Understanding the emission duration through femtoscopy and imaging

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Talk Outline:

- Background
- Simple model emission function
- Source functions in this model
- Can we see this in experiment?
- CorAL Status

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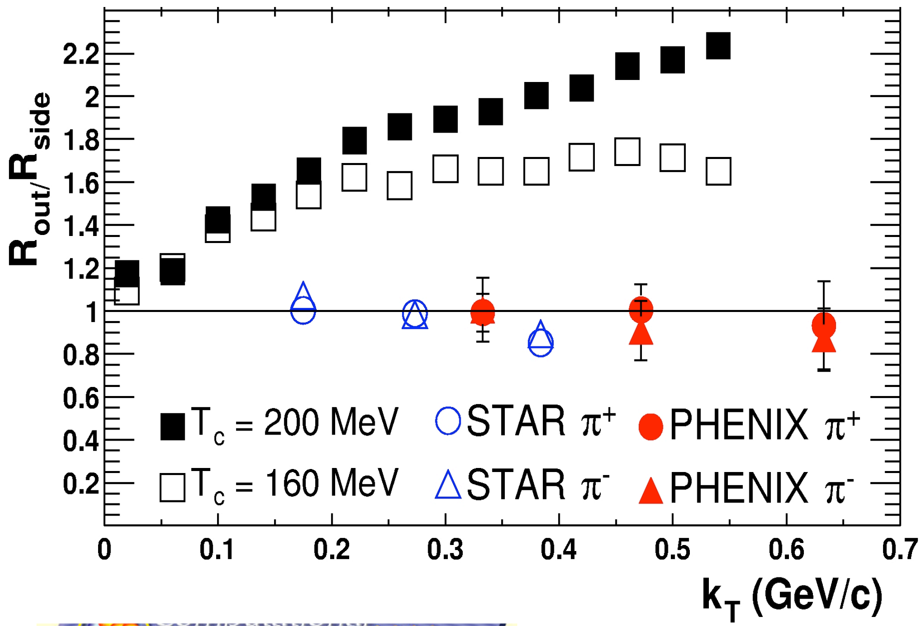
Mike's HBT Puzzle #1

What does QGP do to HBT radii? (Rischke, Gyulassy Nucl. Phys. A **608**, 479 (1996), many others):

- If there is a phase-transition, hydro evolution will slow in mixed phase.
- Will lead to long-lived source
- Huge difference in Outward/Sideward radii



R_{out}/R_{side} == 1! (in LCMS)

Experiment analysis wrong?

- Coulomb correction?

Theory screwed up?

Interpretation confused?

- Instant freeze-out?
- Gaussian fits?

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The Koonin-Pratt Formalism

$$C(\vec{q}) = \int d^3r \, |\Phi_{\vec{q}}(\vec{r})|^2 \, S(\vec{r})$$

The pair's final state wave-function, we can compute this if we know the interaction. This is the kernel $K(q,r)$.

The *source function*, it gives the probability of producing a pair a distance r apart in the pair CM frame

We invert to get S(r) directly

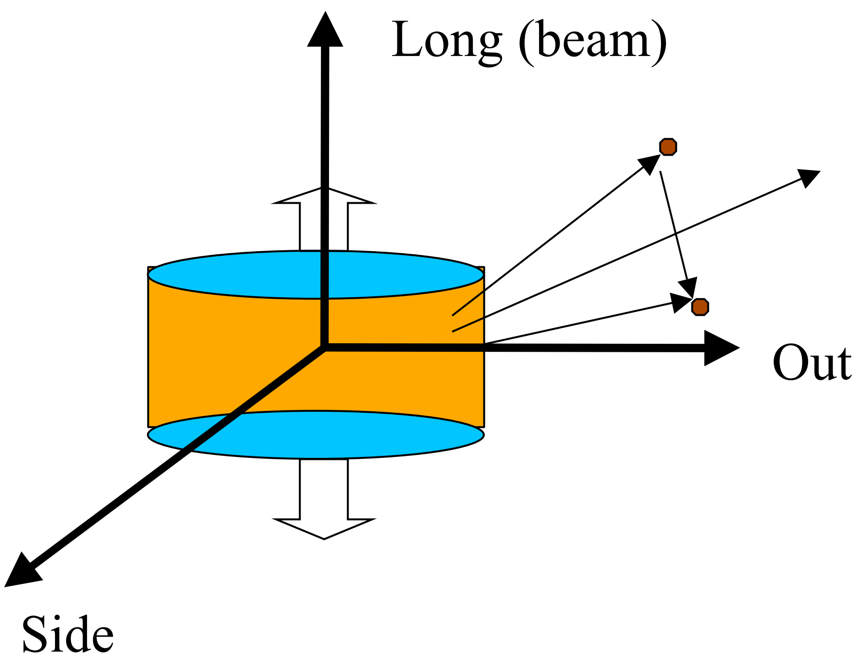
Source function related to emission function:

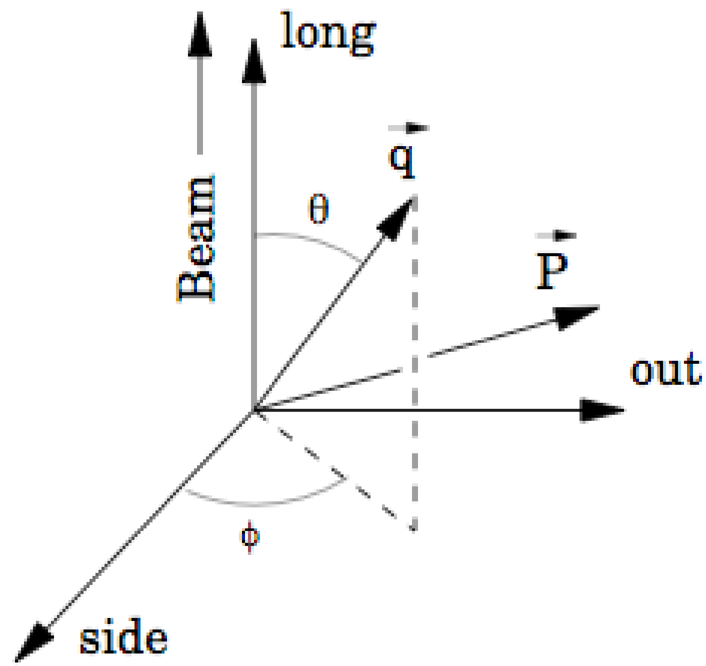
$$S_{\vec{P}}(\vec{r'}) = \int dr'_0 \int d^4R \, D(R + r/2, \vec{P}/2) \, D(R - r/2, \vec{P}/2)$$

We work in Bertsch-Pratt coords., in pCM

Our coordinate choice

Work in Bertsch-Pratt coordinates in pair CM frame:

