FRACTAL SOURCES
INTERMITTENCY
LEVY DISTRIBUTIONS

A. BIAŁAS

Proc. XXVII ICHEP
GLASGOW (94) p. 1287

(1) SELF-SIMILARITY & INTERMITTENCY

(2) SCALING - CASCADES - FLUCTUATIONS

(3) POWER LAWS IN DATA

(4) PARTON CASCADE → PROBLEM

(5) HBT HELPS

(6) POWER LAWS IN SPACE-TIME ?

(7) STABLE DISTRIBUTIONS

(8) CONCLUSIONS

INTERMITTENCY:

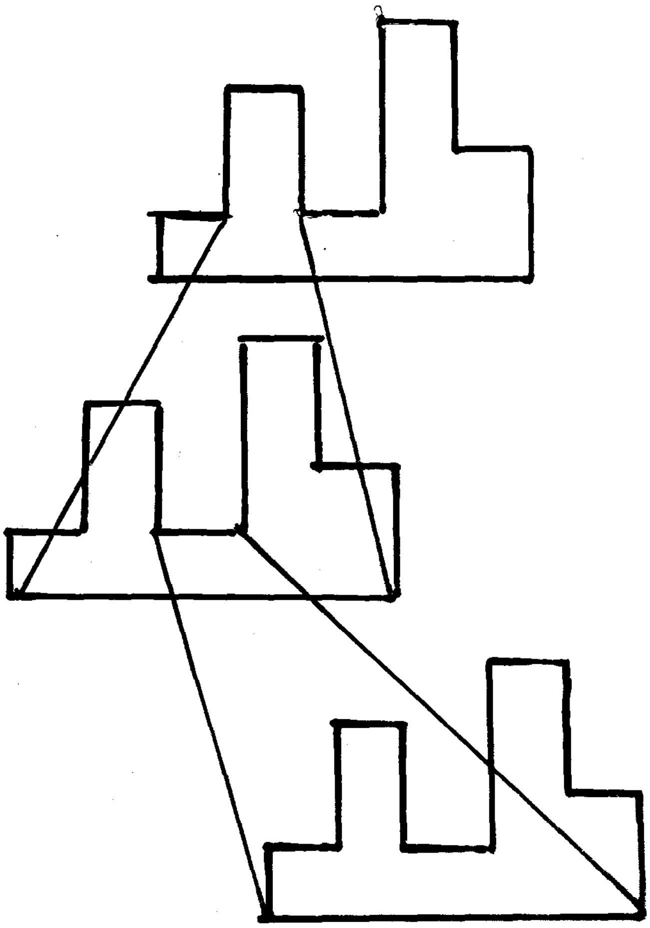
SELF-SIMILARITY

OF THE SPECTRA

⇓

SCALING

⇓



NORMALIZED MULTIPARTICLE DENSITY SCALES:

$$\int_{\delta} \rho(x_1, \ldots x_k)\, dx_1 \cdots dx_k = a_k \boxed{\delta^{-f_k}}$$

R. PESCHANSKI & AB (86)

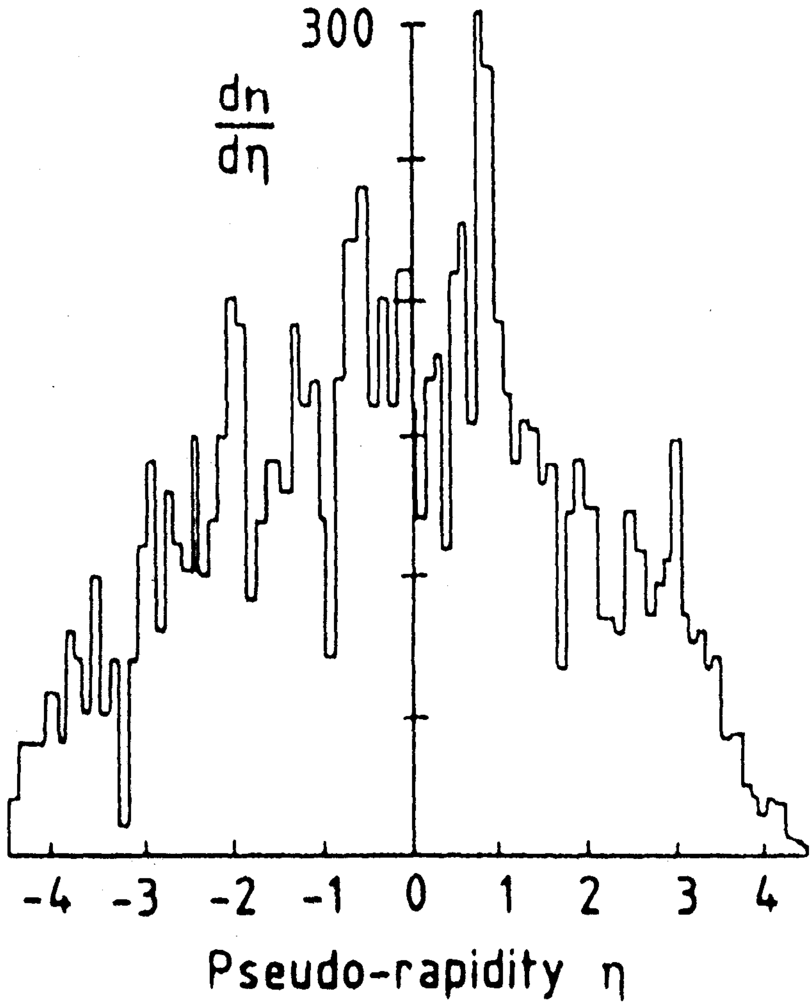
⇓

$$F_k \equiv \frac{1}{\langle n \rangle^k} \langle n(n-1) \cdots (n-k+1) \rangle = a_k \boxed{\delta^{-f_k}}$$

