Flavour Physics 4

Flavour and CP beyond the Standard Model

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General Remarks

- Currently there is no convincing idea for "Flavour", which allows for a quantitative analysis.
- Currently there is no indication of flavour violation beyond CKM.
- Any "New Physics" Scenario involves additional degrees of freedom
- $\rightarrow$ More coupling constants, potentially complex
- Additional CP violation, Additional FCNC's
- Generically also CP violation in the flavour diagonal sector

- Horizontal Symmetry or Flavour Symmetry:
  - Put Quarks and Leptons of different Families into Multipletts
  - Discrete and Continuous symmetries have been discussed
  - Symmetry must be broken: Goldstone Modes?
  - $\longrightarrow$ Gauged Horizontal Symmetry?
- Most of BSM ideas do not have much to say about Flavour
- Often a simple triplication

Two Higgs Doublet Models

- Two $SU(2)$ Doubletts:

$$\Phi^{(i)} = \begin{pmatrix} \phi_+^{(i)} \\ \phi_0^{(i)} \end{pmatrix} \quad i = 1, 2$$

- Charge Conjugate Fields $\widetilde{\Phi}^{(i)}$: also $SU(2)$ Doubletts
- Two neutral Scalars H_1^0 and H_2^0, One Pseudoscalar H_0^3 and a pair of charged Bosons $H^\pm$
- General Yukawa Couplings:

$$\mathcal{L}_{\text{Yuk}} = -\sum_{ij} \left[y_{ij}^{(1)} \bar{Q}_i \Phi^{(1)} u_{R,j} + z_{ij}^{(1)} \bar{Q}_i \widetilde{\Phi}^{(1)} d_{R,j} + y_{ij}^{(2)} \bar{Q}_i \Phi^{(2)} u_{R,j} + z_{ij}^{(2)} \bar{Q}_i \widetilde{\Phi}^{(2)} d_{R,j} \right]$$

- Mass Matrices:

$$\mathcal{M}_u = y^{(1)} + y^{(2)} \qquad \mathcal{M}_u = z^{(1)} + z^{(2)}$$

- Diagonalization .... This does not diagonalize $y^{(1)}$, $y^{(2)}$, $z^{(1)}$ and $z^{(2)}$ individually!
- Neutral Higgses have tree-level FCNC Interactions
- $\to$ “most” 2HDM’s have problems with data
- Type I: Only one of the Higgs doubletts couples to quarks: $z^{(2)} = y^{(2)} = 0$
- Type II: One doublett couples only to up-type quarks, the other only to down type quarks: $z^{(1)} = y^{(2)} = 0$ $\to$ Supersymmetry

- Possibility of Spontaneous CP Violation: e.g. Branco
- Higgs Potential can be chosen such that the two VEV's acquire a relative phase:

$$v_1 = \text{real} \quad v_2 = |v_2| e^{i\delta}$$

- Induces in general also CP violation in the neutral sector: EDM Problem!

Flavour Sector of Supersymmetric Models

- "Soft breaking" of SUSY generates in general many new sources of flavour violation
- general SUSY Model: $\sim$ 120 Flavour Parameters
- 44 CP violating Parameters
- Creates many Problems, e.g.:
  - Supersymmetric ϵ Problem:
    (too) big contributions from Quark-Squark mixing:
    Imaginary part of $K - \overline{K}$ Mxing: Factor 10^4 too big.
  - blue Supersymmetric EDM Problem:
    (too) big contributions e.g. to the electric dipole moment of the e^-

$$d^e_{\rm SUSY} \sim 10^{-25} e\, cm \qquad d^e_{\rm exp} \leq 1.6 \times 10^{-27} e\, cm$$

Grand Unification and Flavour

- Usually GUT's are formulated for one family:
  $\longrightarrow$ triplication to obtain the famiiy structure
- Leptons and Quarks are in one multiplet:
  Relations between Lepton- and Quark Masses are possible
- GUT's including a family symmetry have often also Higgs fields with family quantum numbers:
  $\longrightarrow$ Several neutral Higgs fields,
  $\longrightarrow$ Problems with FCNC's