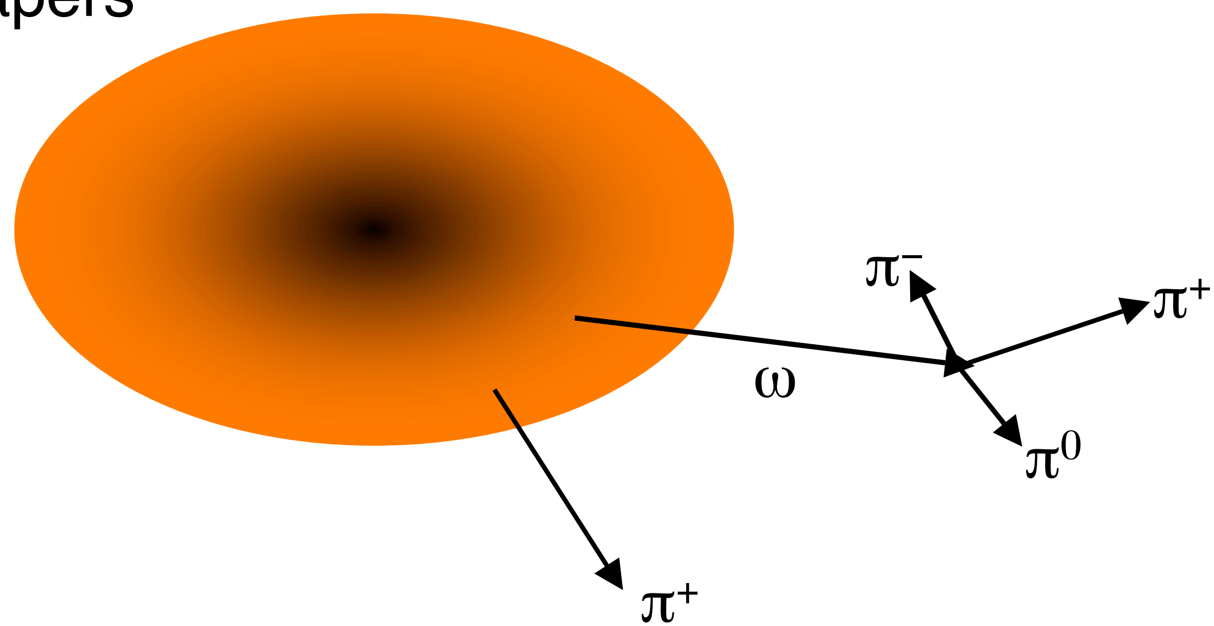




Boost from lab → pair CM means lifetime effects transformed into Outwards/Longitudinal direction.

Core Halo Model

Nickerson, Csörgo˝, Kiang, Phys. Rev. C **57**, 3251 (1998)
+ other papers



$$D(\mathbf{r},t,\mathbf{p}) = f D_{\mathrm{core}}(\mathbf{r},t,\mathbf{p}) + (1-f) D_{\mathrm{halo}}(\mathbf{r},t,\mathbf{p})$$

f is fraction of π's emitted directly from core,
effectively $\lambda = f^2$ in source function

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Core Halo Model, cont.

- From exploding core, with Gaussian shape:

$$D_{\rm core}(\mathbf{r},t,\mathbf{p}) \propto e^{-(E-\mathbf{p}\cdot\mathbf{v}_{\rm flow})/T} D_{\rm gauss}(\mathbf{r}) e^{-t/\tau_{fo}}$$

- Flow profile simple, but adjustable:

$$\mathbf{v}_{\rm flow} = \begin{cases} \alpha\mathbf{r} & \text{if } |\mathbf{r}| < R_{\rm Au} = 6.98 \text{ fm} \\ \frac{2}{3}c\hat{\mathbf{r}} & \text{otherwise} \end{cases}$$

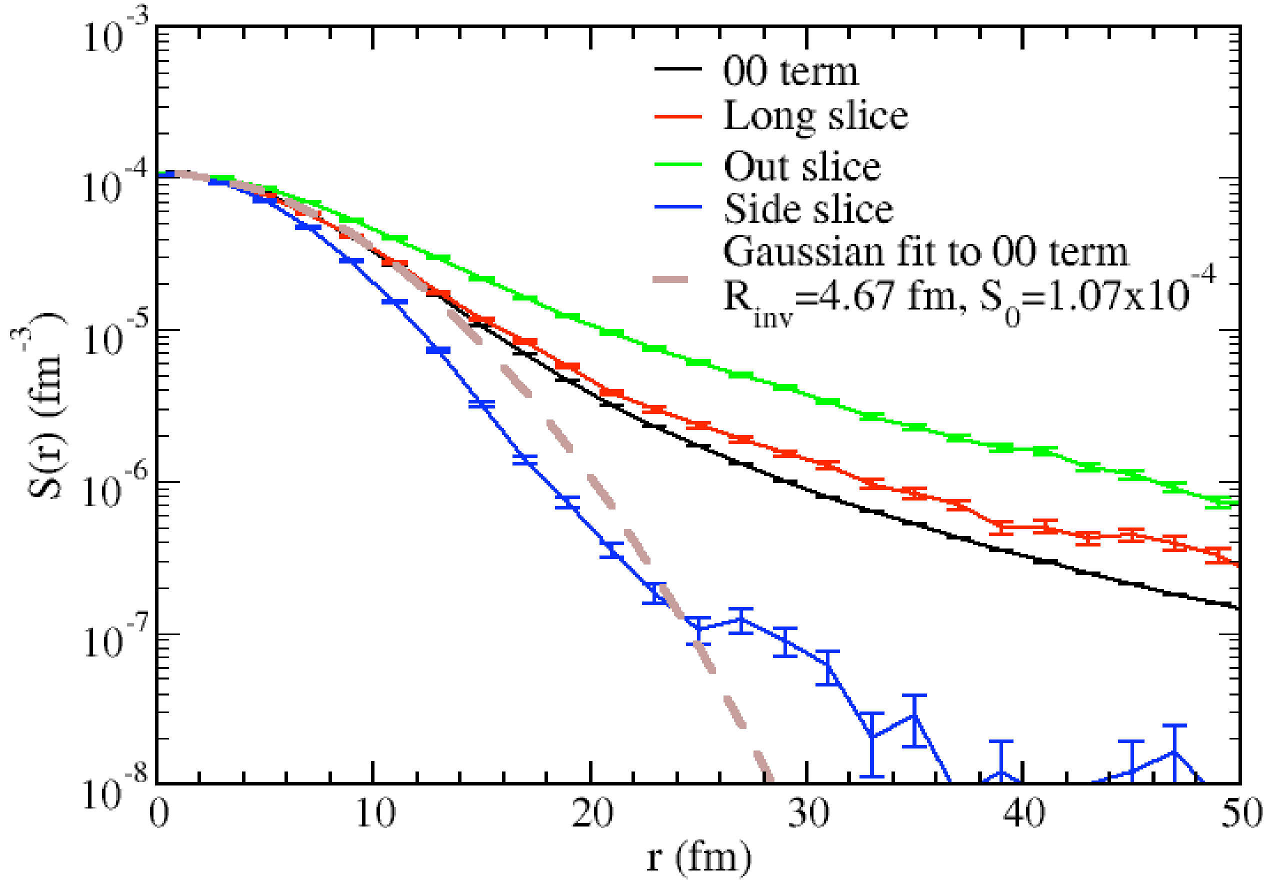
- From decay of emitted resonances, ω has most likely lifetime (23 fm/c)

$$D_{\rm halo}(\mathbf{r},t,\mathbf{p}) \propto \int d\Delta t\, d^3p_\omega\, P(\mathbf{p}_\omega,\mathbf{p}) e^{-\Delta t/\tau_\omega} \times D_{\rm core}\left(\mathbf{r} - \frac{\mathbf{p}_\omega}{E_\omega}\Delta t, t-\Delta t, \mathbf{p}_\omega\right)$$