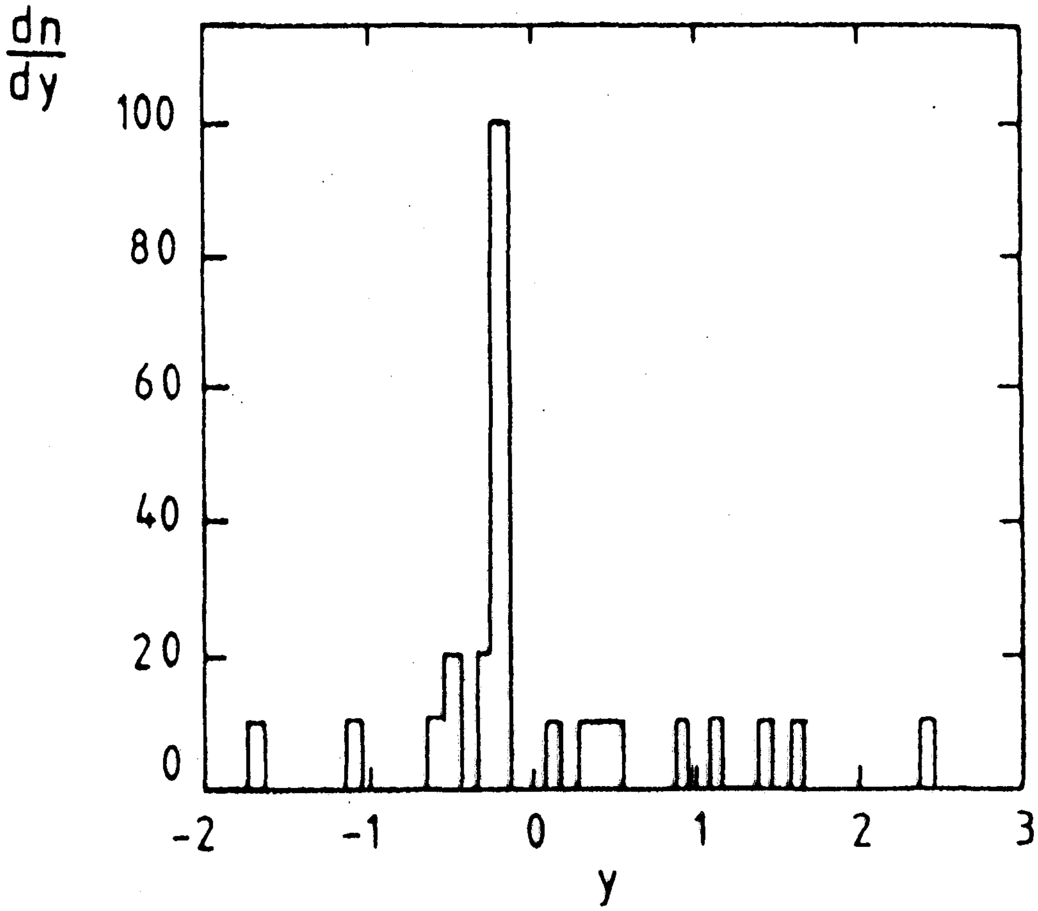
SIMILAR RELATIONS FOR CORRELATION FUNCTIONS & FACTORIAL CUMULANTS

FLUCTUATIONS AT ALL SCALES

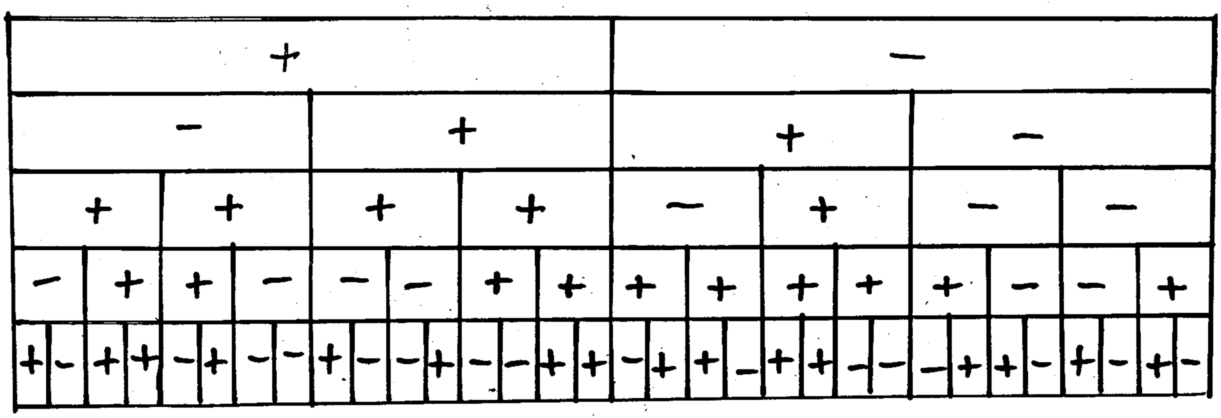
a) JACEE event



b) NA22 event



SELF-SIMILAR CASCADE



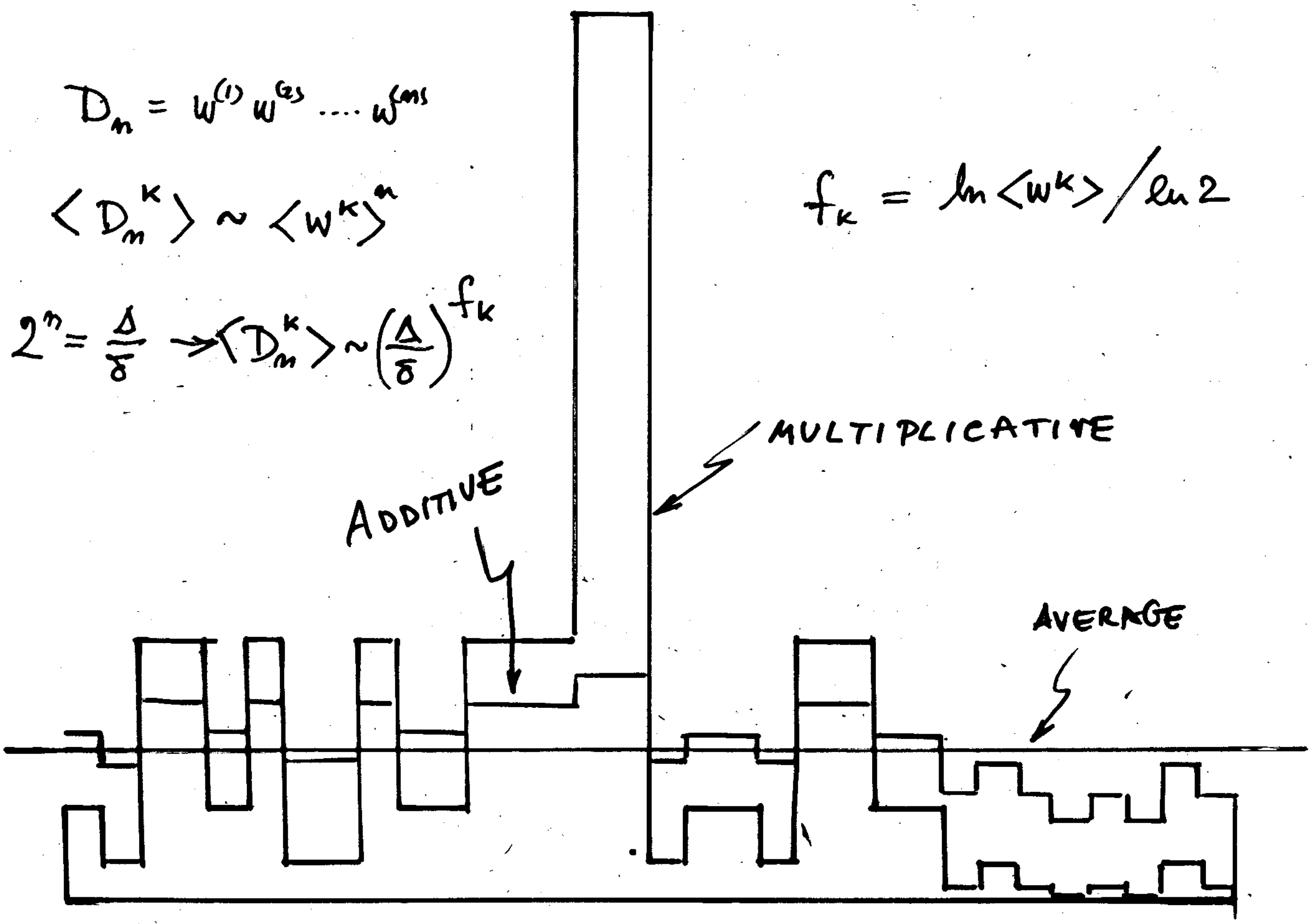
$w_- = \frac{1}{2} \qquad w_+ = \frac{3}{2} \qquad p_+ = \frac{1}{2} = p_-$

$D_n = w^{(1)} w^{(2)} \dots w^{(n)}$

$\langle D_n^k \rangle \sim \langle w^k \rangle^n$

$2^n = \frac{\Delta}{\delta} \rightarrow \langle D_n^k \rangle \sim \left(\frac{\Delta}{\delta}\right)^{f_k}$