Bottom-τ Unification in simple $SU(5)$

- 15 Fermions of one family:

$$u_L \, u_L \, u_L \quad d_L \, d_L \, d_L \quad e_L \, \nu_L$$
$$u_R \, u_R \, u_R \quad d_R \, d_R \, d_R \quad e_R$$

- Grouping into $SU(5)$ Multipletts: $\bar{5}$ (and 10)

$$\Psi_L = ((d_R^C), (d_R^C), (d_R^C), e_L, -\nu_L)_L$$

- Relevant for Flavour Physics:
  Down Quark and Lepton in the same multiplett
- Simplest Higgs Sector (Only a single Higgs Multiplet)

$$m_b = m_\tau \quad m_s = m_\mu \quad m_d = m_e$$

- Renormalization Group Running:

$$(m_b = m_\tau \text{ @ } \mu = M_{\text{GUT}}) \longrightarrow (m_b \approx 3m_\tau \text{ @ } \mu = M_W)$$

- This is a reasonable Prediction!
- ... but only for the third generation.
- Avoid the “bad prediction” for the first and second generation and keep the “good prediction” for the third:
  More complicated Higgs sector ...

Remarks on Left-Right Symmetry

- Left-Right Symmetry: Discrete Symmetry $L \leftrightarrow R$
- (Electroweak) Left-handed Doubletts $\longrightarrow$ Right-handed Doubletts: $SU(2)_L \times SU(2)_R$
- Example: $SO(10)$ GUT: two ways to produce the SM:
  - $SO(10) \longrightarrow SU(5) \times U(1) \longrightarrow SM$
  - $SO(10) \longrightarrow SU(2)_L \times SU(2)_R \times U(1)_{B-L} \longrightarrow SM$
- $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$: Pati-Salam Model
- Resembles our $L \leftrightarrow R$ Ansatz for the SM

- Right-handed up and down Quarks are in the same multiplet:
  - Custodial $SU(2)$ Symmetry
  - No mixing between the families
  - Equal up and down masses
- Spontaneous breaking of custodial $SU(2)$ = Spontaneous breaking of custodial $SU(2)_L \times SU(2)_R$
- Yields a large mass for the right handed Neutrino: See Saw

Minimal Flavour Violation

- Flavour Violation of TeV “new physics” must be very close to one of the Standard Model
- Concept of “minimal flavour violation”:
  All Flavour Violation (and CP violation) is CKM like
  (D’Ambrosio et al. ’02, Ciuchini et al. ’98, Buras et al. ’00)
- More precise definition
  D’Ambrosio et al., hep-ph/0207036
- Leptonic Sector has also been considered as well
  Grinstein et al., hep-ph/0507001, hep-ph/0601111
- Standard Model is Minimally Flavour Violating per definition

Flavour Symmetry: Quarks

- Largest Quark Flavour Symmetry commuting with the Gauge Group of the Standard Model

$$G_F = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$$

with

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix} \sim (3,1,1) \quad U_R \sim (1,3,1) \quad D_R \sim (1,1,3)$$

- G_F is *explicitely* broken by the Yukawa couplings

$$\mathcal{L}_{\text{Yuk}} = \bar{Q}_L H Y_D D_R + \bar{Q}_L \widetilde{H} Y_U U_R$$

- Diagonalization of the Yukawa Couplings

$$Y_D^{\text{diag}} = V_{DL}^\dagger Y_D V_{DR} \quad Y_U^{\text{diag}} = V_{UL}^\dagger Y_U V_{UR}$$

- Leads after Spontaneous Symmetry Breaking to diagonal Mass Matrices for the Quarks
- Note that $V_{UR} \in SU(3)_{U_R}$ and $V_{DR} \in SU(3)_{DR}$
- ... but both V_{UL} and V_{DL} should be $\in SU(3)_{Q_L}$
- this leads to a relative and observable mismatch and

$$V_{\text{CKM}} = V_{U_L}^\dagger V_{D_L}$$

- Using mass eigenstates, V_{CKM} appears as the matrix of charged current couplings.