use full 3-body decay kinematics

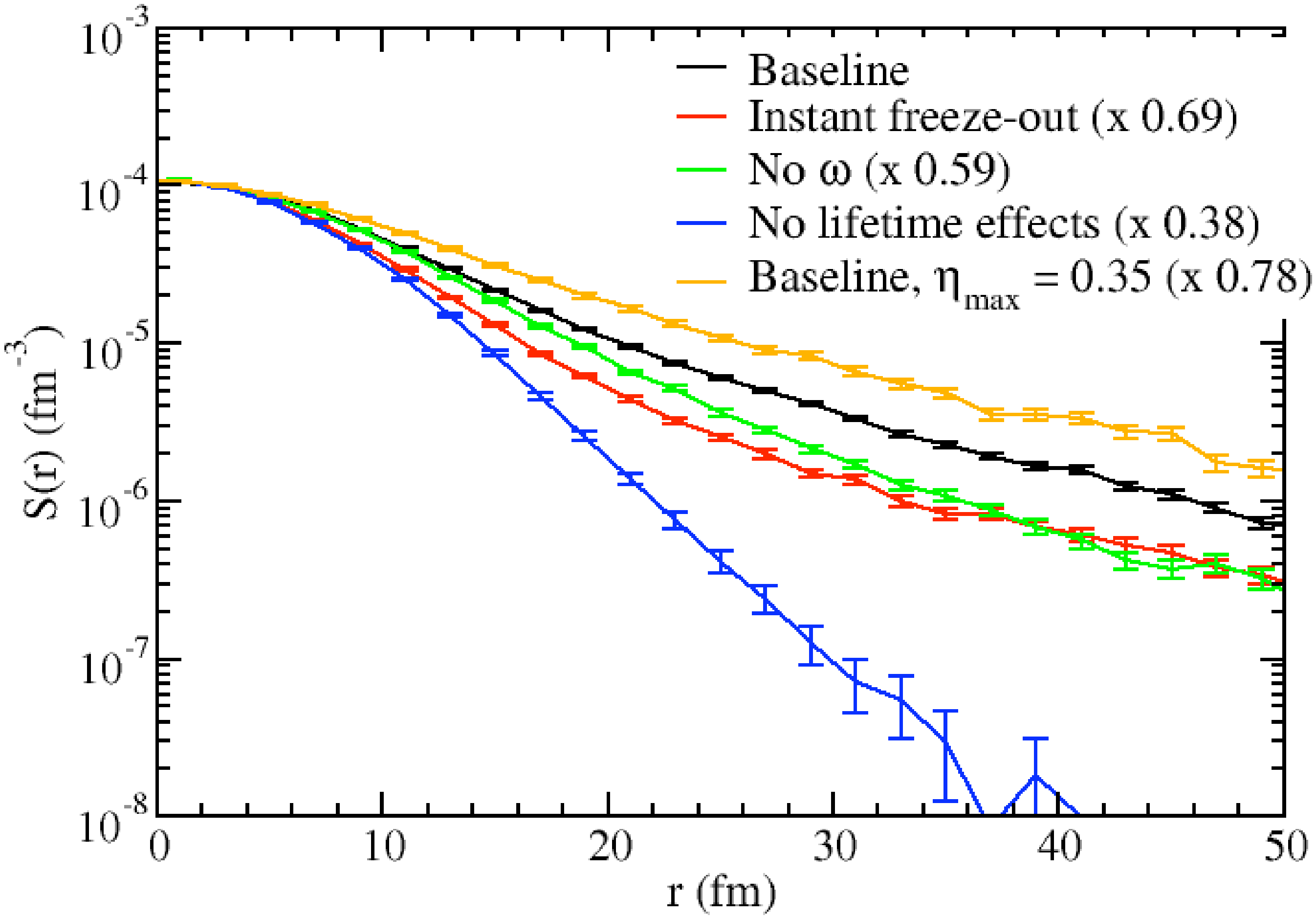
Simple Model, Interesting Results

Set $R_x = R_y = R_z = 4$ fm, $\tau_{f/o} = 10$ fm/c, $T = 175$ MeV, $f = 0.56$



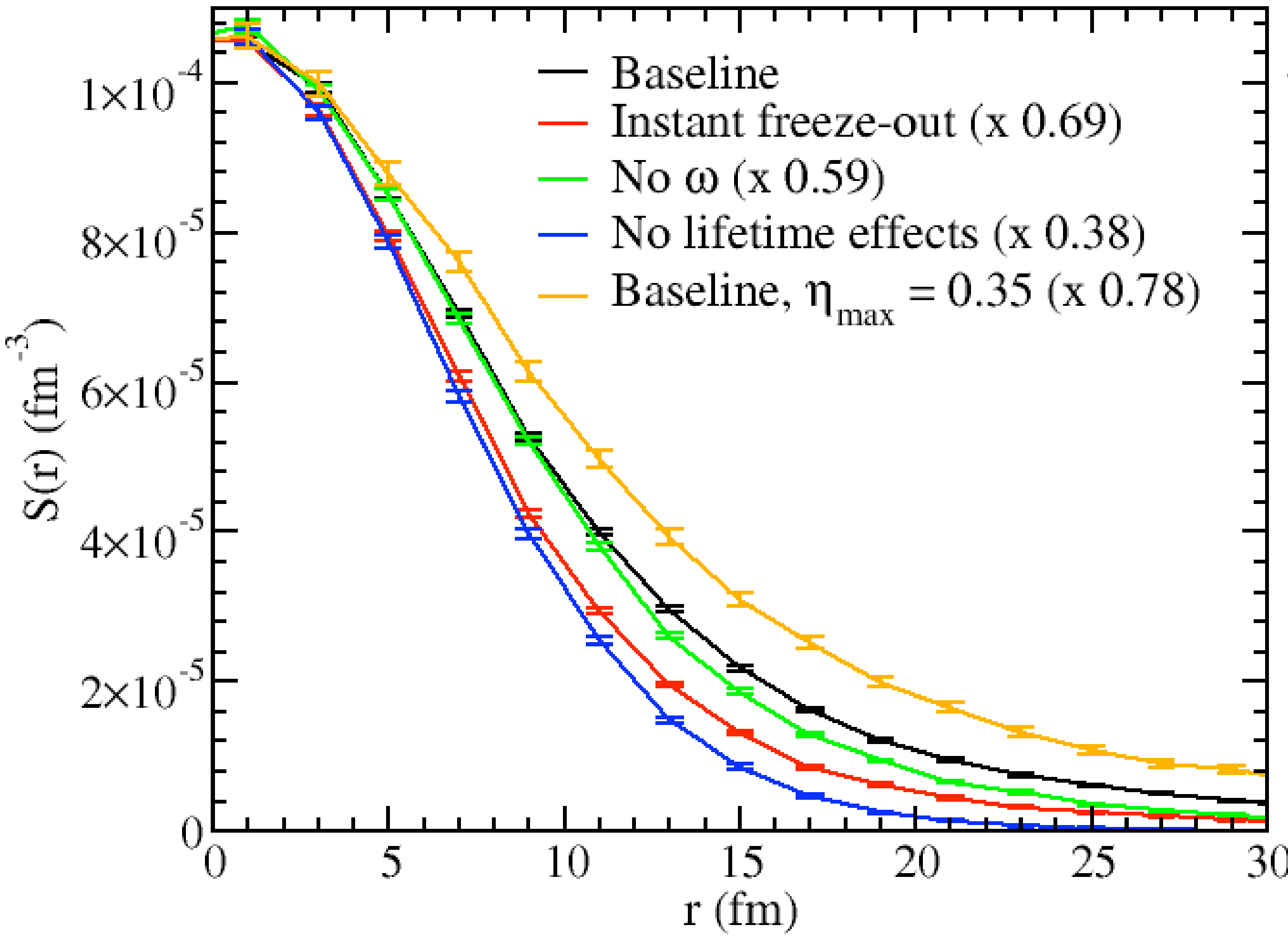
High velocity π's get bigger boost. Boost + finite $\tau \Rrightarrow$ tail, *but only modest core increase*, in L,O directions.

