Rewiew of Bose-Einstein/HBT Correlations

in high energy heavy ion physics

T. Csörgő
(MTA KFKI RMKI, Budapest, Hungary)

- **Introduction and History in a nutshell:**
  - 50 years of the Hanbury Brown - Twiss effect
  - Motivation: know the past to have a brighter future
- **Successfull models of the Bose-Einstein/HBT radii at RHIC**
  - Comparison of Au+Au HBT radii with models
  - Less unpromising models
  - Evidence for (directional) Hubble flow
- **Outlook to new directions and new signatures of QGP**

Discovering New Laws

"In general we look for a new law by the following process.
First we guess it.
Then we compare the consequences of the guess to see
what would be implied if this law that we guessed is right.
Then we compare the result of the computation to nature,
with experiment or experience, compare it directly with observation,
to see if it works.
If it disagrees with experiment it is wrong.

In that simple statement is the key to science.
It does not make any difference how beautiful your guess is.
It does not make any difference how smart you are,
who made the guess, or what his name is —
if it disagrees with experiment it is wrong.

/R.P. Feynman/"

A New Look at the Stars

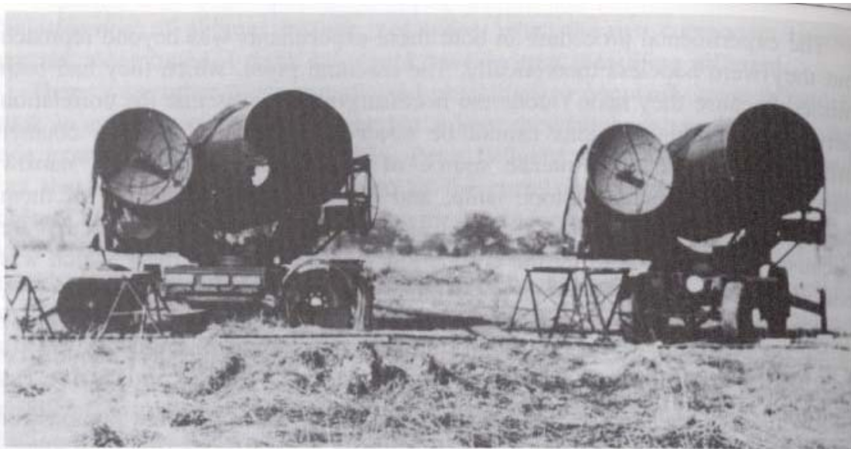


Figure 10.1 The first stellar intensity interferometer; the pilot model of the stellar intensity interferometer at Jodrell Bank in 1955. Two Army searchlights were used to make the first measurement of the angular diameter of a main sequence star (Sirius).

Pilot stellar intensity interferometer
Jodrell Bank, England, (1955)
- First measurement of the angular diameter of a main sequence star (Sirius).
- Stellar Intensity Interferometer, Narrabi, Australia (1963)

Figure 11.1 Narrabri Observatory in 1963, showing the circular railway track (618 foot diameter), the two optical reflectors (22 foot diameter), the central mast and control room, and the very large garage for the reflectors.

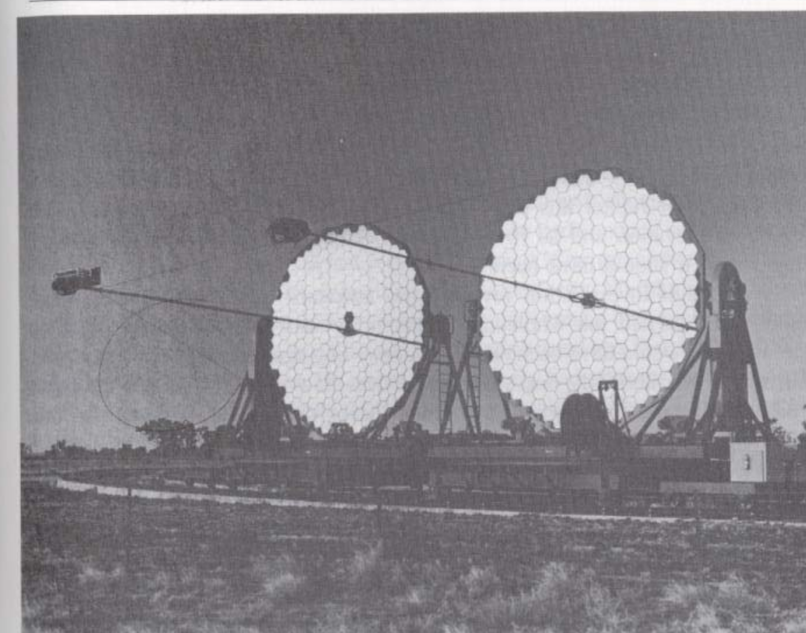


Figure 11.2 The two giant reflectors at Narrabri Observatory; the two optical reflectors of the Stellar Intensity Interferometer. Their reflecting surface was parabolic (diameter 22 feet and focal length 36 feet) and composed of 252 glass hexagonal mirrors front-aluminised and coated with silicon. Each mirror was heated electrically to prevent dew forming.

Goldhaber Goldhaber Lee and Pais (GGLP)

ANTIPROTON-PROTON ANNIHILATION PROCESS

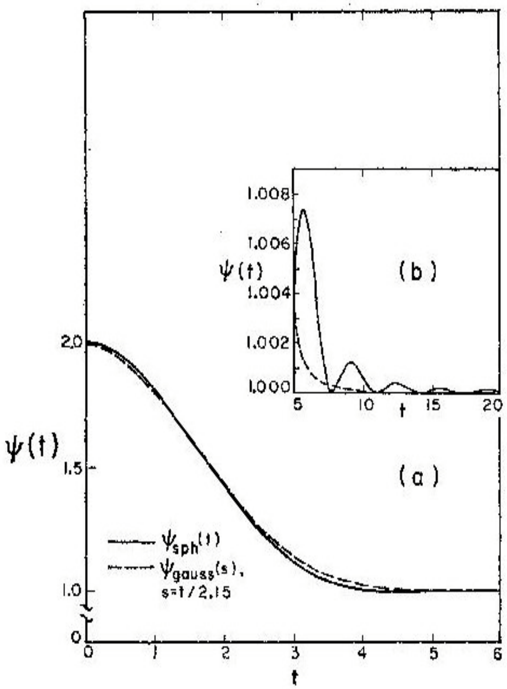


Fig. 1. Evaluation of the correlation functions as a function of the argument. Here $\psi_{sph}^{(t)}$ and $\psi_{gauss}^{(s)}$ correspond to the spherical and Gaussian models, respectively. As can be seen from the figure, the curves corresponding to the two models differ by about 2% at most. Note that the insert [Fig. 1(b)] is enlarged by a factor of 100 vertically and is reduced by a factor of 5 horizontally.

BE effects will be confined more and configurations where two participatin more and more equal to each other. H of the configurations affected by the smaller and smaller and can be ignoi considered, so that also for $\rho\to\infty$ w values. Hence an optimum finite ρ exis BE effects are most marked. This is tively below.

We use $\Phi_N^l(y)$ and $\Phi_N^u(y)$ to denote in $y=\cos\theta$ of pion pairs of like and respectively (θ is the pair angle in the For $\rho\to 0$, both these functions appro limit of the SM distribution denoted ratio of pairs emitted in the backwar those in the forward is denoted by γ. γ^u, and γ^{SM} denotes this ratio for the ca unlike pairs, and the statistical mod relation functions, respectively. In discussion ψ means the relativistic except in Eq. (29).

