Saturation with Dipole Correlations

Robi Peschanski[a]
(SPhT, Saclay)

Workshop on Low x Physics, Prague, September 15-19

- Saturation: Hierarchy of Non-Linear Equations:

  Dipole Reduction

- One Step beyond BK:

  Saturation Equations including 2-Dipole-Correlations

- A Factorized Solution:

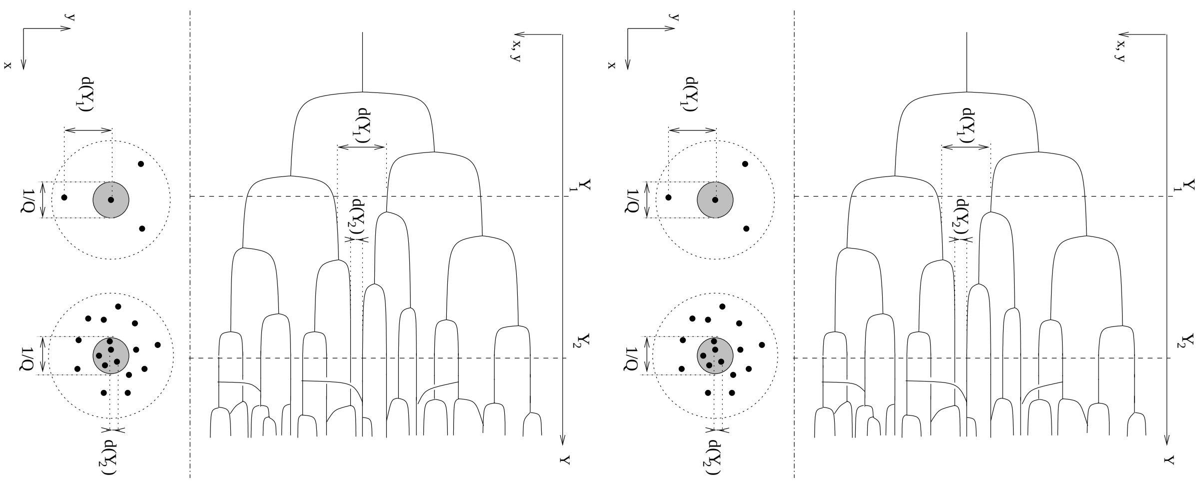
  One Step beyond

  ...and beyond One Step (R.Janik)

[a]with Romuald Janik (Theoretical Physics, Cracow):ph/0407007

Saturation: BK and Beyond

Without Correlations: BK



$d(Y) \to 0 =$ Non-Linear Density effects

Exponential regime: BFKL

Independent Probes: BK Equation

Non-Independent probes: n-Dipole Correlations ?

The Balitskii-Kovchegov Equation

(Balitsky's Framework)

- **The Dipole Operator:**

$$\mathcal{D}_{ij} = \mathrm{tr}\, U_{x_i} U^{\dagger}_{x_j} \ ,$$

$x_{i,j}$: Dipôle Coordinates

U_{x_i} : Wilson Lines

- **The Non-Linear Equation for $\mathcal{D}$:**

$$\frac{\partial}{\partial Y} \langle \mathcal{D}_{01} \rangle = \frac{g^2}{8\pi^3} \int d^2 x_2 \left[\langle \mathcal{D}_{02} \mathcal{D}_{21} \rangle - N_c \langle \mathcal{D}_{01} \rangle \right] K_{021}$$

- **The BK Equation:**

$$\langle \mathcal{D}_{02} \mathcal{D}_{21} \rangle - \langle \mathcal{D}_{02} \rangle \langle \mathcal{D}_{21} \rangle \equiv d_{0221} = 0$$

Equation valid for uncorrelated probes

- Beyond BK: Can we derive d_{0221}?

