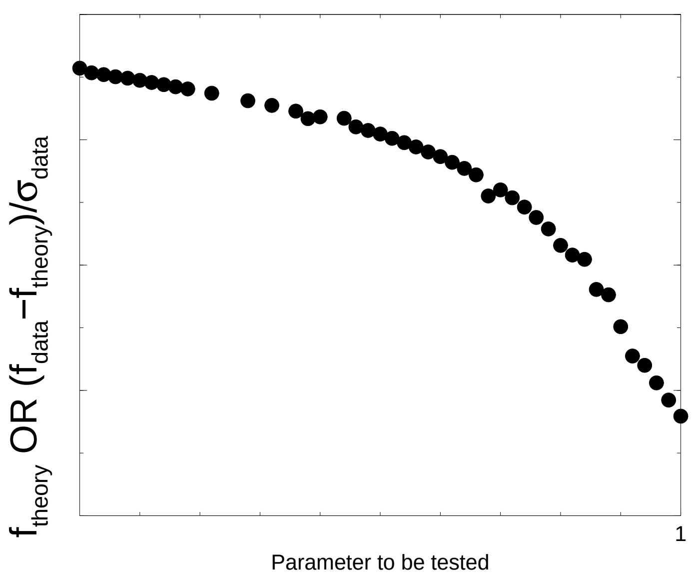


Different models have a different DoF $\longrightarrow$ Use Statistical significance (P_{true}) to judge fit quality

Non-trivial correlations/data-point sensitivity can be analyzed by Profiles in statistical significance