B. Calculation of the Correlati

1. $\bar{p}+p\to 2\pi^+ + 2\pi^-$

We have

$$R_4(2^+,2^-)\approx\int\frac{d\mathbf{p}_1\cdots d\mathbf{p}_4}{\omega_1\cdots\omega_4}\psi(12)\psi(34)$$

Noted the similarity of Gaussian and spherical sources, deviation on 2 % level

Searching for the ρ meson in $\overline{p}$+p annihilation

Instead of ρ, a more interesting result: increased probability of emitting like-charged pions in the same hemisphere

Interpreted the result in terms of Bose-Einstein symmetrization effects:
INFLUENCE OF BOSE-EINSTEIN STATISTICS ON THE ANTI-PROTON PROTON ANNIHILATION PROCESS
Phys.Rev.120:300-312,1960
by modifying Fermi's thermal model(!)

-> The GGLP effect

"The energy dependence of the angular correlations may provide valuable clues for the validity of our model."

70':Kopylov, Podgoretskii, Cocconi, Shuryak

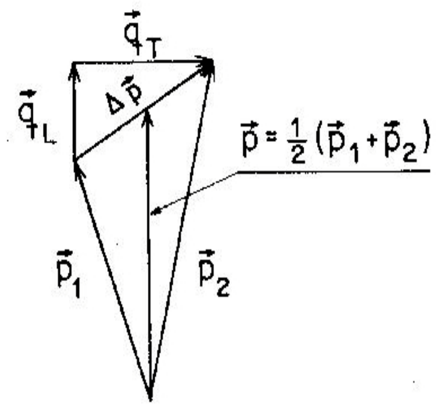


FIG. 6: *Illustration of the Kopylov variables: we see that* $\vec{q_L} \equiv \vec{q_\parallel} \parallel \vec{K}$ *and* $\vec{q_T} \equiv \vec{q_\perp} \perp \vec{K}$. *The figure also shows other notations commonly used:* $\vec{q} \equiv \vec{\Delta p}$ *and* $\vec{K} \equiv \vec{p}$.

$$C(k_1, k_2) = 1 \pm [\frac{2J_1(q_T R)}{q_T R}]^2 [1 + (q_0 \tau)^2]^{-1} ,$$

where

$$\begin{aligned} q_\parallel &= \mathbf{q}.\frac{\mathbf{K}}{|\mathbf{K}|} \\ \mathbf{q_T} &= \mathbf{q} - \mathbf{q}_\parallel \\ q_0 &= E_1 - E_2 \approx \frac{1}{2m}(\mathbf{k}_1^2 - \mathbf{k}_2^2) \\ &\approx \frac{1}{2m}(\mathbf{k}_1 - \mathbf{k}_2).(\mathbf{k}_1 + \mathbf{k}_2) \propto q_\parallel . \end{aligned}$$

Interpretation of GGLP as a Fourier-transformed analogue of the HB-T effect

Kopylov variables and Kopylov model:
Phys. Lett. B50:472-474,1974

- uniformly illuminated sphere.
- Bessel functions (non-Gaussian)
- Use interferometry as a tool!

2 parameters, radius and life-time.

Kopylov and Podgoretskii:Yad. Fiz. 18: 656-666, 1973
boost-invariant formulation for moving sources.

E. V. Shuryak: Phys.Lett. B44 (1973) 387
Birth of Wigner-function formalism

Cocconi: Phys.Lett. B49 (1974) 459
Introduces the concept of thickness (opacity)

P. Grassberger: Nucl.Phys. B120 (1977) 231
First investigation of resonance decays

Andersson, Bowler, Glauber, Gyulassy, Weiner ...

Observation of the effect in all kinds of reactions, including e^+e^- annihilation (G. Goldhaber) λ

M. Deutschmann *et al.* Nucl. Phys. B **204**, 333 (1982).

intercept parameter λ introduced

R. Glauber:

coherent state formalism,overcomplete basis, exp of a thermal density matrix using coherent states

R. J. Glauber, Phys. Rev. **130** (1963) 2529.
R. J. Glauber, Phys. Rev. **131** (1963) 2766.

M. Gyulassy, S. K. Kauffmann and L. W. Wilson, Phys. Rev. C **20** (1979) 2267.

based theory on Klein-Gordon equation and density matrix formalism, ensemble of curren

M. Gyulassy – S. S. Padula et al:

development of the covariant current formalis

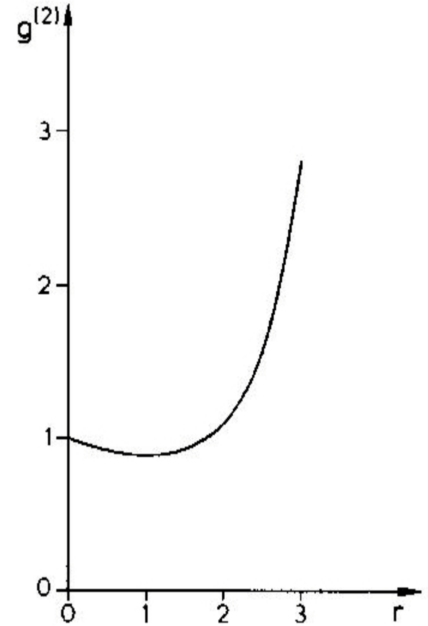


FIG. 1. Second-order correlation $g^{(2)}$ as a function of the squeezed parameter r for pure squeezed states with $\langle n \rangle = 5$.

B. Andersson and W. Hofmann, Phys. Lett. B **169** (1986) 364.

Andersson-Hoffmann model: Bose-Einstein correlations for decaying Lund strings
Bowler and Artru: Hadronic string models for Bose-Einstein correlations,
Bowler: Coulomb correction for resonance halos

A. Vourdas and R. M. Weiner, Phys. Rev. D 38, 2209-2217 (1988)
squeezing: $-1 < \lambda <$ infinity; Quantum optics, partial coherence

QGP and Hydrodynamics

Directional dependence of radii emphasized first by Y. Hama and S. S. Padula

Y. Hama and S. S. Padula, Phys. Rev. D **37** (1988) 3237.

Predictions by S. Pratt: For a strong first order phase transition, $R_{out} >> R_{side}$

S. Pratt, Phys. Rev. Lett. **53** (1984) 1219.

S. Pratt, Phys. Rev. D **33** (1986) 1314.

Similar predictions by G. F. Bertsch, Gong, Tohiyama...

G. F. Bertsch, Nucl. Phys. A **498** (1989) 173C.

G. Bertsch, M. Gong and M. Tohyama, Phys. Rev. C **37** (1988) 1896.

"Observation" of QGP by NA35 in Pb+Pb @ CERN SPS: $R_{out} / R_{side} >> 1$

T. J. Humanic *et al.* [NA35 Coll.], Z. Phys. C **38** (1988) 79. (a 3 sigma effect!)

QGP explanation by Padula, Gyulassy et al: Nucl. Phys. A **498** (1989) 555C.

fits to preliminary $R_{out} >> R_{side}$ data. Strong k_t dependences seen in simulations

Second generation experiments, Pb+Pb @ CERN SPS:

$R_{out} \sim R_{side} \sim R_{long}$ (NA44) Nucl. Phys. A **661** (1999) 435.