Dipole Reduction

Truncated Equation for

$$\langle \mathcal{D}_{02}\mathcal{D}_{2'1}\rangle - \langle \mathcal{D}_{02}\rangle \langle \mathcal{D}_{2'1}\rangle = d_{022'1} = N_c^2 \ N_{022'1}$$

- **Neglecting** $\left\langle \mathrm{tr}\ U^6 \right\rangle$ **and** $\left\langle \mathrm{tr}\ U^4 \right\rangle$

$$\frac{\partial}{\partial Y} \langle \mathcal{D}_{02}\mathcal{D}_{2'1}\rangle = \frac{g^2}{8\pi^3} \int d^2x_3 \left[\langle \mathcal{D}_{03}\mathcal{D}_{32}\mathcal{D}_{2'1}\rangle - N_c \langle \mathcal{D}_{02}\mathcal{D}_{2'1}\rangle \right] K_{032} + [..] K_{132'}$$

- **Keeping 2-dipole Correlations**

$$\langle \mathcal{D}_{03}\mathcal{D}_{32}\mathcal{D}_{2'1}\rangle = d_{03}d_{32}d_{2'1} + d_{0332}d_{2'1} + d_{322'1}d_{03} + d_{032'1}d_{32}$$

- **New Equations**

$$\frac{\partial}{\partial Y} N_{022'1} = \frac{g^2 N_c}{8\pi^3} \int d^2x_3 \left\{ \left[N_{322'1} + N_{032'1} - N_{022'1} \right] K_{032} + [3 \ terms ... N\right.$$

$$\frac{\partial}{\partial Y} N_{01} = \frac{g^2 N_c}{8\pi^3} \int d^2x_2 \left[N_{02} + N_{21} - N_{01} - N_{02}N_{21} - N_{0221} \right] K_{021} \ .$$

One Step beyond BK

Study of the Coupled System

- **Fourier Transform to Momenta**

$$N_{01} = x_{01}^2 \int \frac{d^2k}{2\pi}\, e^{ik\cdot x_{01}}\, N_k$$

$$N_{022'1} = x_{02}^2 x_{2'1}^2 \int \frac{d^2q}{2\pi}\frac{d^2q'}{2\pi} d^2Q\, e^{iq\cdot x_{02}}\, e^{iq'\cdot x_{2'1}}\, e^{iQ\cdot(x_{02'}+x_{21})}\, N_{qq',Q}$$

- **Equations in Fourier Space**

$$\frac{\partial}{\partial Y} N_q = \frac{g^2 N_c}{4\pi^2}\left\{2\chi(-\partial_L)N_q - N_q^2 - \int d^2Q\, N_{(q-Q)(q-Q),Q}\right\}$$

$$\begin{aligned}\frac{\partial}{\partial Y} N_{qq',Q} = \frac{g^2 N_c}{4\pi^2}\, &\{2\chi(-\partial_L)N_{qq',Q} + 2\chi(-\partial_{L'})N_{qq',Q} \\ &- (N_{q+Q} + N_{q-Q} + N_{q'+Q} + N_{q'-Q})N_{qq',Q}\}\end{aligned}$$

- **If:** $N_{qq',Q} = N_{qq'}\delta^2(Q)$

$$\frac{\partial}{\partial Y} N_q = \frac{g^2 N_c}{4\pi^2}\left\{2\chi(-\partial_L)N_q - N_q^2 - N_{qq}\right\}$$

$$\frac{\partial}{\partial Y} N_{qq'} = \frac{g^2 N_c}{4\pi^2}\left\{2\chi(-\partial_L)N_{qq'} + 2\chi(-\partial_{L'})N_{qq'} - 2(N_q + N_{q'})N_{qq'}\right\}$$

A Factorized Solution

Ansatz: $N_{qq'} = \lambda N_q N_{q'}$

- **Two-Step System** $\lambda = 1$

$$\begin{aligned} N_{qq'} &= N_q N_{q'} \\ \frac{\partial}{\partial Y} N_q &= \frac{g^2 N_c}{4\pi^2} \left\{ 2\chi(-\partial_L) N_q - 2N_q^2 \right\} \end{aligned}$$

- **Back in Position Space**

$$\begin{aligned} N_{022'1} &= N_{02} N_{2'1} \\ \frac{\partial}{\partial Y} N_{01} &= \frac{g^2 N_c}{8\pi^3} \int d^2x_2 \left[N_{02} + N_{21} - N_{01} - 2N_{02}N_{21} \right] K_{021} \end{aligned}$$