All other parameters at their best fit value for point in abscissa





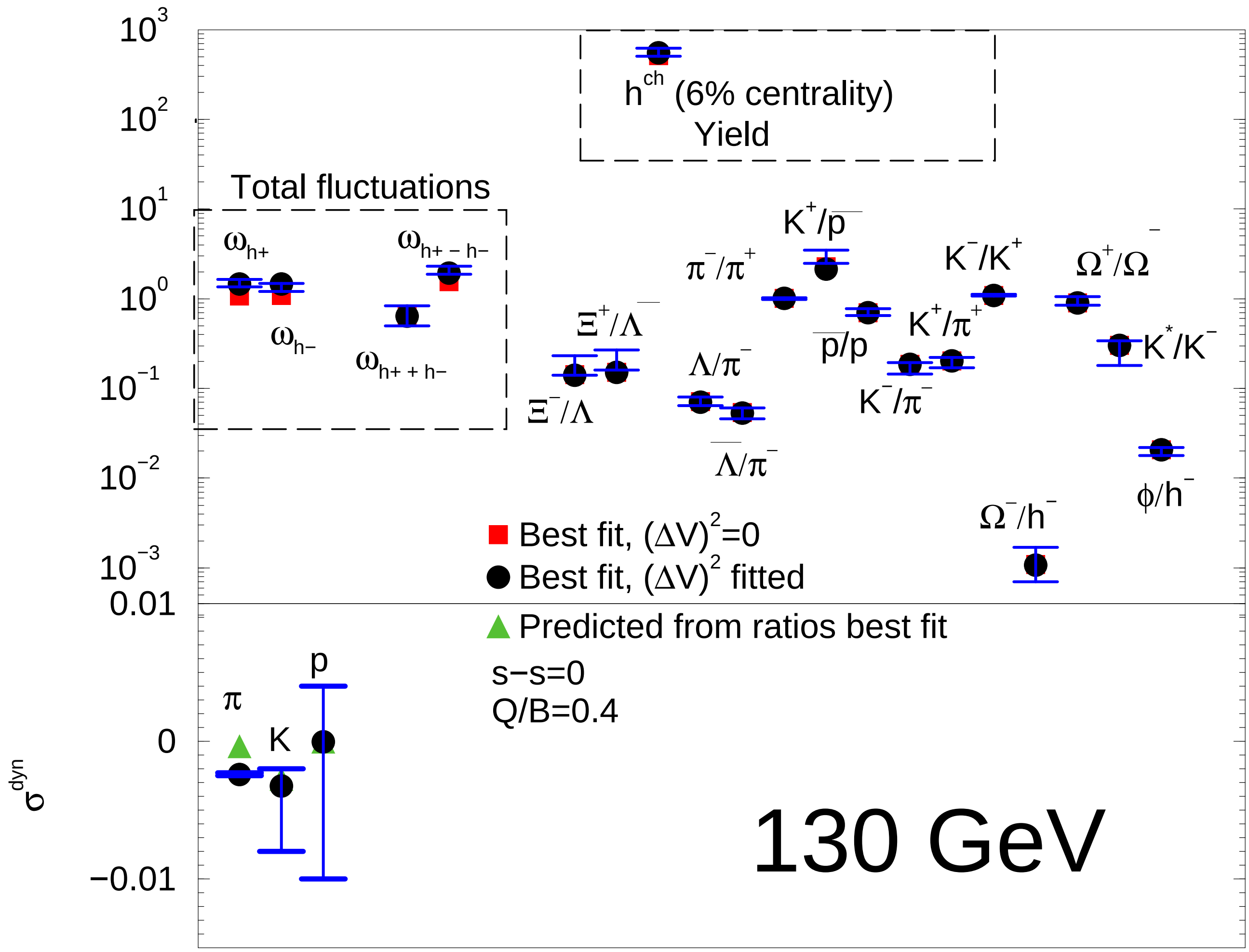
Fits at 130 GeV

fluctuations

- $\sigma^{dyn}_{\pi^+/\pi^-}, \sigma^{dyn}_{K^+/K^-}, \sigma^{dyn}_{\overline{p}/p}$: STAR
  PRC 68, 044905 (2003), nucl-ex/0307007
  Insensitive to Volume fluctuations but require $< V >$ (6% centrality)