$f_k = \ln \langle w^k \rangle / \ln 2$



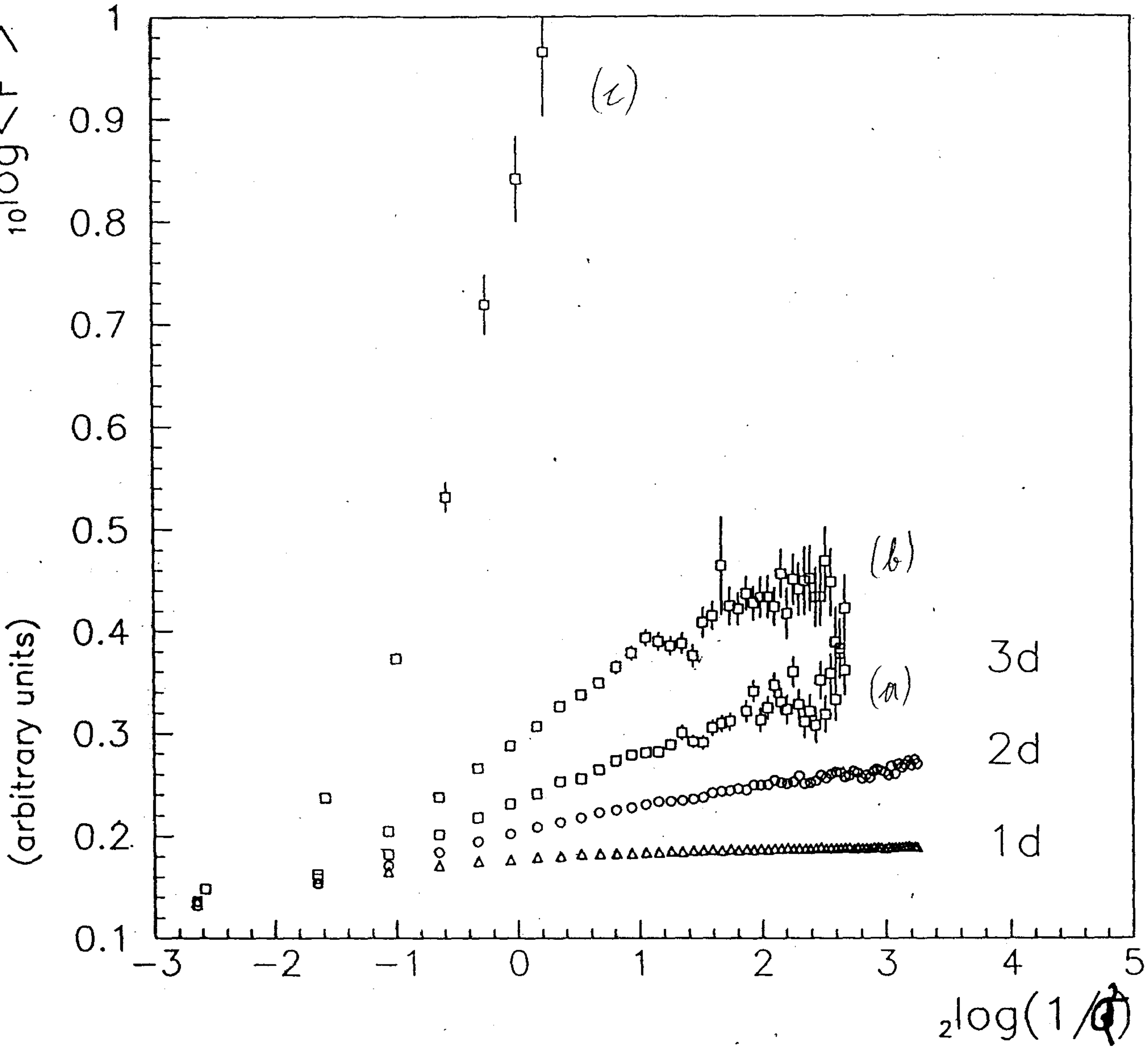


Fig. 1 Comparison of the 1-, 2- and 3-d analysis of UA1data.

THE NATURAL IDEA: QCD PARTON CASCADE

(SATZ & SARCEVIC)

IT IS NATURAL BECAUSE IT IS KNOWN TO WORK FOR GENERAL PROPERTIES OF THE SPECTRA (MC CODES!)

NUMERICAL & ANALYTIC CALCULATIONS WERE PERFORMED CONFIRMING SELF-SIMILARITY OF THE PARTON SPECTRA FOR FIXED α_s AND ESTIMATING VIOLATIONS OF SCALING CAUSED BY RUNNING α_s.

DIFFICULTY:

PARTON CASCADE PREDICTS STRONG CORRELATIONS AT VERY SMALL ΔQ BETWEEN PARTONS. BUT THE EXPERIMENT OBSERVES CORRELATIONS BETWEEN HADRONS.

PROBLEM: WHY THE TRANSITION

PARTONS $\longrightarrow$ HADRONS

DOES NOT DESTROY CORRELATIONS AT $\Delta Q \lesssim 50$ MeV?

DISCOVERY: HBT DOMINANCE

SUGGESTION: CARRUTHERS; GYULASSY (89)

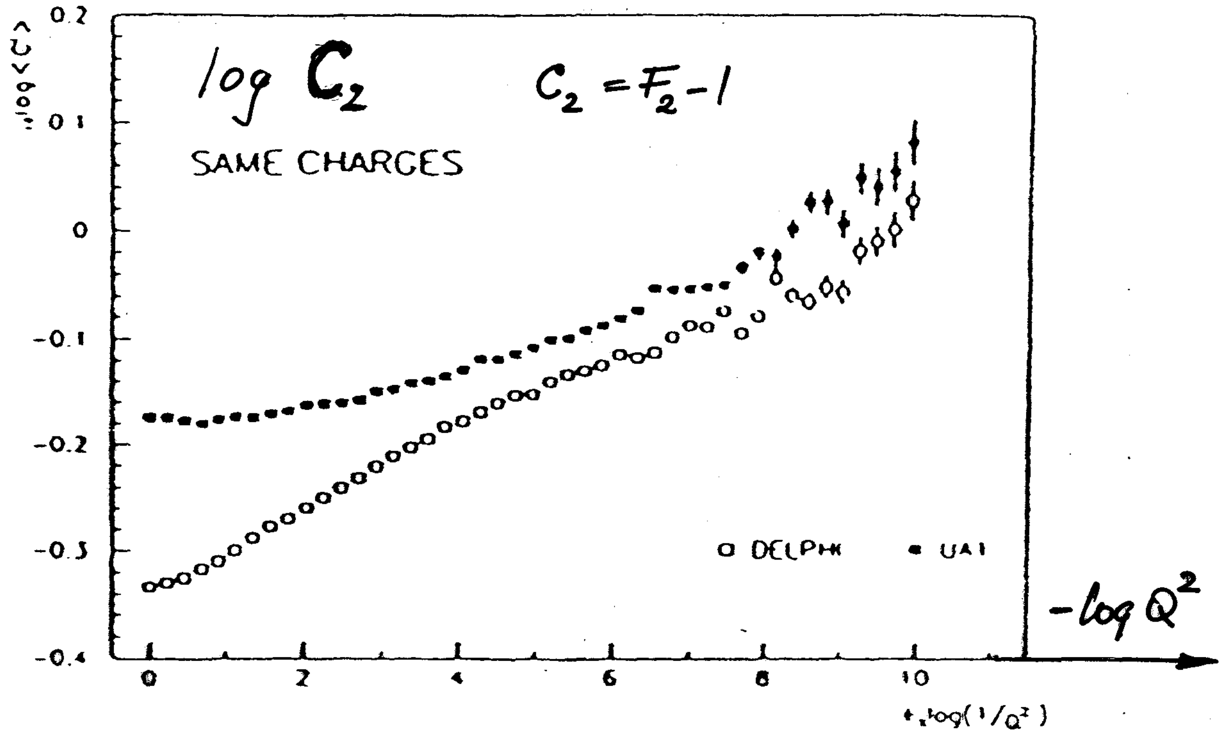
PRECIZE DATA SHOWED THAT THESE SHORT-RANGE CORRELATIONS ARE LARGELY DOMINATED BY HBT PHENOMENON I.E. BY QUANTUM INTERFERENCE.