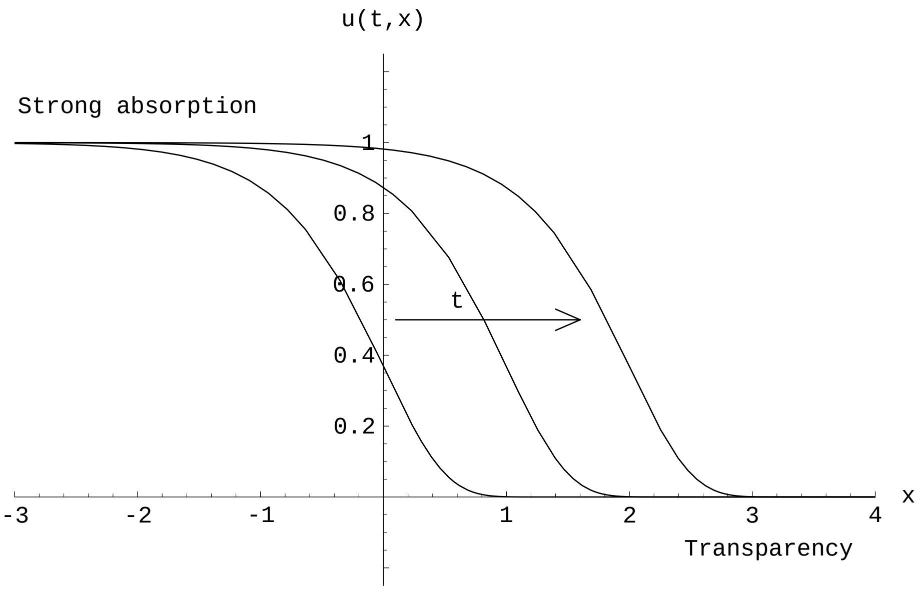
- **Two-Step Solution:**

$$N_{01} = \frac{1}{2} N_{01}^{BK}$$

$$\frac{\partial}{\partial Y} S_{01} = \frac{g^2 N_c}{8\pi^3} \int d^2x_2 \left[2(S_{02} - 1/2)(S_{21} - 1/2) - (S_{01} - 1/2) \right] K_{021}$$

Traveling wave Solutions

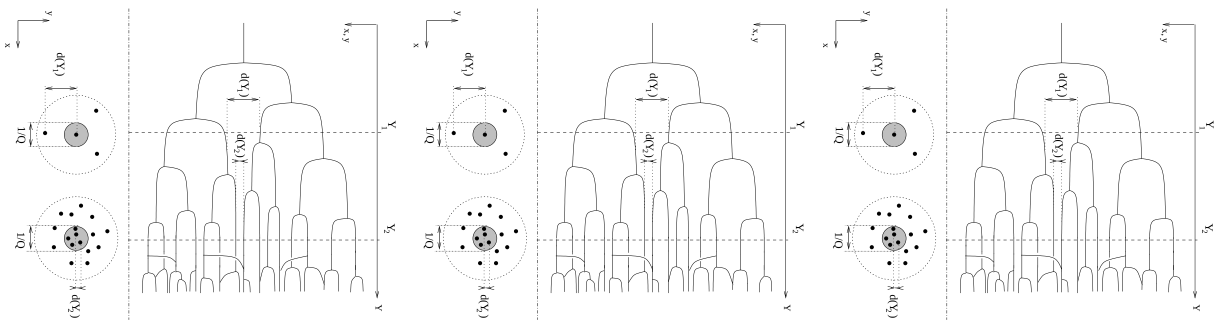
$$u(t,x)^{two-step} \to \frac{1}{2}u(t,x)^{BK}$$



- $\rightarrow$ Geometrical Scaling with same saturation scale
- $\rightarrow$ Full Saturation when $S \rightarrow 1/2$

Saturation: Beyond two steps...

Romuald Janik, to appear



$d(Y) \to 0$ = Multi-Dipole Correlation effects

- Balitsky-JIMWLK Hierarchy for dipoles

- Generalized Factorized Solution

$$\langle \mathcal{N}_1 \mathcal{N}_2 \ldots \mathcal{N}_n \rangle = c_n \langle \mathcal{N}_1 \rangle \langle \mathcal{N}_2 \rangle \ldots \langle \mathcal{N}_n \rangle \equiv c_2^n N_1 N_2 \ldots N_n$$

Conclusions and Prospects :

Results

- Saturation Equations including two-dipole correlations
- Factorized solution with enhanced average correlation
- Generalization to n-dipoles: R.Janik

Prospects

- The factorized Solution: "A dielectric Constant"?
- Beyond Mean-Field of Correlations?
- Is Geometrical scaling really violated by fluctuations?