Spurions

- Trick to parametrize explicit symmetry breaking: Introduce “Spurions”
- Spurion: Field with a well defined transformation under the symmetry to be explicitely broken.
- Write all terms that are allowed by the symmetry with a finite number of insertions of the spurion field(s)
- “Freeze” the spurion field(s) to a nonzero value: “vacuum expectation value”
- Explicit Symmetry breaking = Spontaneous Symmetry Breaking without the Higgs degrees of freedom
- Small symmetry breaking: Power counting for the spurion insertions is needed.

Yukawa Couplings as Spurions

- Interpret the Yukawa couplings as spurion fields transforming as

$$Y_U \sim (3, \bar{3}, 1) \quad Y_U \sim (3, 1, \bar{3})$$

- In this way the Yukawa terms become formally invariant under G_F
- "Freezing" the Yukawa couplings to the observed values breaks G_F explicitely.
- Minimal Flavour Violation: The two spurions Y_U and Y_D are the only sources of flavour violation.

Example $B \to X_s \gamma$ in MFV

- The $b \to s\gamma$ decay is a $D_R \to D_L$ transition.
- $\bar{Q}_L D_R$ is not invariant under G_F
- $\bar{Q}_L Y_D D_R \to \bar{D}_L m_d^{\mathrm{diag}} D_R$ is flavour diagonal.
- $\bar{Q}_L Y_U Y_U^\dagger Y_D D_R \to \bar{D}_L V_{\mathrm{CKM}}^\dagger (m_u^{\mathrm{diag}})^2 V_{\mathrm{CKM}} m_d^{\mathrm{diag}} D_R$ minimal number of spurions for a flavour transition.
- Leading term in $b \to s\gamma$: $\bar{s}_L V_{ts}^* V_{tb} m_t^2 m_b b_R$
- Right handed helicities suppressed by a power of the quark mass
- FCNC require at least two CKM matrix elements, at least one of which is off diagonal
- GIM: no FCNC's in case of degenerate quark masses

Flavour Symmetry: Leptons

- "Minimal field content" (no right handed neutrino)

$$E_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad E_R = e_R$$

- Smaller flavour group for the leptons

$$\tilde{G}_F = SU(3)_{E_L} \times SU(3)_{E_R}$$

- Transformations under $\tilde{G}_F$:
  $E_L \sim (3, 1)$ and $E_R \sim (1, 3)$
- Yukawa term for the leptons

$$L_{\text{Yuk}} = \bar{E}_L H Y_E E_R$$

- Y_E can be diagonalized by a $\tilde{G}_F$ transformation
  No flavour mixing for leptons !

Lepton Flavour Violation: Higher Dimensional Operators

- Dim-5 operator leading to a ν Majorana Mass Term

$$\mathcal{L}_{\rm Maj} = \frac{1}{2\Lambda_{\rm LN}} \left(N^T g N \right)$$

$$\text{with} \quad N = \left(T_3^{(R)} + \frac{1}{2} \right) H^\dagger L$$

$$\text{and} \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h_0 + i\chi_0 & \sqrt{2}\phi_+ \\ -\sqrt{2}\phi_- & v + h_0 - i\chi_0 \end{pmatrix}$$

- $\Lambda_{\rm LN}$: Scale of lepton number violation
- g: New Spurion field transforming as $(\bar{6}, 1)$ under $\tilde{G}_F$
- Y_E, g can (in general) not be diagonal simultaneously

New Physics in MFV: Quarks

- Generic point of view: Consider the Standard model as an effective theory, valid at the electroweak scale
- "New Physics" enters below M_W through power suppressed operators with dimensions ≥ 6
- Assume that Y_U, Y_D (and Y_E) are still the only spurions explicitely breaking flavour
- The flavour transitions of the new-physics contributions are still suppressed by the same CKM factors and masses as in the Standard Model
- Focus first on quarks ...