Phys. Rev. Lett. **78**, 2080 (1997).

NA44: m_t scaling of the HBT radii + scaling of particle spectra

$R_{out} / R_{side} >> 1$ predicted for Au+Au@RHIC (D. Rischke, M. Gyulassy) nucl-th/9509040

as a signal of a strong 1st order QGP-> hadrons transition nucl-th/9606039

Observed $R_{out}/R_{side} \sim 1$ @ RHIC is not reproduced with relativistic hydrodynamics,

if they do not use the lattice QCD equation of state

(even if they use it but particle emission is continuous).

Impressionism and Hydrodynamics

Analytic solution of non-rel. hydrodynamics:
Zimányi, Bondorf and Garpman -> parameterizations

Relativistic generalization, formulation of Blast-wave model:
Siemens and Rasmussen (relativistic shell)

Blast-wave parameterization (boost invariant, relativistic, 3d expanding cylinder)
E. Schnedermann and U. Heinz

Buda-Lund hydro model (non boost invariant, relativistic, 3d expansion, analytic)
T. Cs. and B. Lörstad
Predicts R_{out} ~ R_{side} ~ R_{out} in the scaling limit,
other limiting behaviours (static limit) are also possible

Heinz-Wiedemann model (similar to Buda-Lund, but temperature is kept constant)

Cracow hydro model (sudden freeze-out, Hubble flow, includes thermal model)
W. Florkowski and W. Broniowski

Renk (Duke) Time evolution of the parameters of the transv. and long. expansio
at freeze-out similar to Buda-Lund or Blastwave

40 years in 4 minutes: an impossible task

The field is established (i.e. there is a book written about it):

R. M. Weiner, Introduction to Bose-Einstein Correlations and Subatomic Interferometry
Publisher: John Wiley & Sons; 1st edition (March 1, 2000), ISBN: 0471969222
W. Kittel and E. A. De Wolf: Soft Multihadron Dynamics
Publisher: World Scientific Publishing Company (June 30, 2005), ISBN: 9812562958

See for details the following excellent review articles:

G. Alexander: hep-ph/0302130
D. Ardouin: Int.J.Mod.Phys. E6 (1997) 391
D. H. Boal,C. K. Gelbke, B. K. Jennings, Rev.Mod.Phys. 62 (1990) 553
T. Cs.: nucl-th/0505019, hep-ph/0001233
T. Cs, B. Lörstad: hep-ph/9901272
U. Heinz: hep-ph/0407360
U. Heinz and B. Jacak: nucl-th/9902020
J. Harris and B. Müller: hep-ph/9602235
W. Kittel: hep-ph/9905394, hep-ph/0111462, hep-ph/0110088
M. A. Lisa, S. Pratt, R. Soltz, U.A. Wiedemann, nucl-ex/0505014
B. Lörstad: Int.J.Mod.Phys.A4:2861-2896,1989
Sandra S. Padula: nucl-th/0412103
W. Bauer, C. K. Geblke and S. Pratt: Ann.Rev.Nucl.Part.Sci.42:77-100,1992
B. Tomasik and U. A. Wiedemann: hep-ph/0210250
U. A. Wiedemann and U. Heinz:nucl-th/9901094
R. M. Weiner: hep-ph/9904389
W. A. Zajc: “A Pedestrian's Guide to Interferometry”, NATO ASI, Il Ciocco, 1992, 435-459

Interesting New Directions

- **Improvement of 2-body Coulomb corrections (CERES, STAR, PHENIX, PHOBOS, Bowler, Sinyukov, ...)**
- **Many-body Coulomb effects (Alt et al)**
- **Non-identical particle correlations (R. Lednicky, S. Panitkin et al, STAR)**
- **Many-body non-identical particle correlations??**
- **Similarities and differences with h+h and e+e- collisions !**

1 New Fit Parameter / 10 years

- **50-es: angular diameter**
- **60-es: intercept parameter λ, radius parameter R**
- **70-es: λ, R and lifetime τ**
- **80-es: λ, R_t, R_l, τ**
- **90-es: λ, R_{side}, R_{long}, R_{out}, $R_{outlong}$**
- **2000-: asHBT, λ, R_{side}, R_{long}, R_{out}, $R_{ij}(\theta, \phi)$**
- **Possible: Non-Gaussian measure: Levy index α**
  - **Non-trivial energy dependence in shape, not in size parameters??**

Interesting New Directions

- Study of continuous emission, escaping probabilities, fluctuating inital conditions (SPHERIO hydro, Hama, Kodama et al, Sinyukov, ...)
- Asimuthally sensivite HBT (STAR, U. Heinz et al, M. Lisa et al)
- Rapidity dependent HBT (PHOBOS)
- Non-Gaussian features (Edgeworth expansion, Levy exponent)
  (S. Hegyi, T. Cs., W. A. Zajc, L3, STAR, ...)
- Imaging, evidence for large resolved scales
  (D. Brown, P. Danielewitz,... -> PHENIX)
- Pion lasers
  (S. Pratt, Q.H. Zhang, J. Zimányi, U. Heinz, Yu.Sinyukov...)
- Mass-modification, back-to-back correlations, squeezing
  (M. Asakawa, T. Cs., M. Gyulassy, Y. Hama, S. Padula, ...)
- Search for axial UA(1) symmetry restoration using $\lambda(m_t)$
  (Kunihiro -> S. Vance, T. Cs., D. Kharzeev, -> PHENIX@QM'05)
- Use penetrating probes : Important New Light On Time Evolution
  - photon interferometry
    (D.K. Srivastava et al, J. Alam, B. Mohanty, WA98, ...)
  - Correlations of lepton pairs (J. Alam, B. Mohanty et al...)
- 3, 4, 5 particle correlations (NA22, UA1, L3, STAR, PHENIX, ...)
- Q-boson interferometry (D. Anchiskin, S. S. Padula, Q.H. Zhang ...)