Focus on Outwards Direction



Lifetime effects have dramatic affect
on non-Gaussian halo in outwards direction.

Focus on Outwards Direction



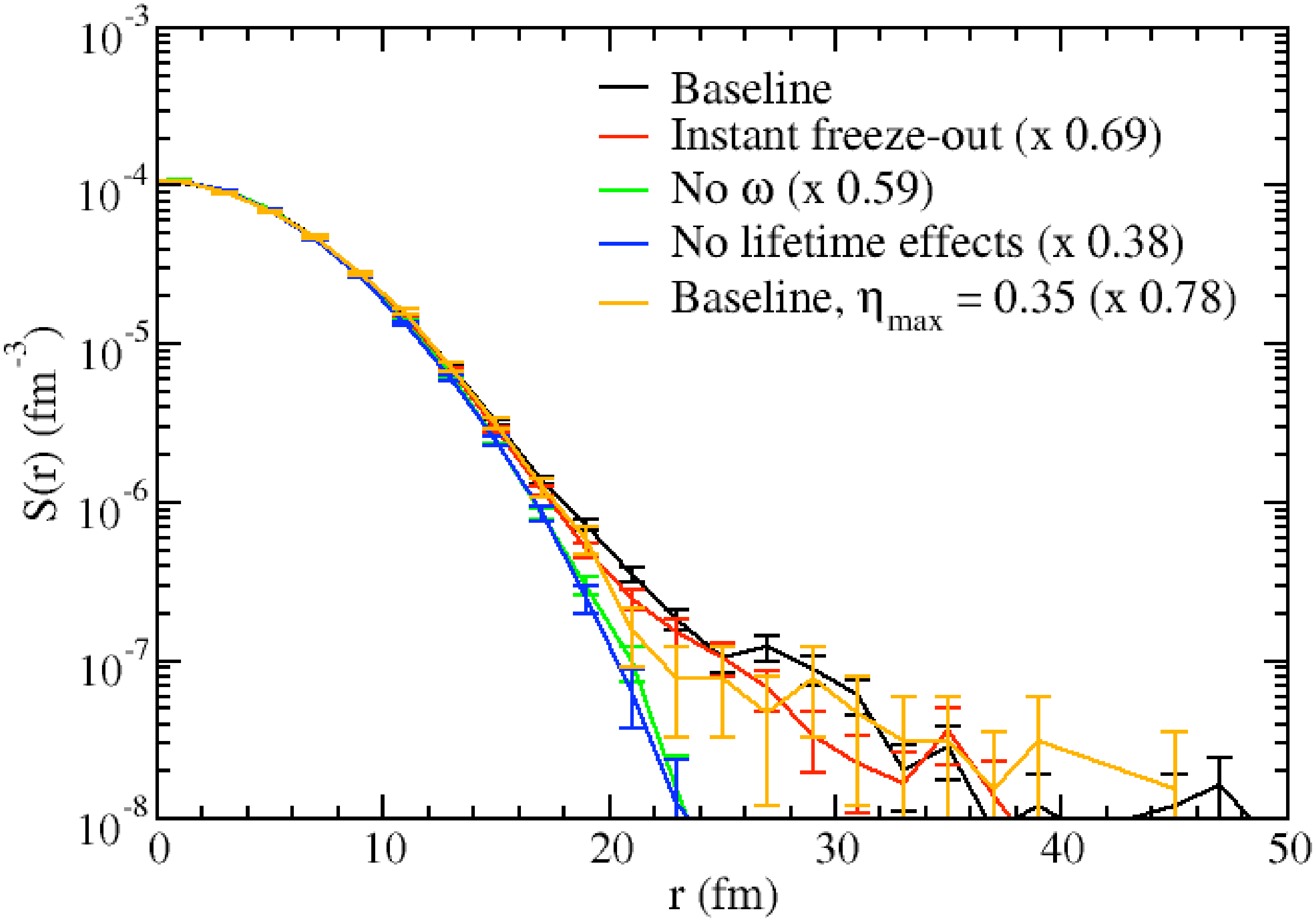
Gaussian R_O fits:

Baseline:	5.63
Instant f/o:	4.78
No ω:	5.35
No lifetime:	4.48
η_{max}=0.35:	6.51

radii in fm,
fit w/ r<15 fm,
fit tolerance 1%

… but only *modest core changes*.

Focus on Sidewards Direction



Minor changes deep in tail reflecting geometry of ω induced halo.
All curves have core radii ~ 4 fm.