- $\omega_{h^+}, \omega_{h^-}, \omega_{h^+-h^-}, \omega_{h^-+h^+}$ STAR
  Nucl. Phys. A 698 (2002) 611
  Need $\Delta V/V$, independent of centrality.
  No error bar, assume Ad Hoc 10%

Yields and Ratios

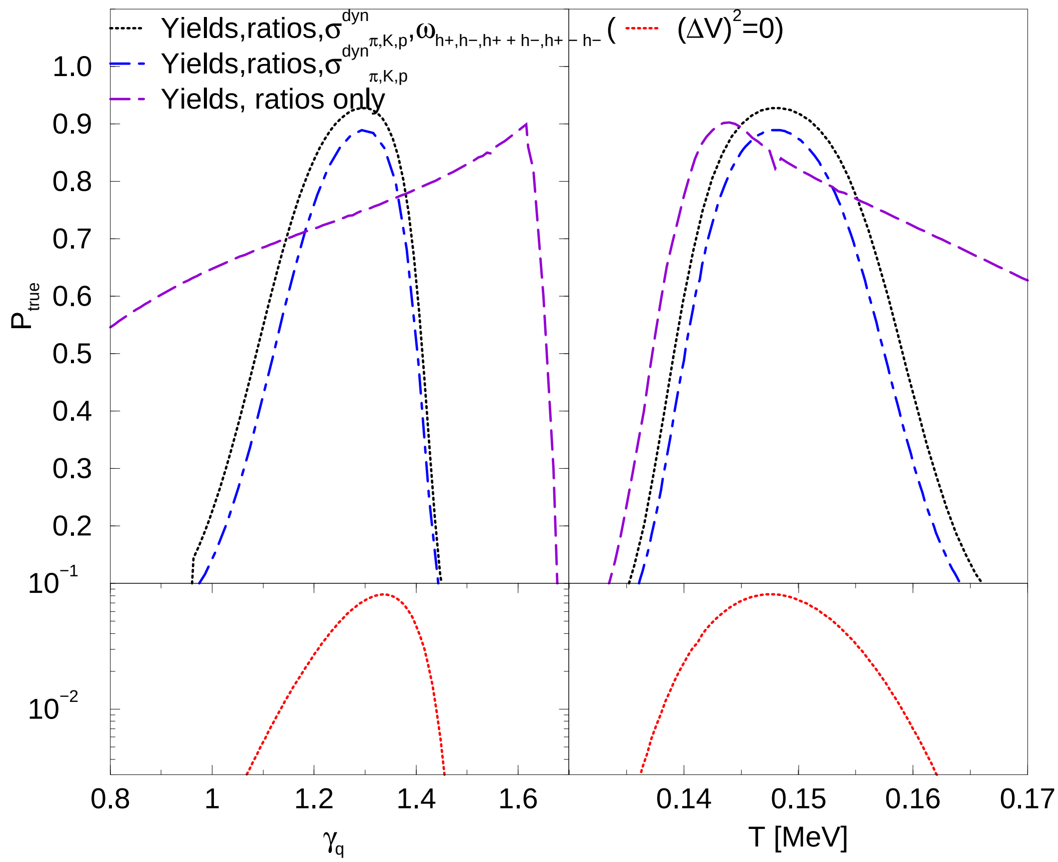
- $h^{charged}$, PHOBOS
  PRL (2003) , nucl-ex/0201005 (6% centrality)
- $\pi, K, p, \Lambda, \Xi, \phi, K^*$ and antiparticles, STAR,
  various (5% centrality)