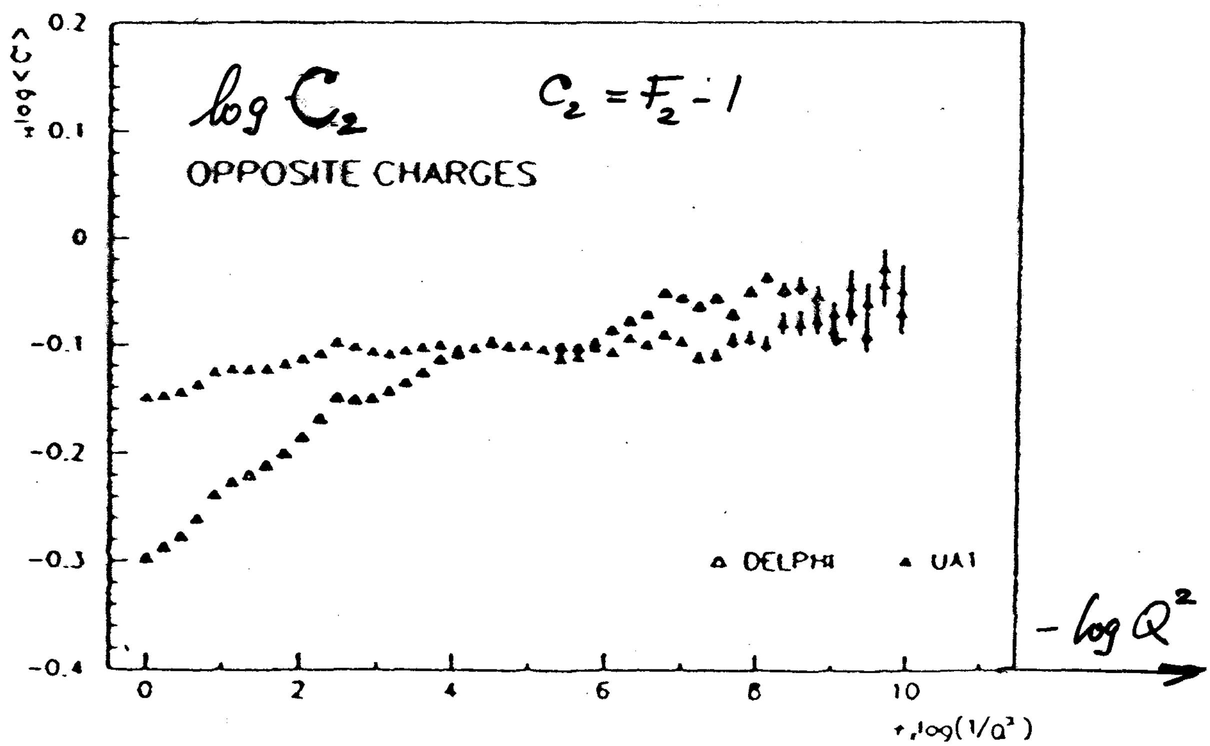
THIS OBSERVATION

(i) CHANGES DRAMATICALLY THE WAY OF THINKING ABOUT THE PROBLEM, AS IT IS NOW CLEAR THE DISTRIBUTION OF PARTICLE SOURCES IN SPACE-TIME ARE ESSENTIAL FOR UNDERSTANDING THE OBSERVED CORRELATIONS

(ii) SOLVES THE PROBLEM OF THE QCD CASCADE. THE CASCADE OF PARTONS IN MOMENTUM SPACE IS SIMPLY IRRELEVANT FOR THE VERY SHORT RANGE CORRELATIONS, AS THEY DO NOT MEASURE CORRELATIONS BETWEEN PARTON MOMENTA BUT THEIR SPACE-TIME DISTRIBUTIONS.