Power Counting and Wolfenstein Parametrization

- Power Counting $\sim$ “small” symmetry breaking
- Implemented by the Wolfenstein parametrization

$$V_{\rm CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \qquad \lambda \sim 0.2$$

- Quark Masses (except top) are small compared to the electroweak scale
- Additional spurion insertions yields more suppression (except for $t \longrightarrow b$ transitions, fllavour diagonal)
- Consider only minimal number of spurion insertions
  - Up to four insertions for right $\longrightarrow$ right transitions

Effective Field Theory Picture of New Physics

- List the various quark transitions:

	U_L	U_R	D_L	D_R
$\bar{U}_L$	$V_{u_L}^\dagger Y_D Y_D^\dagger V_{u_L}$ $= V_{\rm CKM} \hat{m}_D^2 V_{\rm CKM}^\dagger$	$V_{u_L}^\dagger Y_D Y_D^\dagger Y_U V_{u_R}$ $= V_{\rm CKM} \hat{m}_D^2 V_{\rm CKM}^\dagger \hat{m}_U$	$V_{u_L}^\dagger V_{d_L}$ $= V_{\rm CKM}$	$V_{u_L}^\dagger Y_D V_{d_R}$ $= V_{\rm CKM} \hat{m}_D V_{\rm CKM}^\dagger$
$\bar{U}_R$	h.c.	$V_{u_R}^\dagger Y_U^\dagger Y_D Y_D^\dagger Y_U V_{u_R}$ $= \hat{m}_U V_{\rm CKM} \hat{m}_D^2 V_{\rm CKM}^\dagger \hat{m}_U$	$V_{u_R}^\dagger Y_U^\dagger V_{d_L}$ $= \hat{m}_U V_{\rm CKM}$	$V_{u_R}^\dagger Y_U^\dagger Y_D V_{d_R}$ $= \hat{m}_U V_{\rm CKM} \hat{m}_D V_{\rm CKM}^\dagger$
$\bar{D}_L$	h.c.	h.c.	$V_{d_L}^\dagger Y_U Y_U^\dagger V_{d_L}$ $= V_{\rm CKM}^\dagger \hat{m}_U^2 V_{\rm CKM}$	$V_{d_L}^\dagger Y_U Y_U^\dagger Y_D V_{d_R}$ $= V_{\rm CKM}^\dagger \hat{m}_U^2 V_{\rm CKM} \hat{m}_D$
$\bar{D}_R$	h.c.	h.c.	h.c.	$V_{d_R}^\dagger Y_D^\dagger Y_U Y_U^\dagger Y_D V_{d_R}$ $= \hat{m}_D V_{\rm CKM}^\dagger \hat{m}_U^2 V_{\rm CKM} \hat{m}_D$

- Loops may change the number of insertions:
  Suppressed by powers of Wolfenstein λ

New Physics in MFV: Leptons

- Majorana term is a “new physics” contribution
- Distinguish between the scale of lepton flavour violation and lepton number violation
- For dim-6 operators: Possible Spurion combinations

$$g^\dagger \times g \sim \bar{6} \times 6 = 1 + 8 + 27$$

- Bilinears (e.g. $\tau \to \mu\gamma$) are governed by $\Delta = (g^\dagger \times g)_8$ $\to$ predicts e.g. relations between $\tau \to \mu\gamma$ and $\tau \to e\gamma$
- Four fermion operators for e.g. $\tau \to \mu\mu\mu$ can have a contribution of the 27-plet
- Even in MFV no relation between $\tau \to e\gamma$ and $\tau \to e\mu\mu$

Simple Examples

- How to get an Idea about the mass matrices:
  - Guess some matrices with as few parameters as possible:
    "Textures" as many zeros as possible
  - Use some symmetry to obtain (at least qualitatively) some insight into mass matrices
    e.g. a simple horizontal $U(1)$

Textures: Two Family Example

- Find two matrices $\mathcal{M}_u$ and $\mathcal{M}_d$ with less than five parameters
- $\rightarrow$ Relation(s) between m_u, m_c, m_d, m_s and Θ_C
  Simplest guess: Diagonal $\mathcal{M}_u$ and nondiagonal $\mathcal{M}_d$

$$\mathcal{M}_u = \begin{pmatrix} m_u & 0 \\ 0 & m_c \end{pmatrix} \qquad \mathcal{M}_d = \begin{pmatrix} 0 & a \\ a & 2b \end{pmatrix}$$

- Matrix diagonalizing $\mathcal{M}_d$ is already the CKM Matrix
- Four Parameters $\rightarrow$ One relation !
- Model predicts $\tan \Theta_C = \sqrt{\frac{m_d}{m_s}}$ (which is not bad!)