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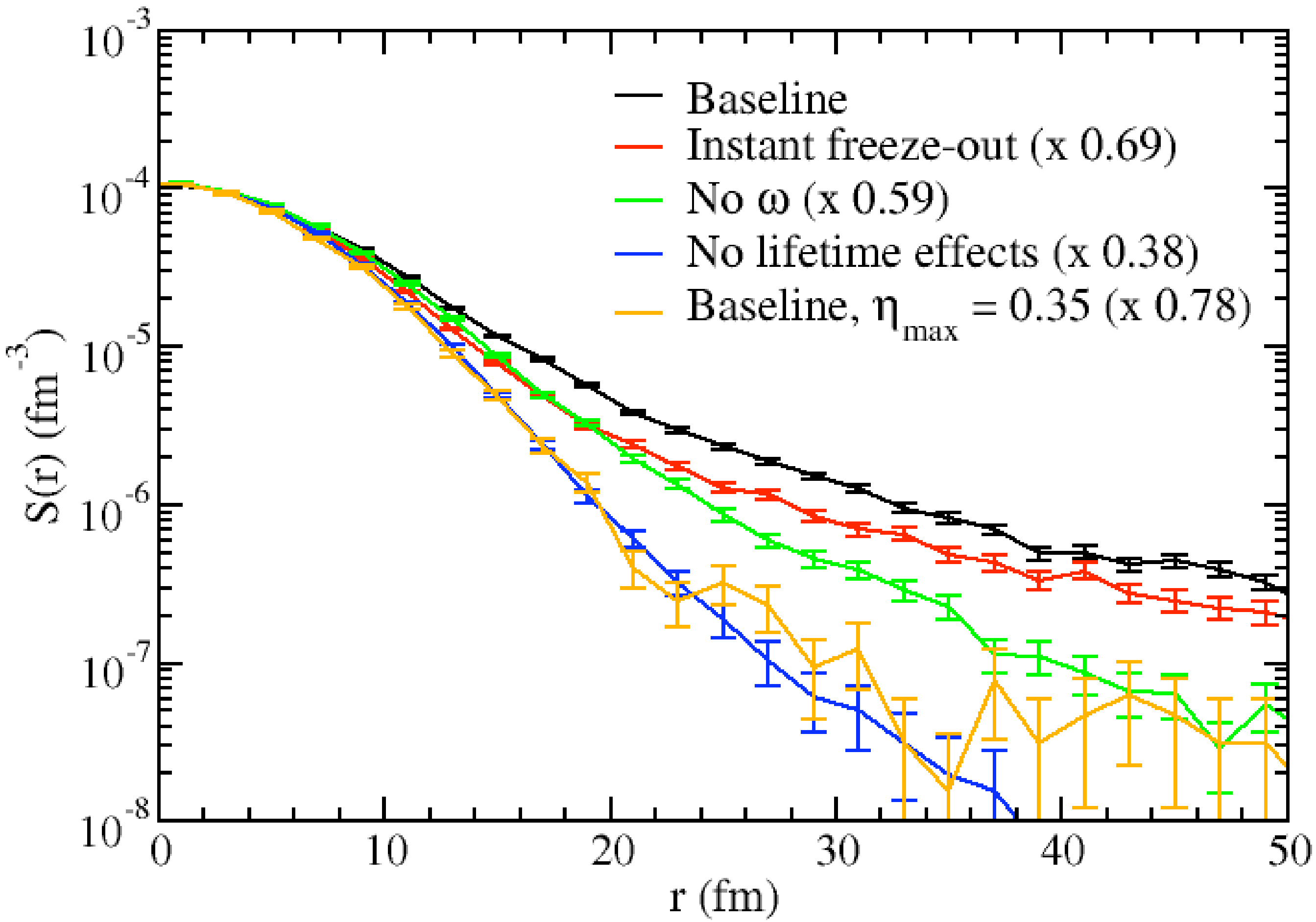
Focus on Longitudinal Direction



Rapidity cut eliminates tail, otherwise similar to outwards direction.

Take Home Messages from Model

* Need emission duration + particle motion to get lifetime effect
* Finite lifetime can create tail, w/o changing core radii substantially
* RHIC HBT puzzle #1 could be Gaussian fit missing tail from long lifetime
* Resonance effects detectable, but similar to freeze-out duration effects
* Maybe need to image 3d kaon correlations to observe system lifetime & determine whether f/o is sudden or not
* Don’t be fooled by acceptance effects

Since source tails hide in Coulomb hole of correlation, imaging RHIC data should shed light on puzzle

Nuclear Physics

Warning:

You are about to see preliminary plots!

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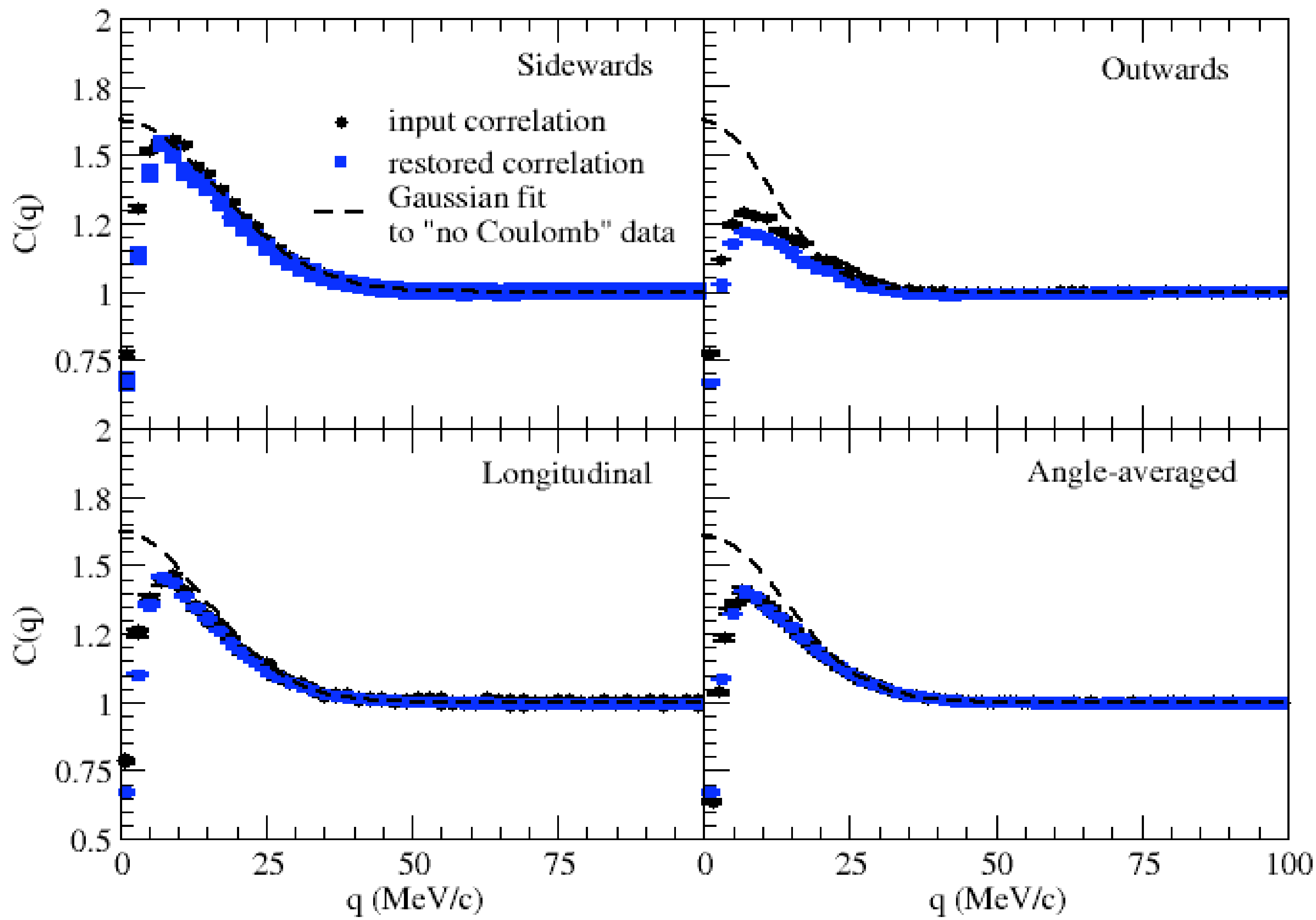
Correlations from Baseline model



Used S. Pratt's CRAB code

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Breaking Problem into 1d Problems

Expand in Ylm's and Legendre polynomials:

$$C_{\ell m}(q) - \delta_{\ell 0} = 4\pi \int_0^\infty dr\, r^2 K_\ell(q,r) S_{\ell m}(r)$$

Where $K(\vec{q},\vec{r}) = \sum_{\ell=0}^{\ell_{max}} (2\ell+1) K_\ell(q,r) P_\ell(\hat{q}\cdot\hat{r})$

$$C(\vec{q}) = \sqrt{4\pi} \sum_{\ell=0}^{\ell_{max}} \sum_{m=-\ell}^{\ell} C_{\ell m}(q) Y_{\ell m}(\hat{q})$$

$$S(\vec{r}) = \sqrt{4\pi} \sum_{\ell=0}^{\ell_{max}} \sum_{m=-\ell}^{\ell} S_{\ell m}(r) Y_{\ell m}(\hat{r})$$