TWO-PARTICLE CORRELATIONS





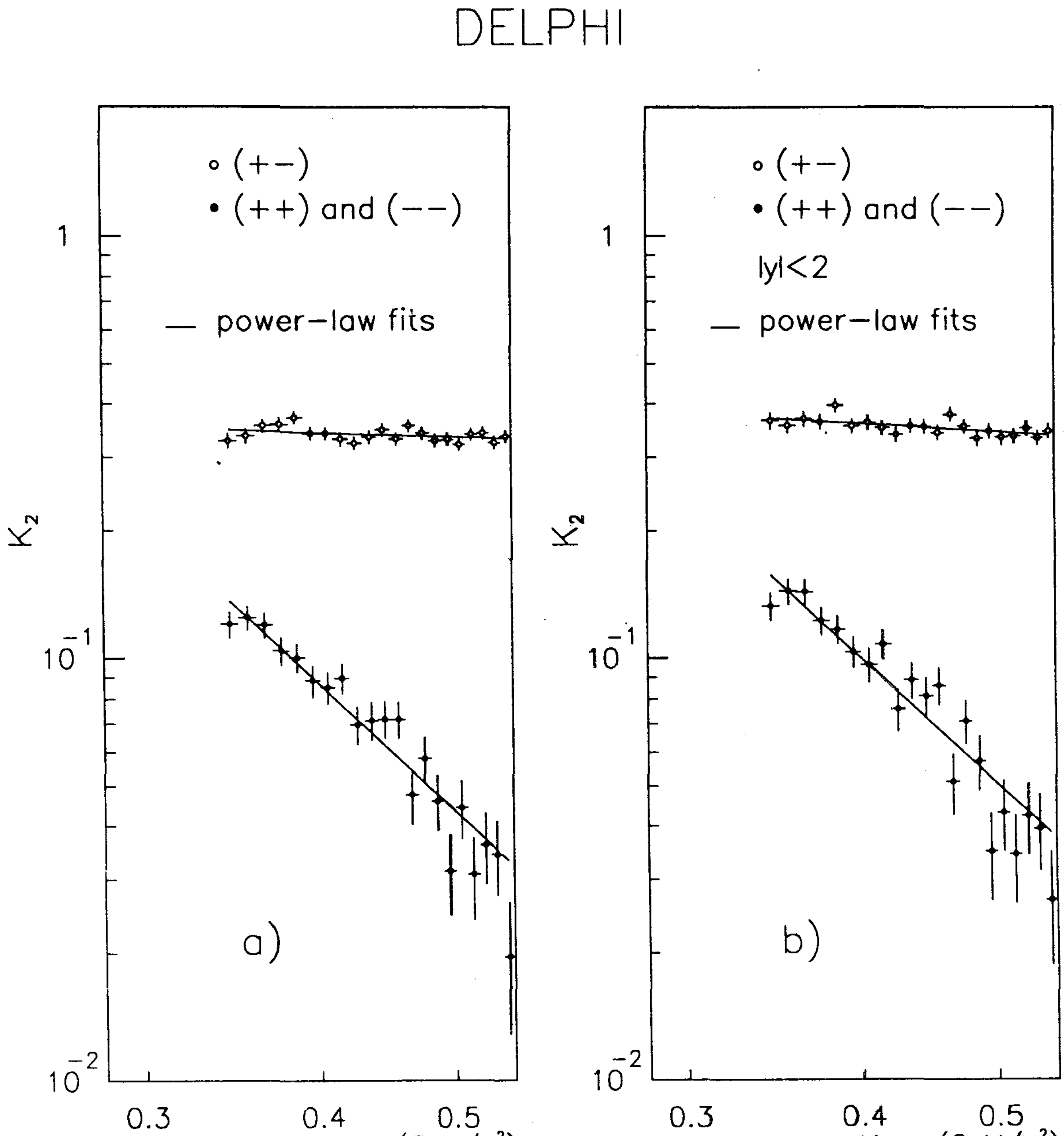
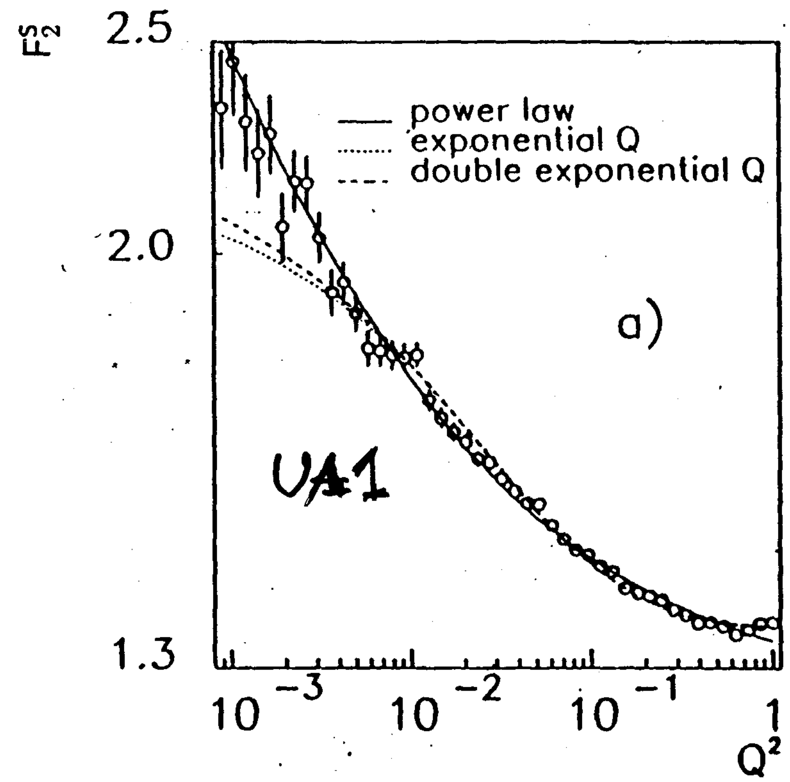
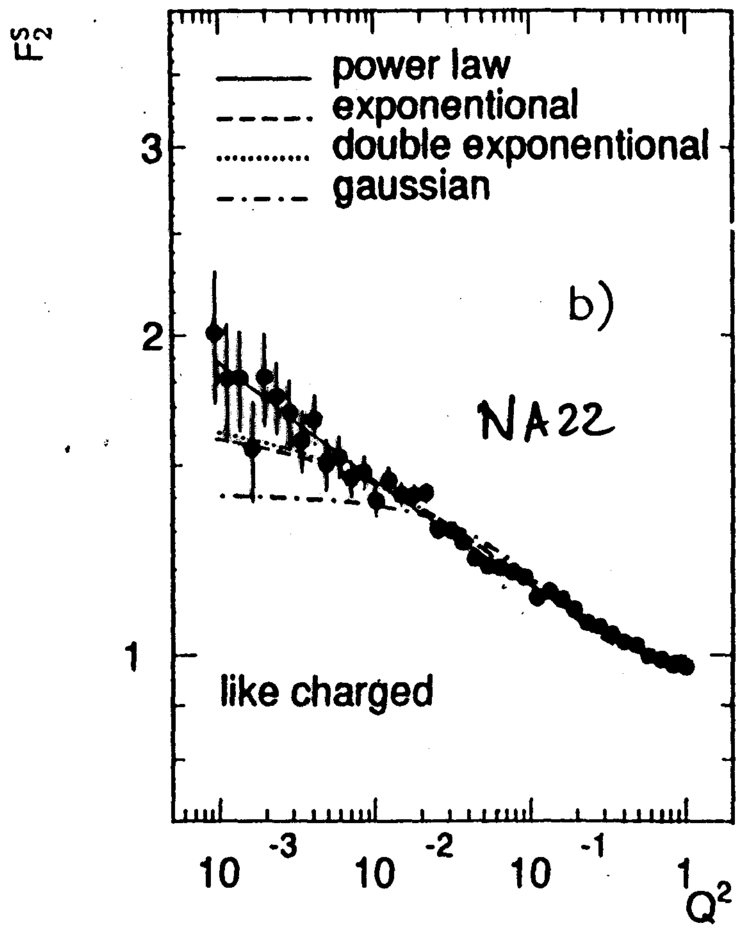

DELPHI
∘ (+−)
• (++) and (−−)
— power−law fits
a)
∘ (+−)
• (++) and (−−)
|y|<2
— power−law fits
b)
K₂
1
10⁻¹
10⁻²
0.3
0.4
0.5
Mass (GeV/c²)

Fig.8

TWO-PARTICLE CORRELATIONS





"HBT EXPLAINS INTERMITTENCY": WRONG!