- Has been done also for three families
- Guesses often supported by assuming (discrete) symemtries
- Typical structure: $\tan\theta_{ij} \sim \sqrt{m_i/m_j}$
- Remains Guesswork, until some deeper understanding of the guesses emerges.

Horizontal $U(1)_H$: Models of the Froggatt-Nielsen type

- Horizontal $U(1)$: The quarks of the different families have different quantum numbers:

$$Q_L \longrightarrow \exp(iT_L\phi)Q_L \quad U_R \longrightarrow \exp(iT_u\phi)D_R \quad D_R \longrightarrow \exp(iT_d\phi)D_R$$

and

$$T_L = \begin{pmatrix} t_L^{(1)} & 0 & 0 \\ 0 & t_L^{(2)} & 0 \\ 0 & 0 & t_L^{(3)} \end{pmatrix} \quad T_u = \begin{pmatrix} t_u^{(1)} & 0 & 0 \\ 0 & t_u^{(2)} & 0 \\ 0 & 0 & t_u^{(3)} \end{pmatrix} \quad T_d = \begin{pmatrix} t_d^{(1)} & 0 & 0 \\ 0 & t_d^{(2)} & 0 \\ 0 & 0 & t_d^{(3)} \end{pmatrix}$$

- Introduce a scalar S breaking $U(1)_H$ by its VEV:

$S \longrightarrow \exp(-i\phi)$ $\quad S$: Singlet under all other symmetries

- Yukawa couplings are interpreted as non-renormalizable terms
- Have to be invariant under $U(1)_H$
- $\to$ A appropriate power of S has to compensate the mismatch in $U(1)_H$ quantum numbers between left and right handed fields:

$$\mathcal{L}_{\rm Yuk}^{\rm nr} = \left(\frac{1}{\Lambda^{n_{AB}}} \bar{Q}_A \lambda_{u,AB} S^{n_{AB}} \Phi_u U_B + \frac{1}{\Lambda^{m_{AB}}} \bar{Q}_A \lambda_{d,AB} S^{m_{AB}} \Phi_d D_B \right) \text{ with}$$

$$n_{AB} = t_L^{(A)} - t_u^{(B)} , \quad m_{AB} = t_L^{(A)} - t_d^{(B)} .$$

- Λ some scale of new physics inducing flavour
- Electroweak and $U(1)_H$ SSB yields $\langle S \rangle = \epsilon\Lambda$ and $\langle \Phi_u \rangle \sim \langle \Phi_d \rangle \sim v$
- order of magnitude estimate for the mass matrices

$$\mathcal{L}_{Mass} = v \left(\bar{Q}_A \epsilon^{n_{AB}} U_B + \bar{Q}_A \epsilon^{m_{AB}} D_B \right) \quad ,$$

$$|V_{us}| \sim \epsilon^{t_L^{(2)} - t_L^{(1)}} \quad , \quad |V_{cb}| \sim \epsilon^{t_L^{(3)} - t_L^{(2)}} \quad , \quad |V_{ub}| \sim \epsilon^{t_L^{(3)} - t_L^{(1)}}$$

$$\boxed{|V_{ub}| \sim |V_{cb}||V_{us}|}$$

$$\frac{m_d^{(A)}}{m_d^{(B)}} \sim \epsilon^{t_L^{(A)} - t_L^{(B)} + t_d^{(A)} - t_d^{(B)}} \quad , \quad \frac{m_u^{(A)}}{m_u^{(B)}} \sim \epsilon^{t_L^{(A)} - t_L^{(B)} + t_u^{(A)} - t_u^{(B)}}$$

- This remains qualitative ...
- The simplest model has no reasonable phenomenology
- What happens to the goldstone boson of $U(1)_H$?
- Gauged $U(1)_H$ symmetry?
- How de we get CP violation in this framework?
- ...

Conclusions

- Near Future: Test the CKM Picture of Flavour Mixing
- Are there small deviations from CKM?
- Lepton Flavour Physics and Lepton number violation e.g. $\tau \to \mu\mu\mu$ at LHC
- Origin of Flavour: Horizontal Symmetries ?
- $\to$ at which scale?
- LHC in combination with (Super)-B Factories will possibly answer a few of these questions ...