Phases of QCD Matter

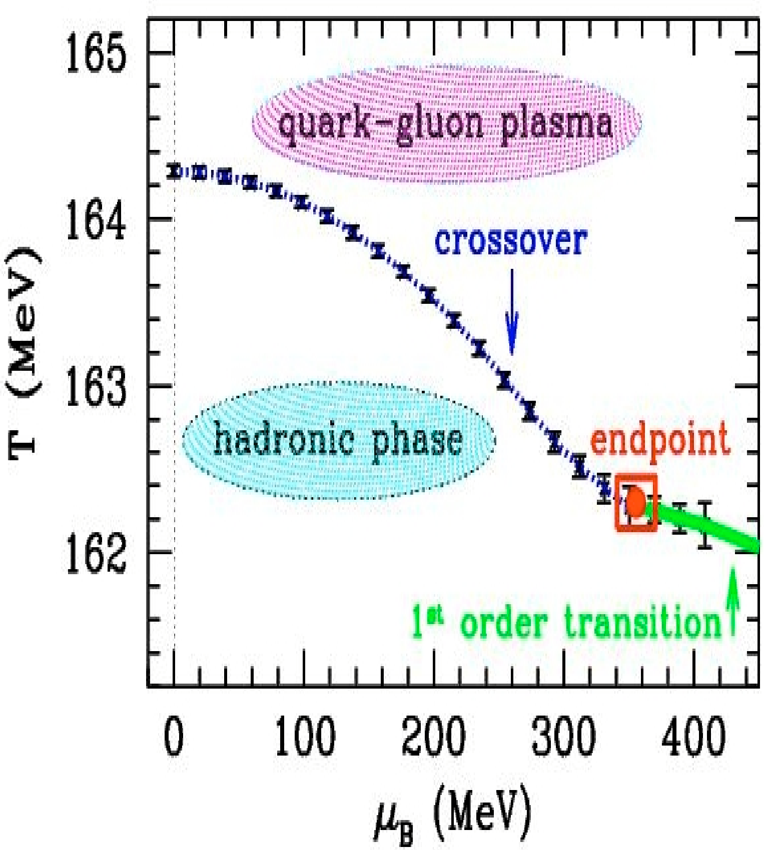
Quark Gluon Plasma

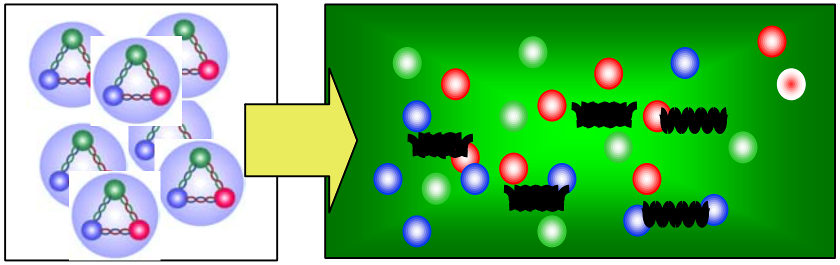
"Ionize" nucleons with heat

"Compress" them with density

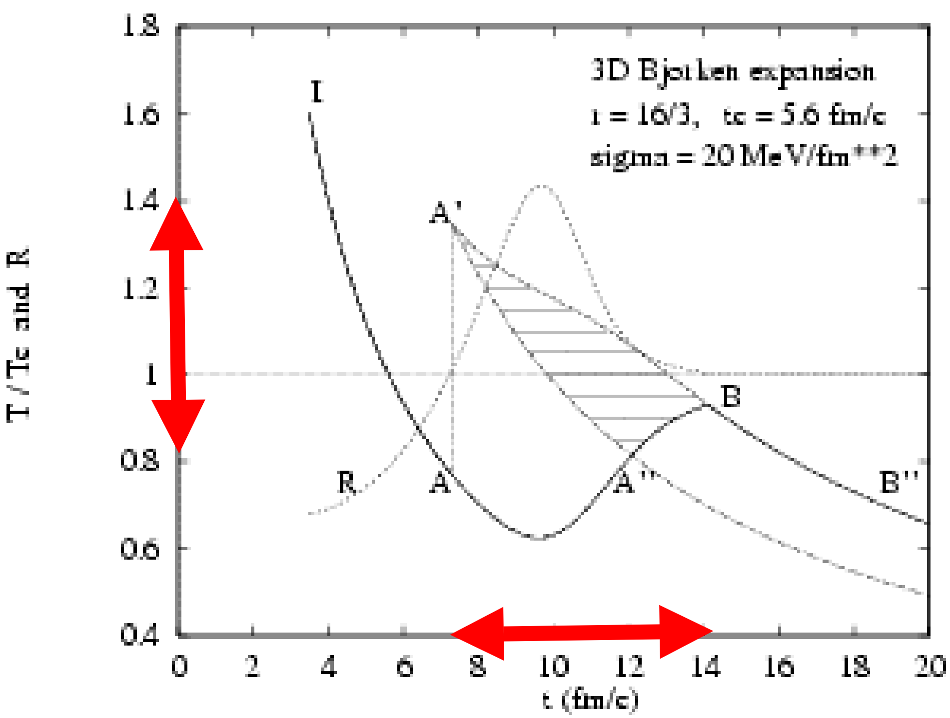
New state(s?) of matter

Z. Fodor and S.D. Katz:
T_c = 164 ± 2 -> cca 190 MeV (QM'05)
even at finite baryon density,
Cross over like transition.
(hep-lat/0106002, hep-lat/0402006)





A successfull RHIC HBT prediction



T. Cs. & L. P. Csernai, hep-ph/9406365, PLB (1994) !

I: Parton cascade (VNI) initial condition for RHIC

A: first possibility to hadronize (pion-flash from a supercooled QGP)
B: end of nonequilibrium mechanism
B'': end of equilbrium freeze-out

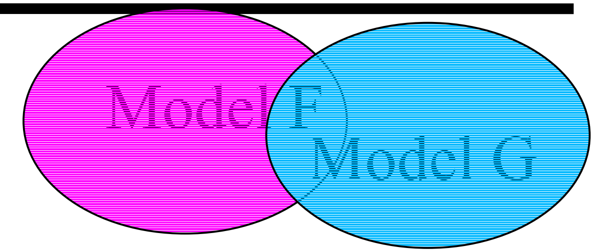
3d Bjorken = Hubble flow

τ predicted: 8-13 fm/c @RHIC
τ from BL data analysis: 7.3 +- 0.6 fm/c @RHIC
$R_{out} \sim R_{side} \sim R_{long}$

successfully predicted also:

- Universality of spectra (same emission S(x,K) for all hadrons)
- Universality (equality) of HBT radii
- Strangeness enhancement remains QGP signal, not modified by hadron flash
- Lack of in-medium hadron modification (no mass-shift of ϕ)

Nature hides her secrets in data (D)



Concern: data points and errorbars

Question 0: Do the models (E,F,G,H) describe the data?

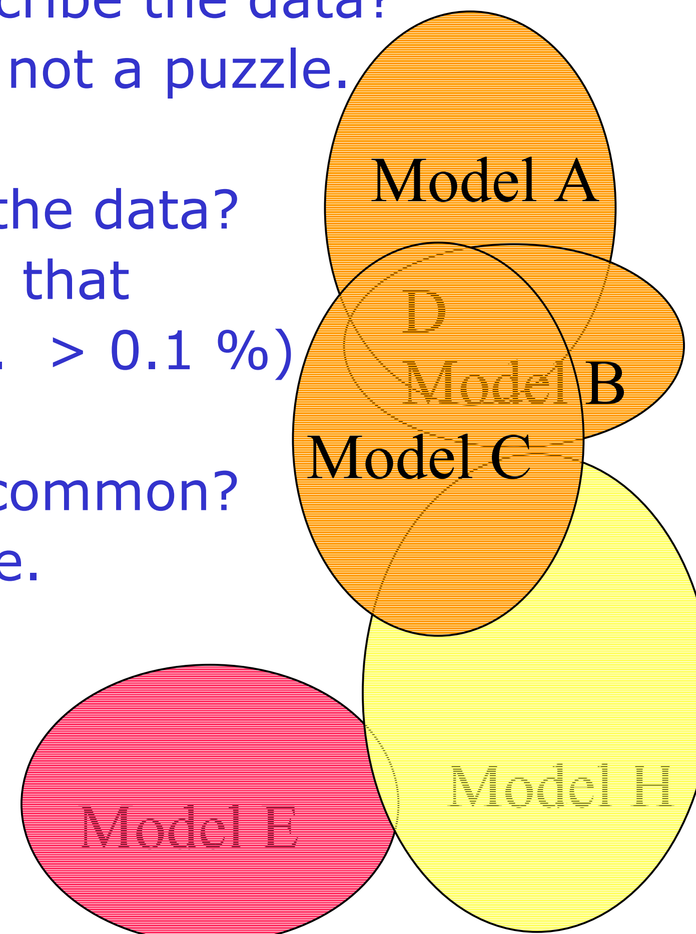
Answer 0: These models fail, but this is not a puzzle.