THE SHAPE IS IMPORTANT:

SHAPE OF THE SPECTRUM IN ΔQ TELLS ABOUT THE SHAPE OF THE EMISSION REGION

IN PARTICULAR,

(i) VERY SHORT RANGE CORRELATIONS IN ΔQ IMPLY VERY LONG TAIL IN THE SPATIAL EXTENSION OF THE SOURCE

(ii) POWER LAW IN ΔQ IMPLIES SOME CONDITIONS ON SHAPE OF THE SOURCE

WHAT CONDITIONS?

A (TWO-DIMENSIONAL) EXAMPLE

CORRELATION FUNCTION
$$C_2(Q) = \frac{1}{(1+Q^2L^2/4)^{2\lambda}} \rightarrow |Q^2|^{-2\lambda}$$

DENSITY DISTRIBUTION
$$d(X) = \frac{1}{\Gamma(\lambda)} \int_0^\infty \frac{dR^2}{L^2} \left(\frac{R^2}{L^2}\right)^{\lambda-1} e^{-R^2/L^2} \frac{e^{-X^2/R^2}}{\pi R^2}$$

SUPERPOSITION OF GAUSSIAN DISTRIBUTIONS WITH VARIOUS SIZES, DISTRIBUTED ACCORDING TO POWER LAW CUT BY A GAUSSIAN

INTEGRATION CAN BE COMPLETED:

$$d(X) = \frac{2}{\pi L^2 \Gamma(\lambda)} \left(\frac{|X|}{L}\right)^{\lambda-1} K_{1-\lambda}(2|X|/L)$$

$$|X| \rightarrow 0 \qquad d(|X|) \rightarrow (|X|^2/L^2)^{\lambda-1}$$

$$|X| \rightarrow \infty \qquad d(|X|) \rightarrow |X|^{\lambda-3/2} e^{-2|X|/L}$$

HIGHER ORDER CORRELATIONS

B. ZIAJA & AB
APP B24 (93) 1509

(1) FLUCTUATIONS OF THE SIZE:

$$D(x_1,\dots,x_M)=\frac{1}{\Gamma(\lambda)}\int_0^\infty \frac{dR^2}{L^2}\left(\frac{R^2}{L^2}\right)^{\lambda-1} e^{-R^2/L^2}\,\frac{e^{-(x_1^2+\dots+x_M^2)/R^2}}{(\pi R^2)^M}$$

$f_k = 1-\lambda$ INDEPENDENT OF **k** !

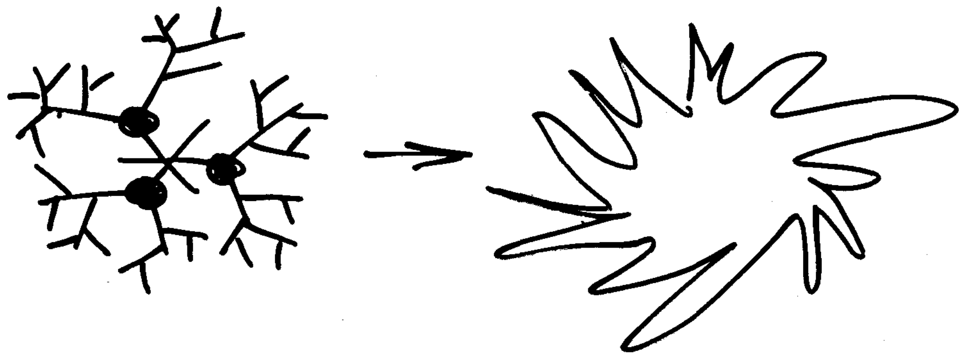
(2) INDEPENDENT PRODUCTION IN SPACE

$$D(x_1,\dots,x_M) = D(x_1)\cdots D(x_M)$$

$f_k = k\gamma$ LINEAR IN k

BOTH DISAGREE WITH DATA → CASCADE ?

HADRONIC FLUCTUATIONS :



ARE "FROZEN" AT HIGH ENERGY
(TIME DILATATION)

THEY ARE "SHAKEN OFF" IN
A SOFT HIGH-ENERGY COLLISION :

AND THEREFORE <u>CAN BE OBSERVED</u>

OBSERVATION OF INTERMITTENCY INDICATES THAT THESE FLUCTUATIONS ARE (AT LEAST PARTLY) SELF-SIMILAR (SCALE-INVARIANT)

VACUUM FLUCTUATIONS ??

MUELLER'S DIPOLE CASCADE IN TRANSVERSE SPACE

R. PESCHANSKI & AB
PL B355 (95) 301

AIM: AT HIGH ENERGY GLUON CONTENT OF A HADRON CAN BE APPROXIMATED BY A CASCADE OF DIPOLES

IDEA: THE CLOUD OF DIPOLES SHOWS UP AFTER INTERACTION

RESULT: IN LEADING ORDER BFKL APPROXIMATION THE DISTRIBUTION OF DIPOLES IN $|x_\perp|$ AT $Y \simeq 0$ HAS A POWER LAW TAIL

$$|X_\perp|^{\gamma_M - 2} \qquad \gamma_M \simeq 0.37$$

$$2\psi(1) - \psi(1-\gamma_M/2) - \psi(1+\gamma_M/2) = 8 \log 2$$

THEREFORE WE EXPECT THE POWER LAW IN $|Q_\perp|$:

$$|Q_\perp|^{2\gamma_M}$$

NA22 EXPERIMENT

N.M. AGABABYAN et al.
PLB 393 (97) 205

$|Y| < 0.5$

$\gamma_M = 0.4 \pm 0.04$

NEW IDEA:

STABLE (LEVY) DISTRIBUTIONS IN X-SPACE

MOMENTUM SPACE:
BRAX & PESCHANSKI
PLB 253 (91) 225.