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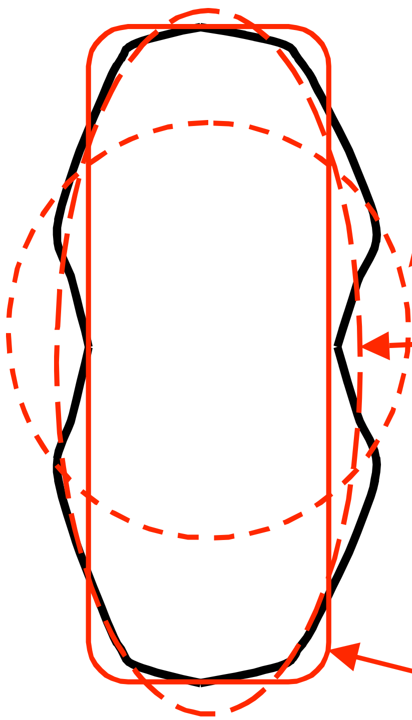
Cartesian harmonics give analogous expressions

What do the terms mean?



- $\ell = 0$: Angle averaged correlation, get access to R_{inv}
- $\ell = 1$: Access to Lednicky offset, i.e. who emitted first (unlike only)
- $\ell = 2$: Shape information: access to R_O, R_S, R_L:
  - $C_{20} \rightarrow R_L$
  - $\alpha C_{00}-(C_{20} \pm C_{22}) \rightarrow R_S, R_O$
- $\ell = 3$: Boomerang/triaxial deformation (unlike only)
- $\ell = 4$: Squares off shape

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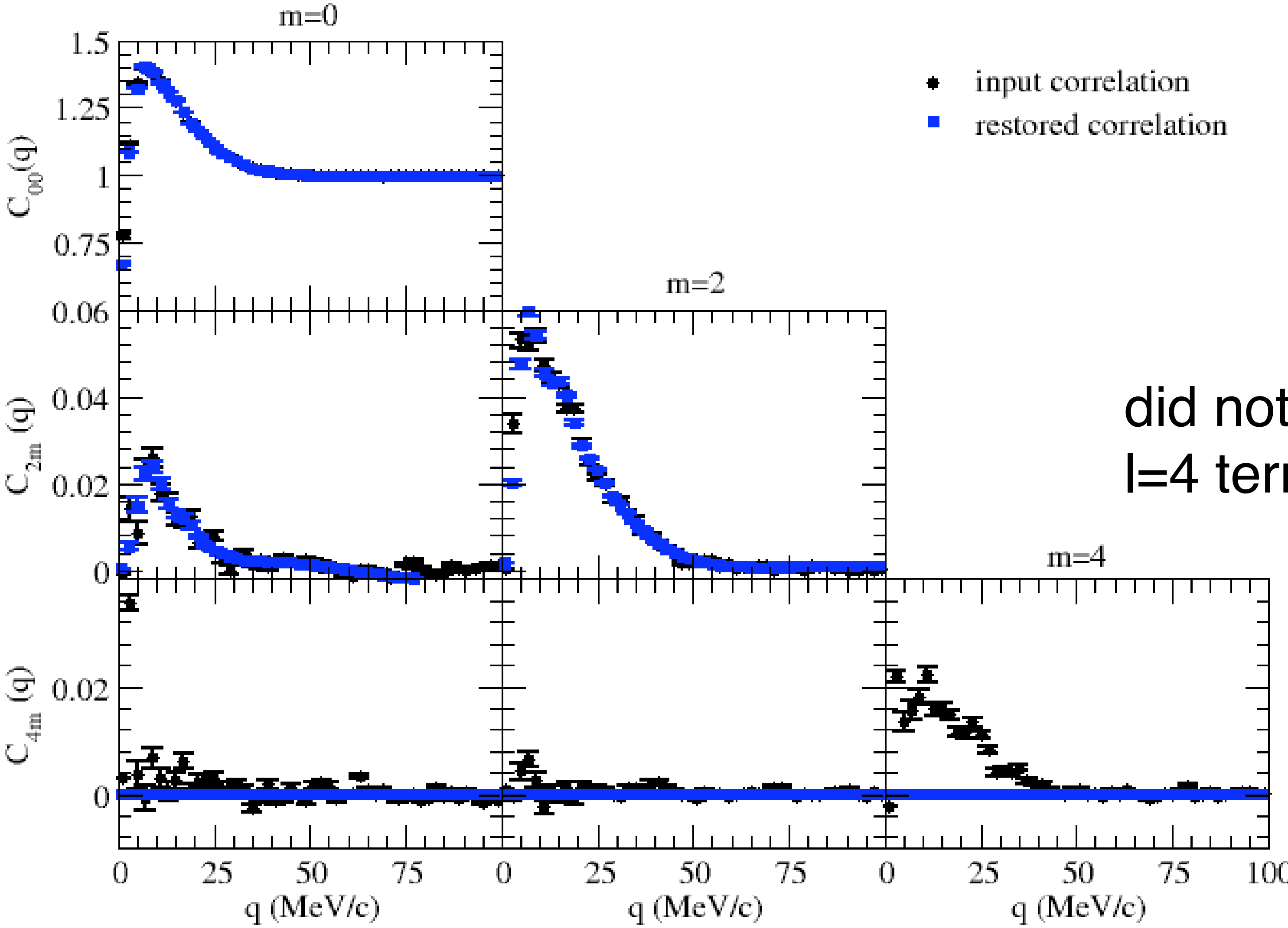
Baseline model's terms



did not image
l=4 terms yet

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Imaging Summary

* Use CorAL version 0.3
* Source radial terms written in Basis Spline representation
  – knots first set using Sampling Theorem from Fourier Theory, then we optimize them to get best χ^2
  – use 3rd degree splines
* Use full Coulomb wavefunction, symmetrized
* Use generalized least-square for inversion
* Use constraints to stabilize inversion
* Cut off at finite *l*, *q* in input correlation