Q. 1: Are any other models that descibe the data?

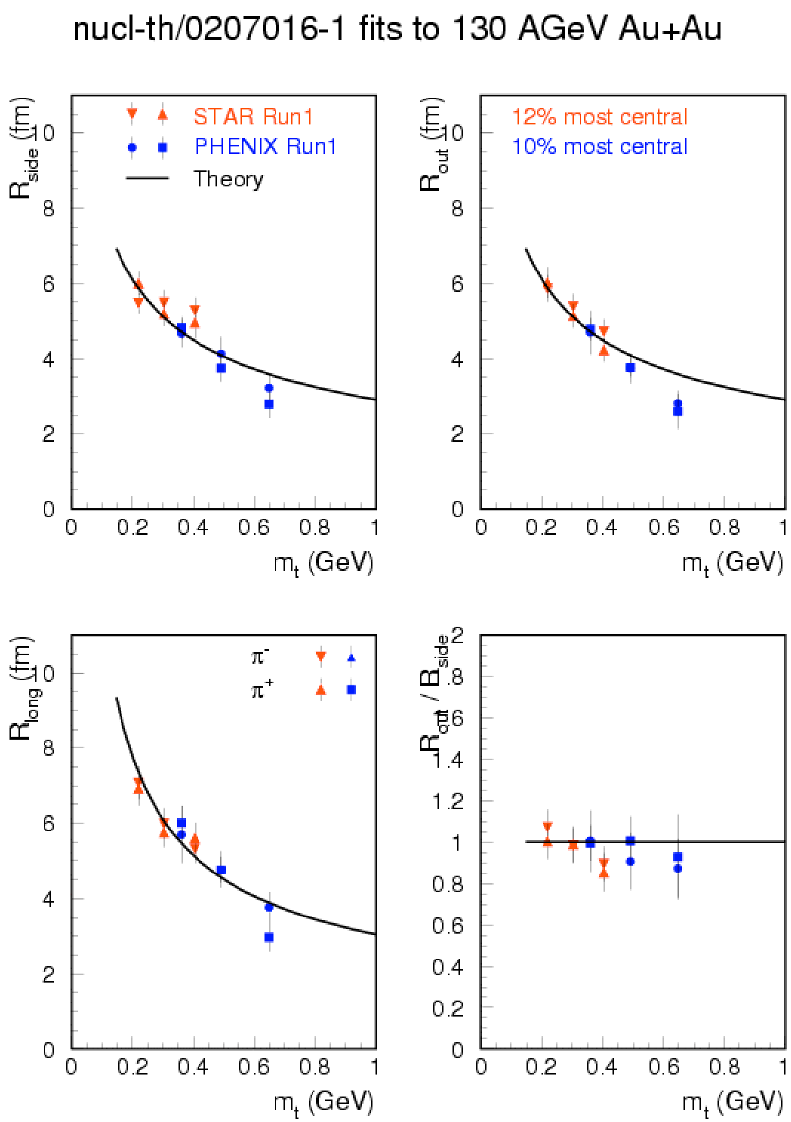
A. 1: Yes, there are three models (A,B,C) that cannot be excluded (Conf. Lev. > 0.1 %)