- When volume fluctuation is used as a fit parameter, both yields and fluctuations fit nicely

- Volume fluctuations needed 4 $\omega_{h^+,h^-,h+h^-}$,not4 $\sigma^{dyn}$and $\omega_{h^+-h^-}$(as understood by Koch,Jeon).

- Fluctuations shift fitted γ_q, T w.r.t. ratios-only fit



T, γ_q not as well determined as it could be

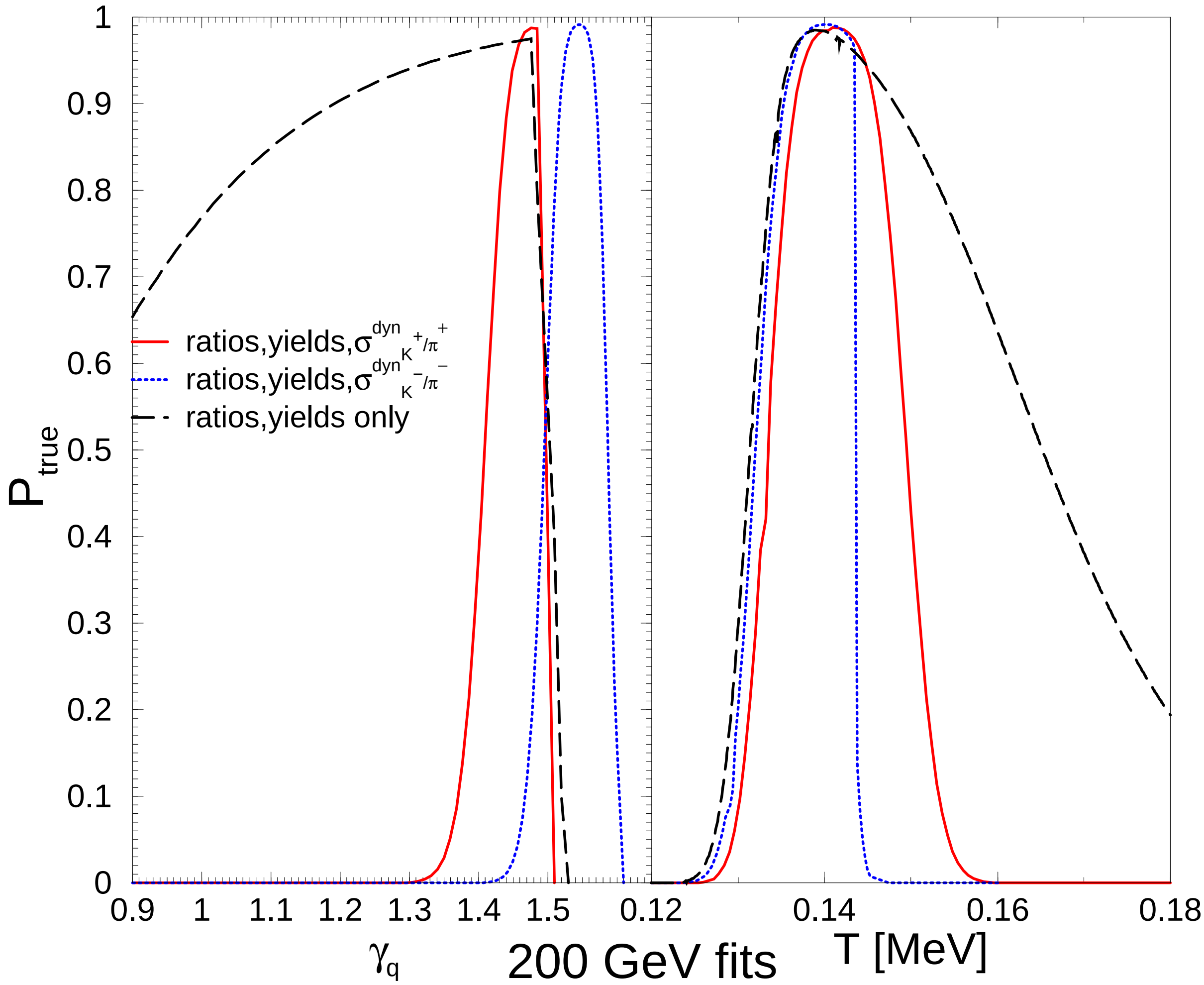
- As we saw earlier, $\sigma^{dyn}_{\pi,K,p}$ are not very good constraines on γ_q
- Volume fluctuation essential for fitting $\omega_{h^+}, \omega_{h^-}, \omega_{h^+ \pm h^-}$(if $\Delta V = 0$, $P_{true} < 0.1$ due to disagreement with these data points), but correlates with normalization, γ_q.