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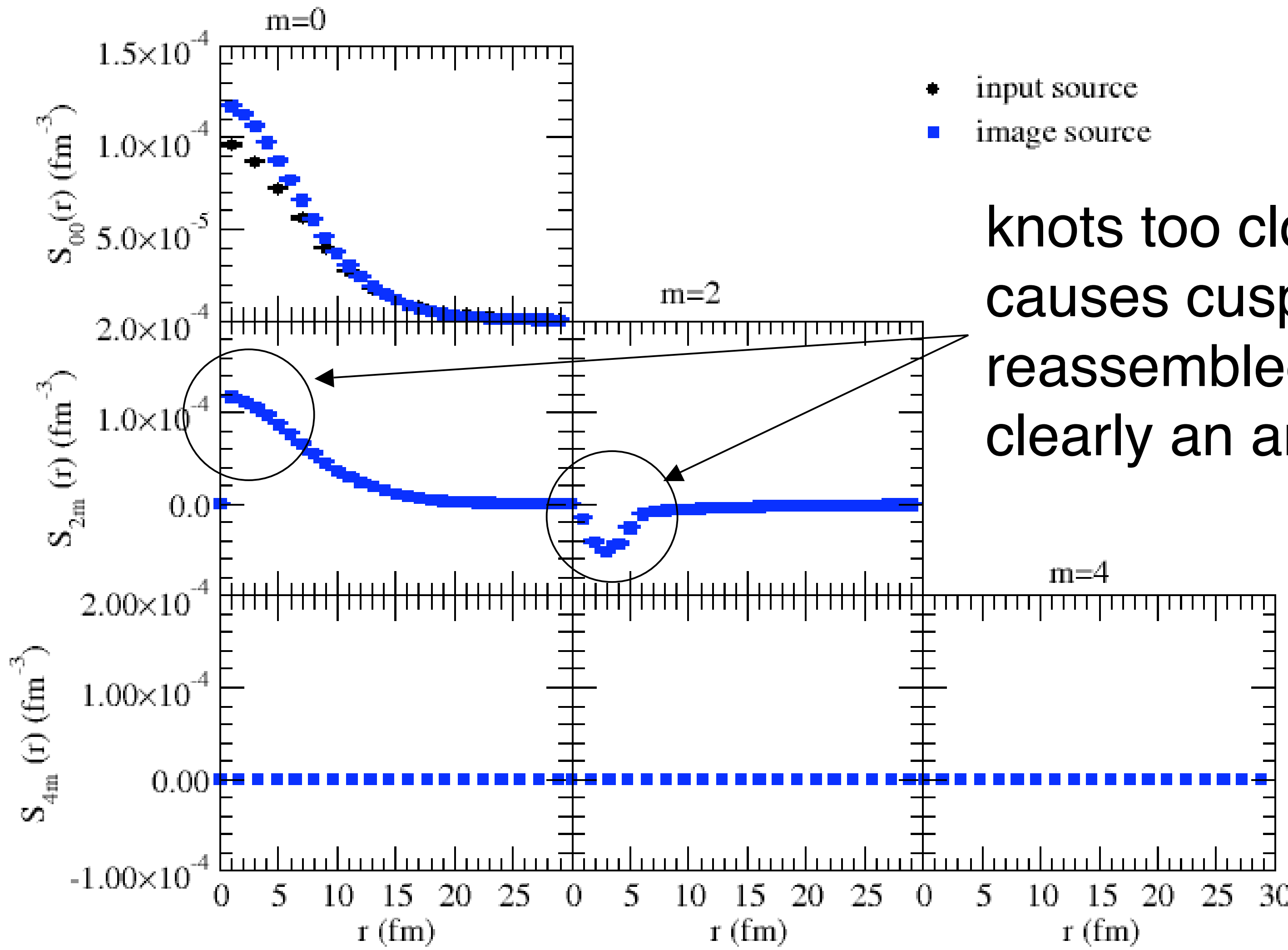
Terms in imaged source



knots too close, causes cusp when reassembled. Is clearly an artifact

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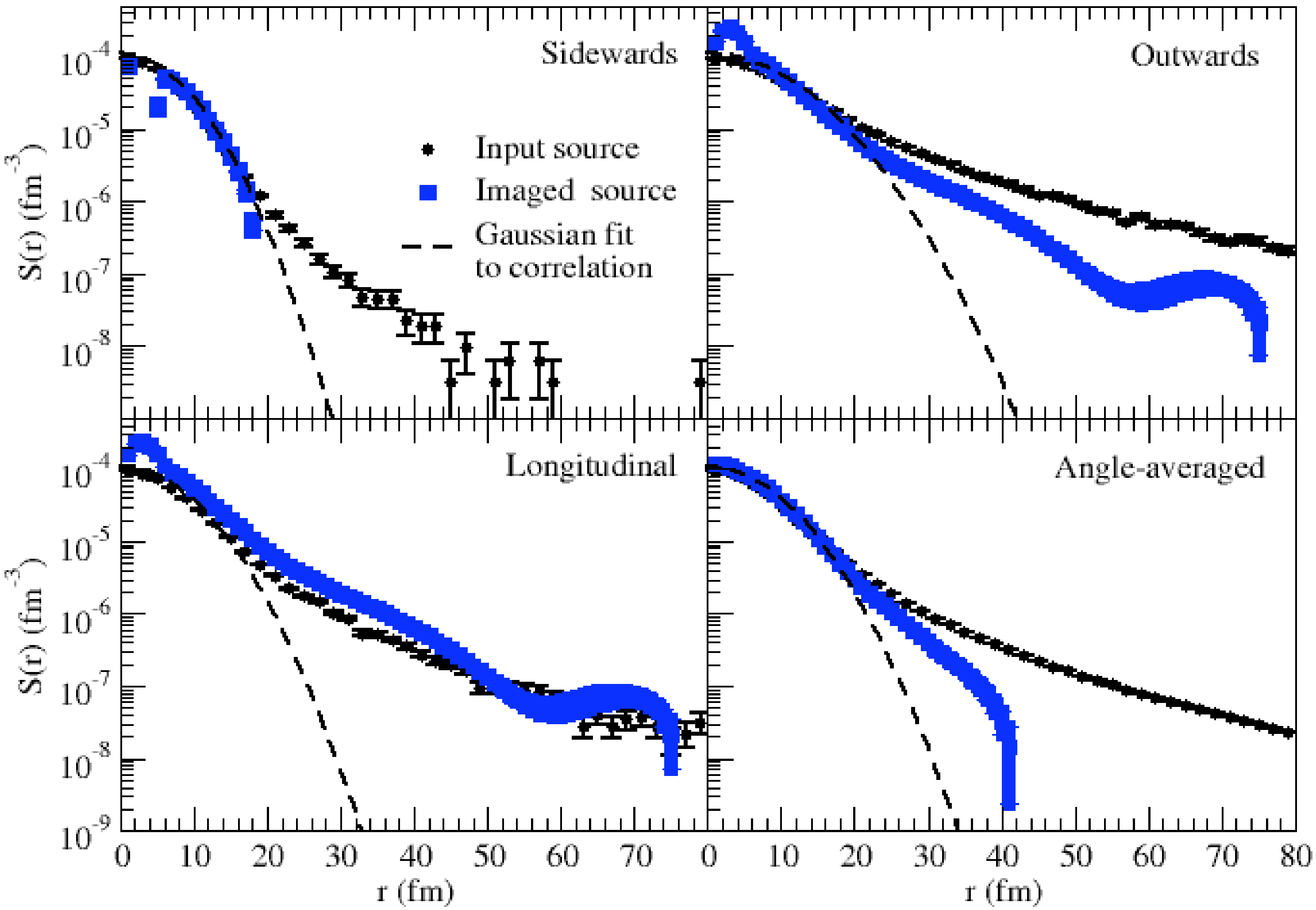
Reassembled imaged source



- Cusp really bad
- Tail hard to resolve
- Gaussian fit actually not bad for core
- lm=00 term needs a lot of tuning (χ^2/ndf=8.4)

Imaging tails in 3d is *hard*…

* Not shown: images of other test cases -- unfortunately they don't look much different from baseline model, yet…
* This is just first pass at imaging these test models, there is a lot of tuning to do:
  – finer q bins in correlation help a lot
  – need those l=4 terms
  – better statistics in model might help
  – newly coded "optimal knot" algorithm needs work
* Can we get the tail's "time constant" from the images? Not easily, $R_{halo} \sim v\tau$, but lot's of different v's comprise same source.