Q. 2: Do these models have anything in common?

A. 2: Yes, and this where the data (D) are. This common part is what Nature is trying to tell us.



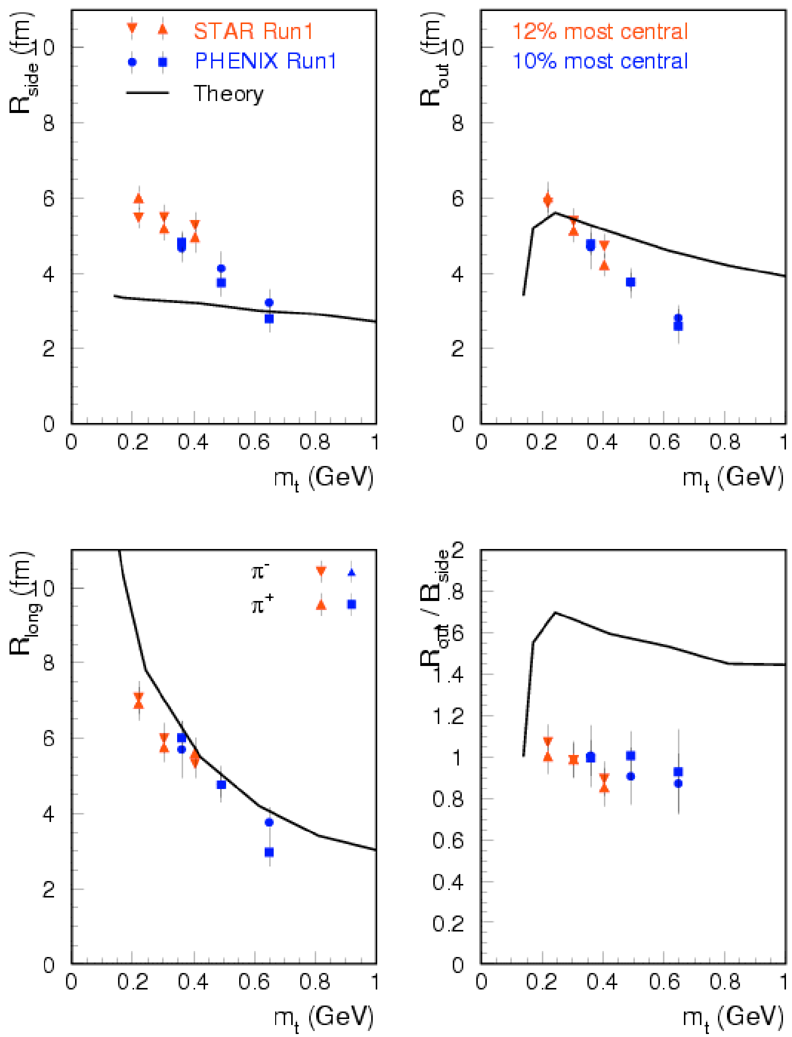
Comparison of results of models



Acceptable

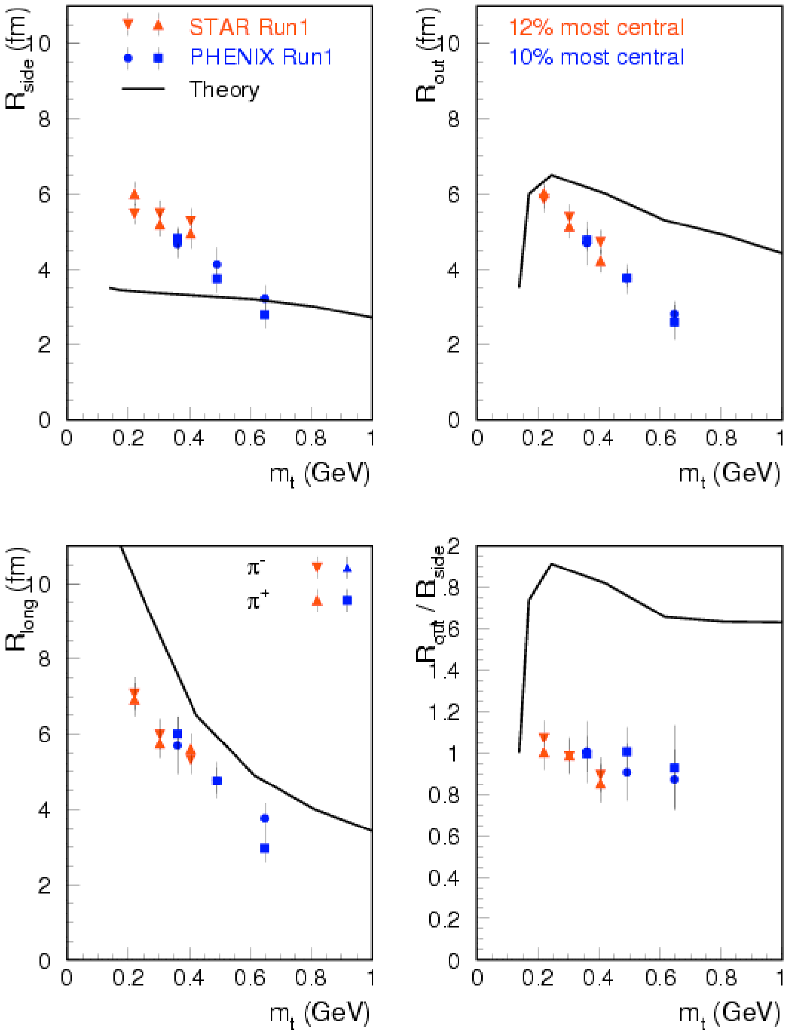
Comparison of results of models

nucl-th/0208068-1 fits to 130 AGeV Au+Au