Fits at 200 GeV

- $\sigma^{dyn}_{K/\pi}$: Supriya Das et al [STAR] nucl-ex/0503023

- Ratios:O. Barannikova et al [STAR] nucl-ex/0403014

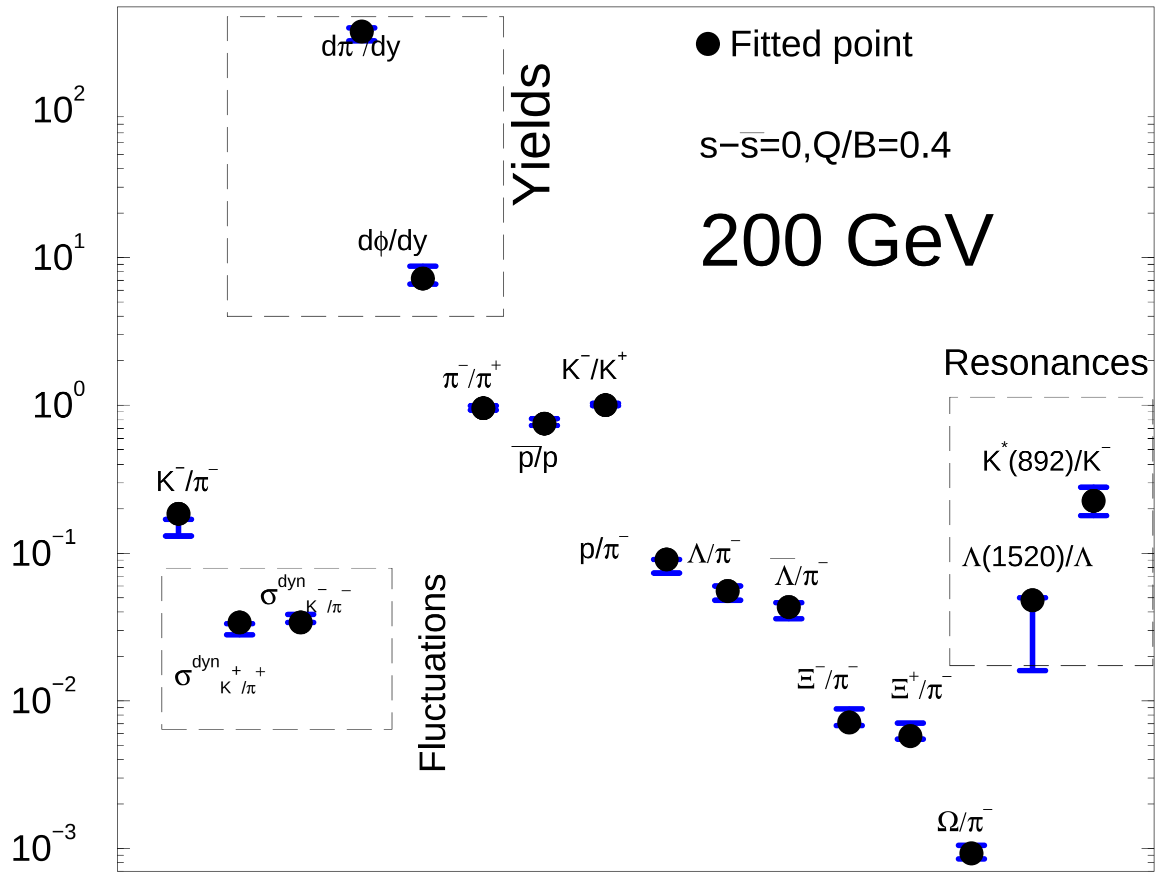
NB: All preliminary



200 GeV fits

With fluctuations, T, γ_q determined
($K/\pi+$ its fluctuation is very sensitive to γ_q).

- firmly in $\gamma_q > 1, T \sim 140$ MeV,
- T, γ_q consistent with 130 GeV considering all preliminary



Some datapoints fail but no exp. systematic errors yet!

σ_{K^+/π^+} vs σ_{K^-/π^-}:	too different
$\overline{\Omega}/\Omega > 1$	not thermal: Bleicher,Liu,Aichelin

If these fitted, best fit unchanged but $P_{true} < 0.1$

Some tentative conclusions

- Ξ/ϕ vs $v(Q), \sigma_{\pi^+/\pi^-}$ fits SHM well, no indication of ρ, σ modification

- Yields, ratios, resonances and fluctuations fit reasonably well provided $\gamma_q > 1$

- very unlikely equilibrium SHM can do it

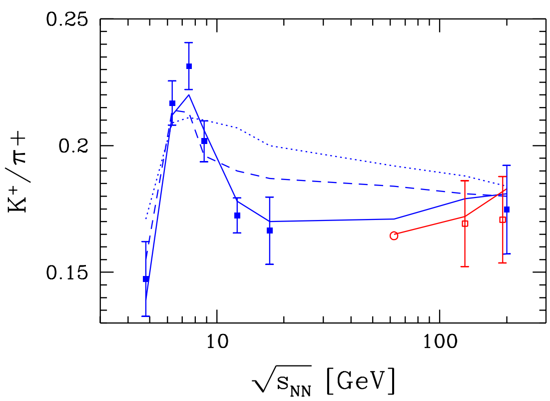
- $\Lambda(1520), K^*$ well accounted for in non-equilibrium. $\sigma_{K/\pi}$ also fits well. Consistent with sudden freeze-out!