CSORGO, HEGYI & ZAJC
nucl-th/0402035
EPJ C36 (04) 67

$$f_K = f_2 \frac{K^\alpha - K}{2^\alpha - 2}$$

$0 \leq \alpha \leq 2$ LEVY INDEX

LEVY DISTRIBUTION IN X-SPACE ⇒

$$\Rightarrow C_2(\Delta Q) = 1 + e^{-|\Delta Q R|^\alpha}$$

FITS OK

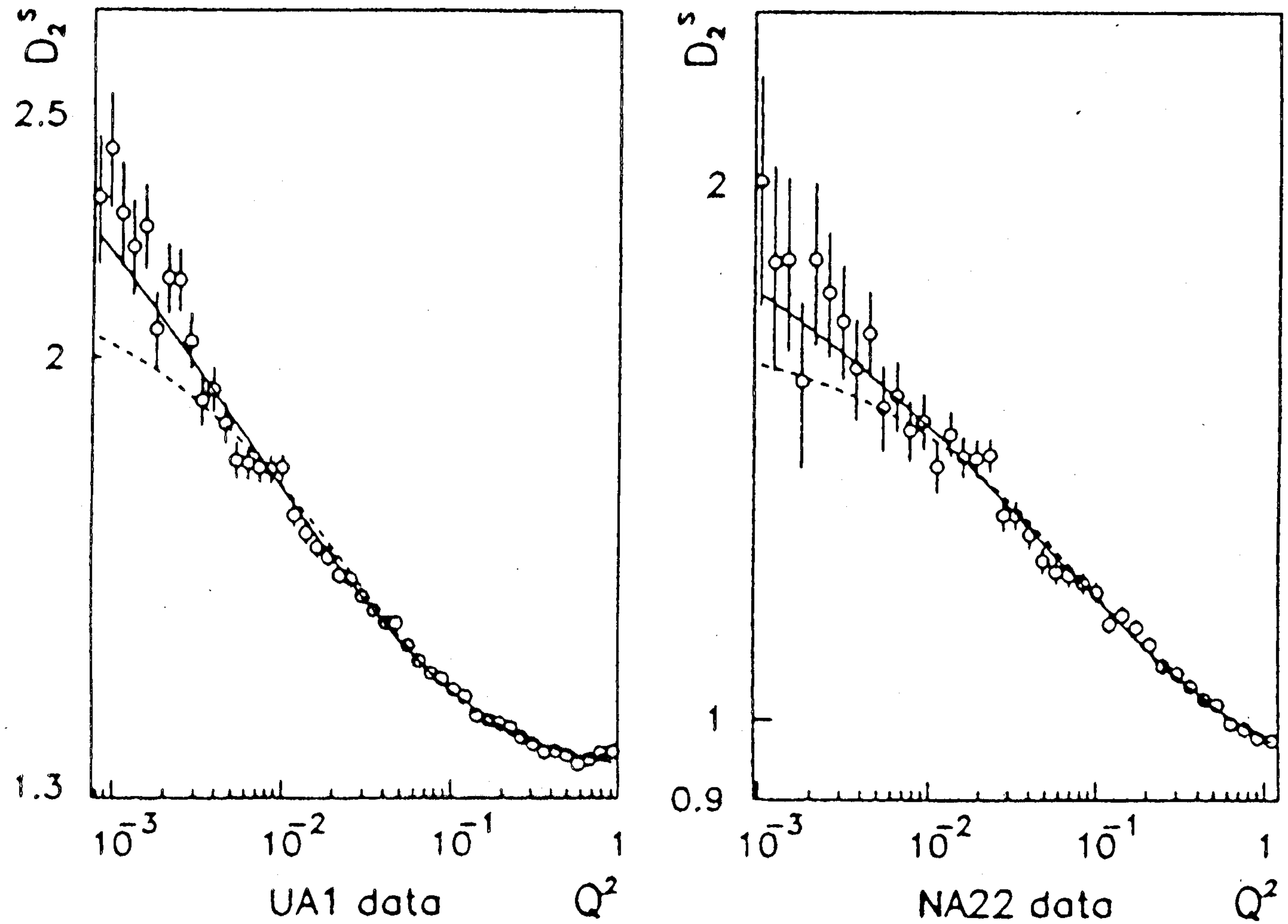
CSORGO, HEGYI, NOVAK, ZAJC:

IN LUND MODEL α IS RELATED TO ANOMALOUS DIMENSION OF QCD ⇒
⇒ DETERMINATION OF α_s FROM HBT (+ MODEL)

NA22: $\alpha_s = 0.24 \pm 0.05$
UA1: $\alpha_s = 0.13 \pm 0.01$

INTRIGUING!

Stretched exponential fit



CONCLUSIONS

(1) APPROXIMATE SELF-SIMILARITY OF PARTON SPECTRA IS A NATURAL CONSEQUENCE OF QCD.

(2) SELF-SIMILAR (FRACTAL) STRUCTURE OF PARTON SPECTRA IN X-SPACE IS RESPONSIBLE FOR (APPROXIMATE?) SCALING OBSERVED IN HADRON SPECTRA AT HIGH RESOLUTION

(3) DILUTE SYSTEMS ARE MORE LIKELY TO SHOW SCALING THAN THE DENSE ONES. pp!

(4) EXCITING NEW POSSIBILITY: LEVY DISTRIBUTION IN X-SPACE. ERA OF TAMAS et al ?