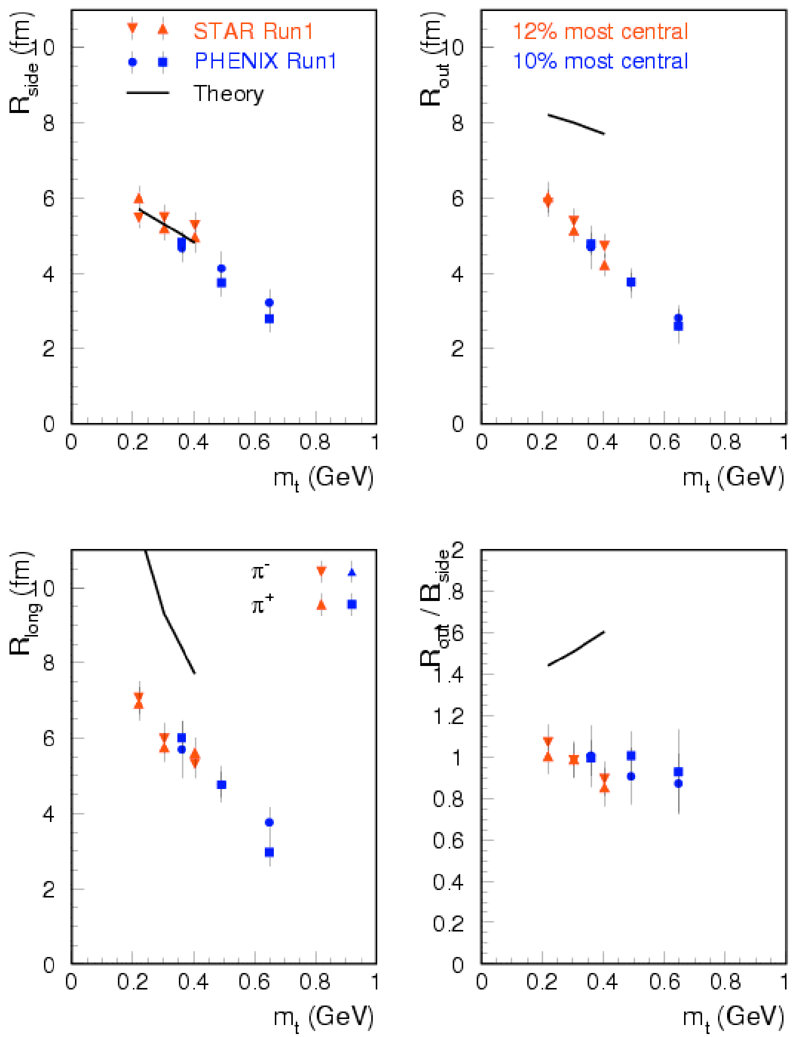
Comparison of results of models

nucl-th/0208068-2 fits to 130 AGeV Au+Au



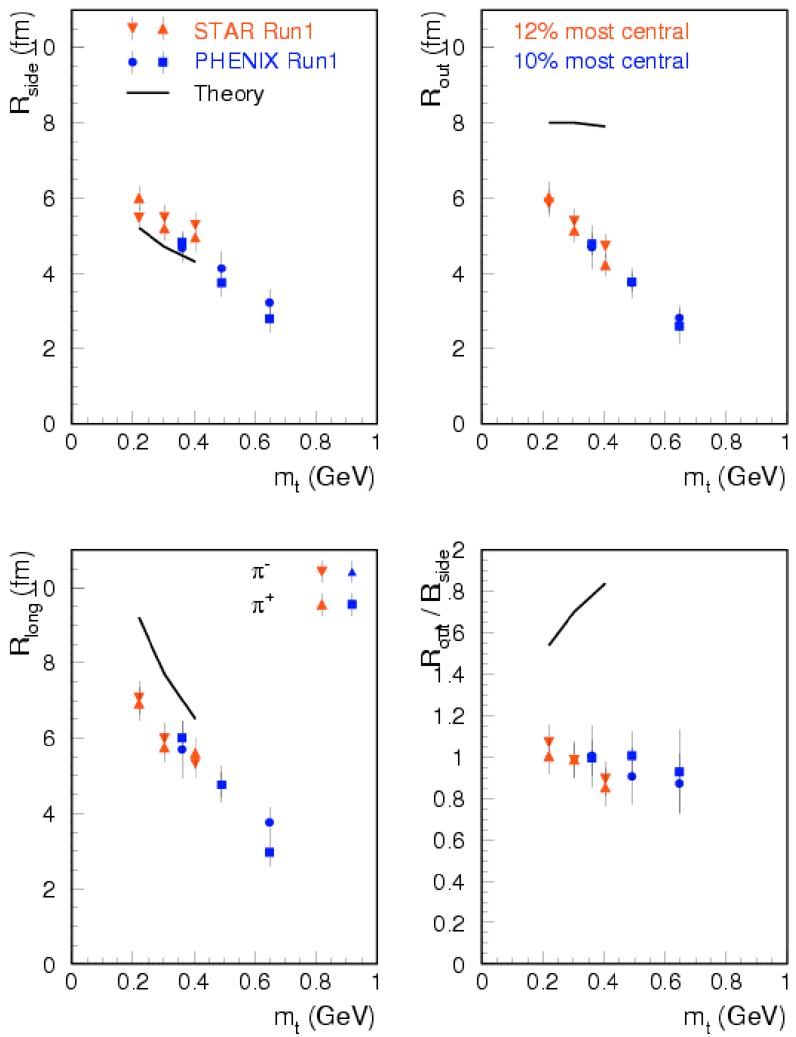
Comparison of results of models

nucl-th/0209055-1 fits to 130 AGeV Au+Au

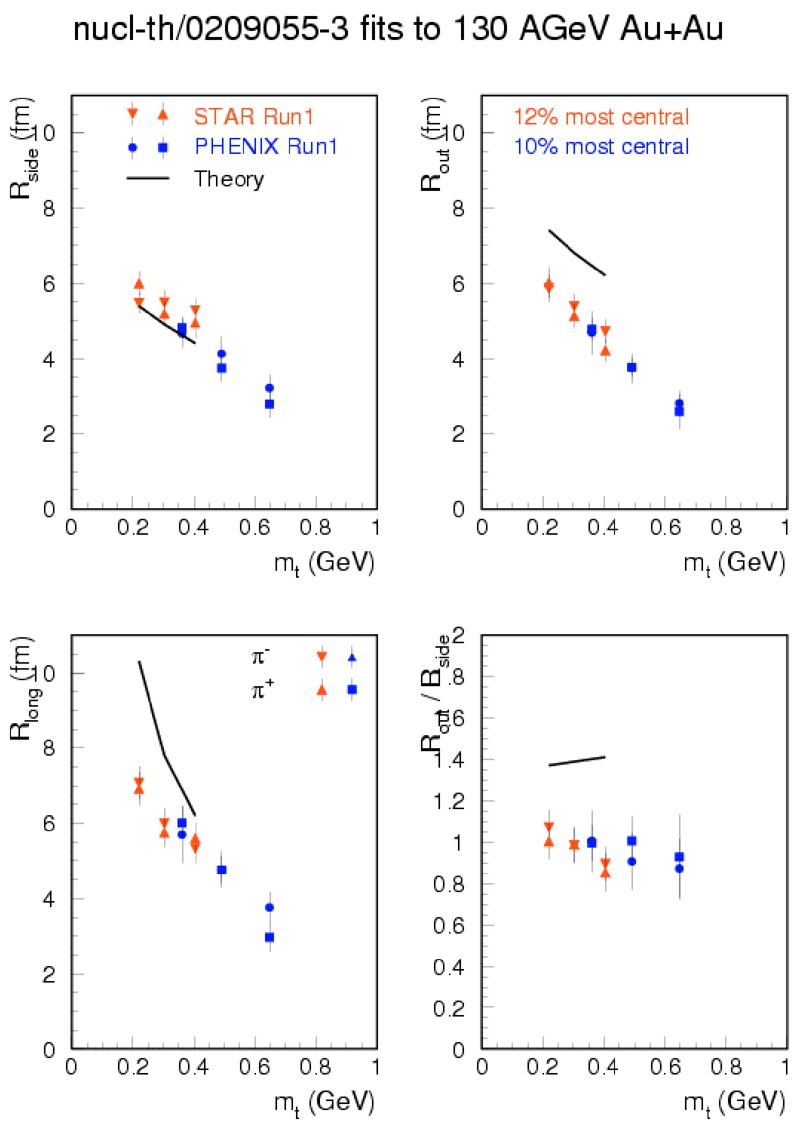


Comparison of results of models

nucl-th/0209055-2 fits to 130 AGeV Au+Au

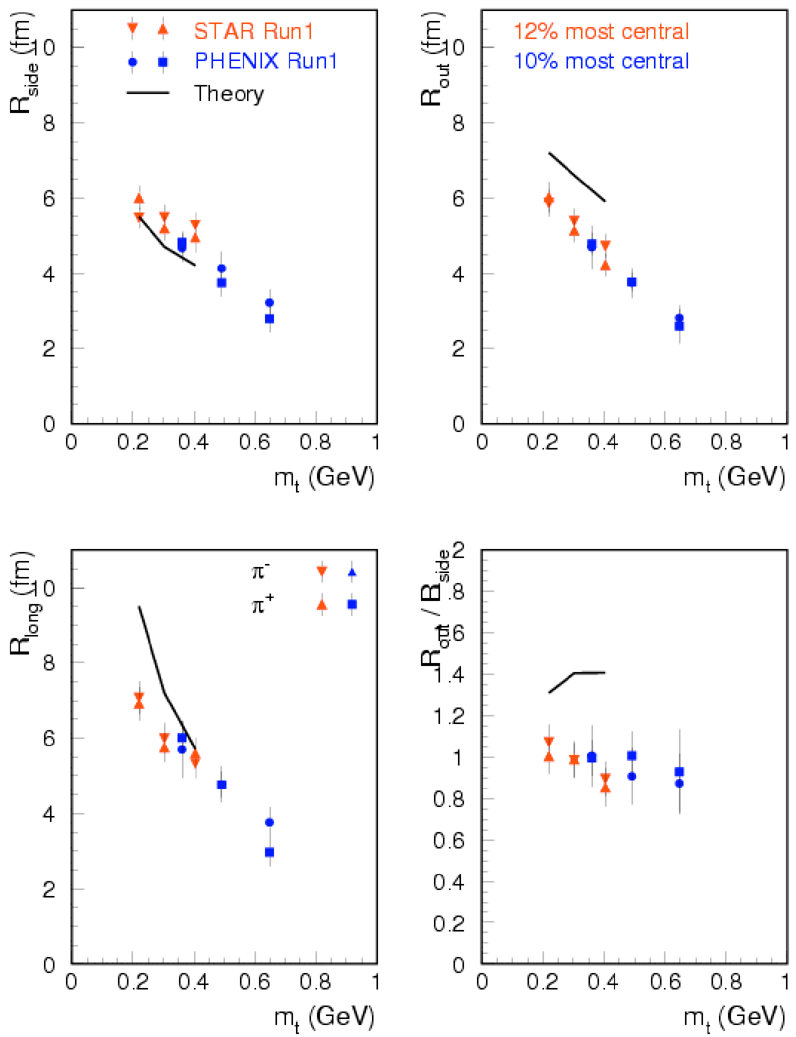