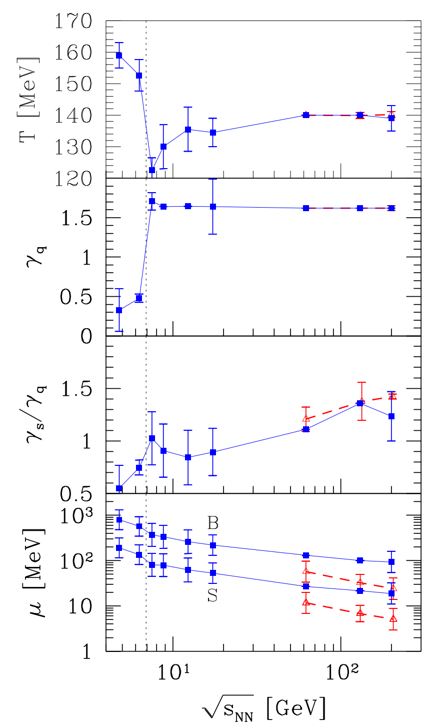
Do it yourself: SHARE

http://www.physics.arizona.edu/~torrieri/SHARE/share.html

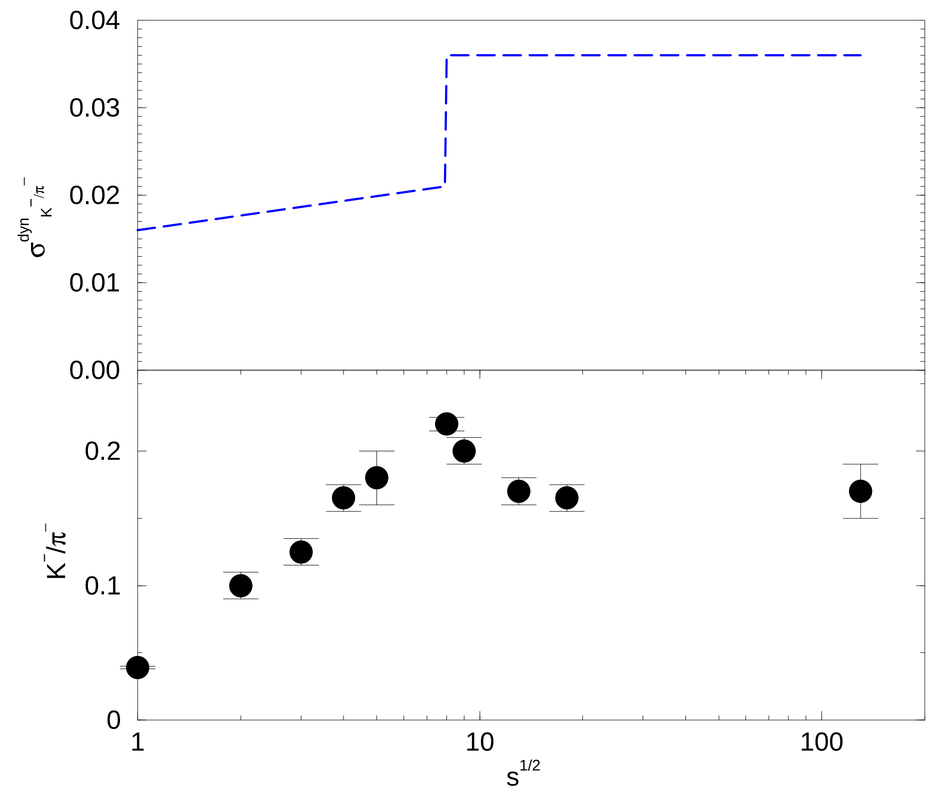
A prediction...

It was recently shown (nucl-th/0504028) that the "horn" in the K^-/π^- yield





Can be explained by the onset of γ_q (sudden freeze-out from a Partonic plasma) at the "horn" energy.



Fluctuations in K^-/π^- should <u>also</u> rise sharply at the horn point.