CorAL Features

* Variety of kernels:
  - Coulomb, NN interactions, asymptotic forms
  - Any combination of p, n, π^{+-0}, K^{+-}, Λ, plus some exotic pairs
* Fit a 1d or 3d correlation w/ variety of Gaussian sources
* Directly image a correlation, in 1d or 3d
* Build model correlations/sources from
  - OSCAR formatted output,
  - Blast Wave,
  - variety of simple models
* Build model correlations/sources in Spherical or Cartesian harmonics

CorAL Status

* Merging of three related projects:
  - original CorAL by M. Heffner, fitting correlations w/ source convoluted w/ full kernel
  - HBTprogs in 3d by D. Brown, P. Danielewicz, imaging sources from correlation data w/ full kernels
  - CRAB (++) by S. Pratt, use models to generate correlations, sources w/ full kernels
* Rewritten in C++, open source, nearly stand alone (depends on GSL only)
* Developed on MacOS X, linux, cygwin
* Time scale for 1.0 release: end of summerish

Extra Slides

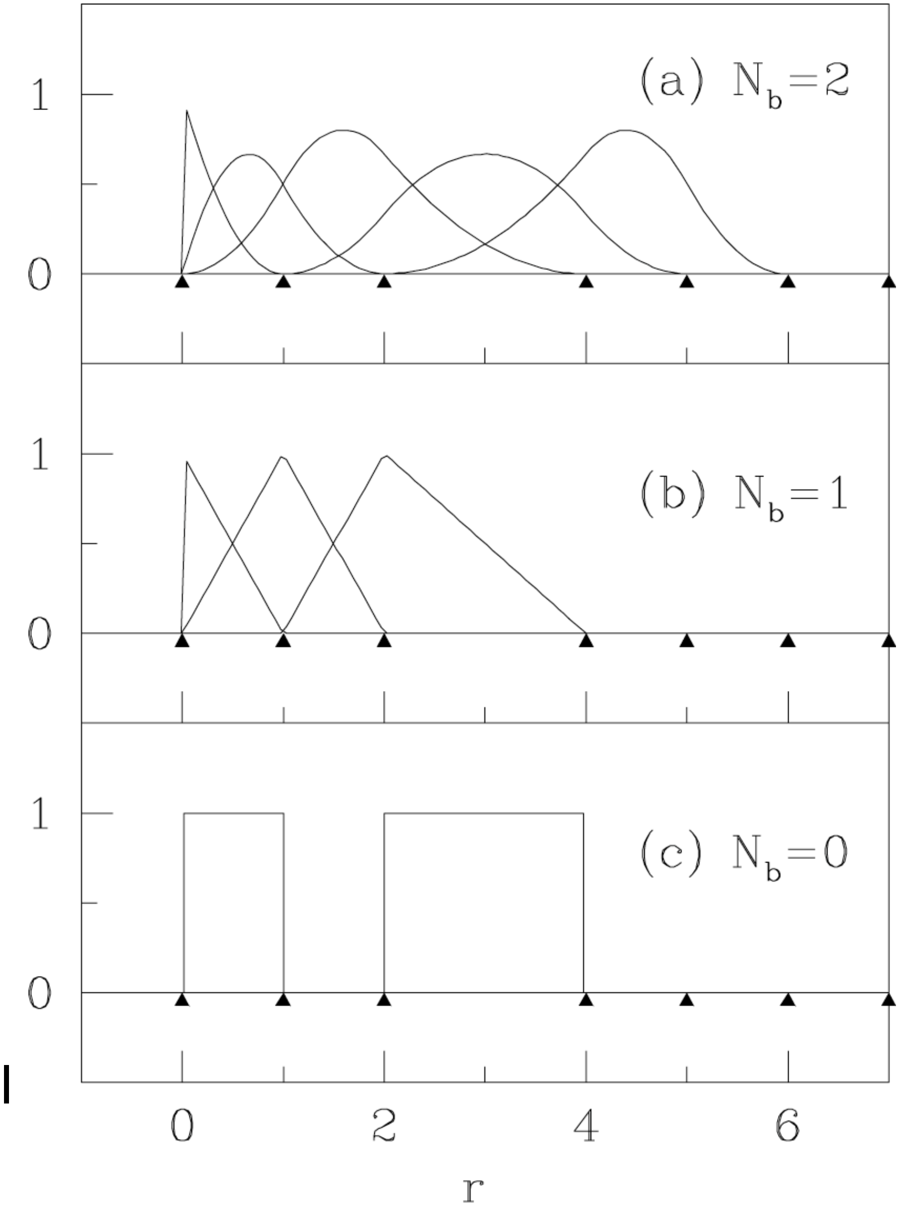
Computational Nuclear Physics

Representing the Source Function

Radial dependence of each term in terms of Basis Splines:

$$S_{\ell m}(r) = \sum_{j=1}^{N_c} S_{j\ell m} B_j(r)$$

- Generalized Splines:
  - N_b=0 is histogram,
  - N_b=1 is linear interpolation,
  - N_b=3 is equivalent to cubic interpolation.
- Recursion relations == fast to evaluate, differentiate, integrate
- Resolution controlled by knot placement
- Form basis for $\boldsymbol{R}^2$, but not orthonormal



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The inversion process

Koonin-Pratt eq. in matrix form: $\mathbf{C} = \mathrm{K} \cdot \mathbf{S}$

this is an ill-posed problem

- kernel not square
- kernel possibly singular
- noisy data
- error propagation

Practical solution to linear inverse problem, minimize χ^2:

$$\chi^2 = \sum_i \frac{(C_i - \sum_j K_j S_j)^2}{\Delta^2 C_{ii}}$$

Most probable source is: $\mathbf{S} = \Delta^2 S \cdot K^T \cdot (\Delta^2 C)^{-1} \cdot \mathbf{C}^{\mathbf{obs}}$

With covariance matrix: $\Delta^2 S = (K^T \cdot (\Delta^2 C)^{-1} \cdot K)^{-1}$

Constraints for 3d problem(s)

Inversion can be stabilized with constraints.
Constraints we use:

Constraint	Purpose	Functional form
$r = 0$ is a maximum of $S(\mathbf{r})$ (for like pairs only)	Constrain the higher ℓ components that are not well controlled due to the r^ℓ dependence of terms in the spherical harmonic expansion.	$\frac{\partial S_{\ell m}}{\partial r}(r \to 0) = 0 \ \ \forall \ell, m$ $S_{\ell m}(r \to 0) = 0 \ \ \forall \ell, m, \ \ell \neq 0$
$S(\mathbf{r}) = 0$ at $r = r_{\max}$	Smooth oscillations in the source at high $\mathbf{r}$ caused by aliasing of statistical and experimental noise in correlation.	$S_{\ell m}(r_{\max}) = 0 \ \ \forall \ell, m$
$S(\mathbf{r}) = 0$ is flat as $r \to r_{\max}$	Smooth oscillations in the source at high $\mathbf{r}$ caused by aliasing of statistical and experimental noise in correlation.	$\frac{\partial S_{\ell m}}{\partial r}(r_{\max}) = 0 \ \ \forall \ell, m$

TABLE I: Equality constraints on the Basis Spline representation for non-spherically symmetric sources.

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