Comparison of results of models

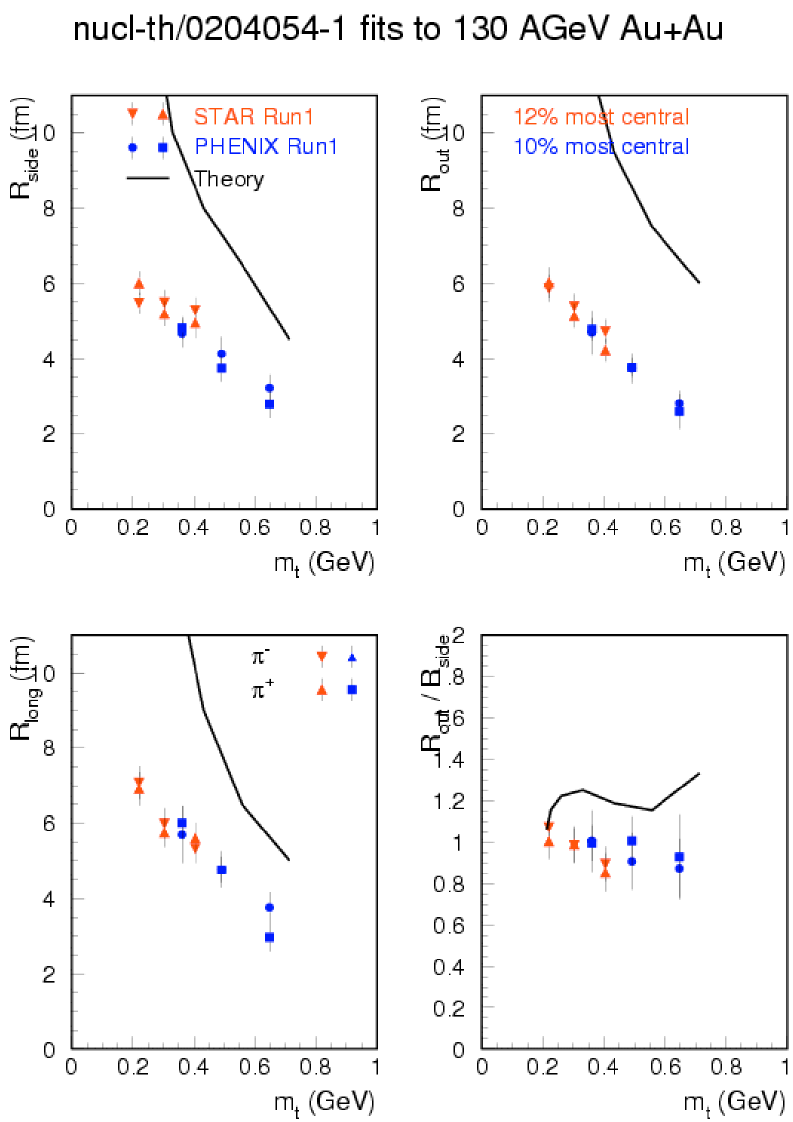


Comparison of results of models

nucl-th/0209055-4 fits to 130 AGeV Au+Au

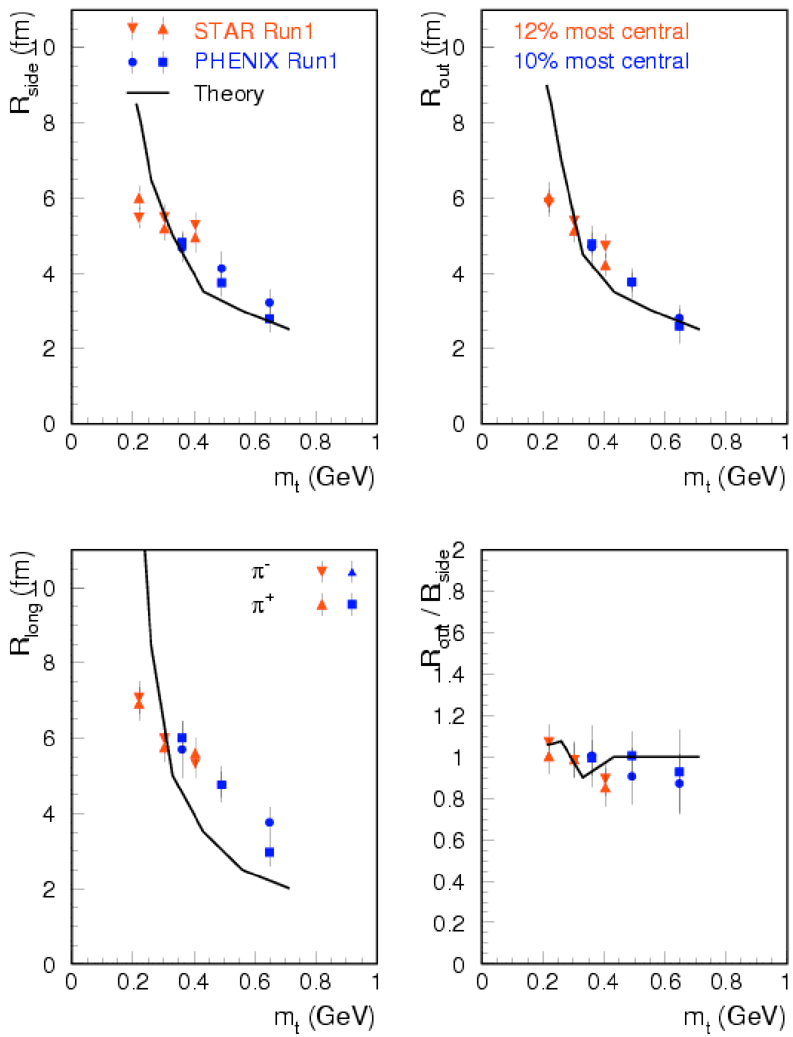


Comparison of results of models

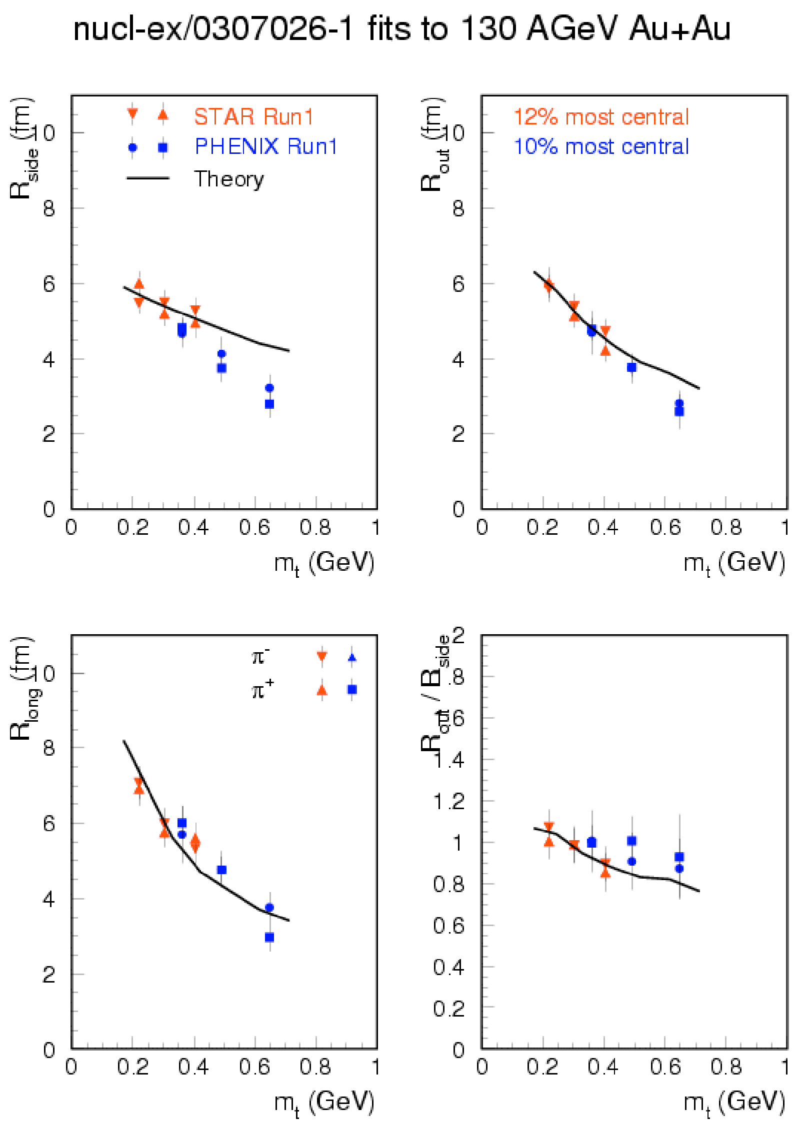


Comparison of results of models

nucl-th/0204054-2 fits to 130 AGeV Au+Au



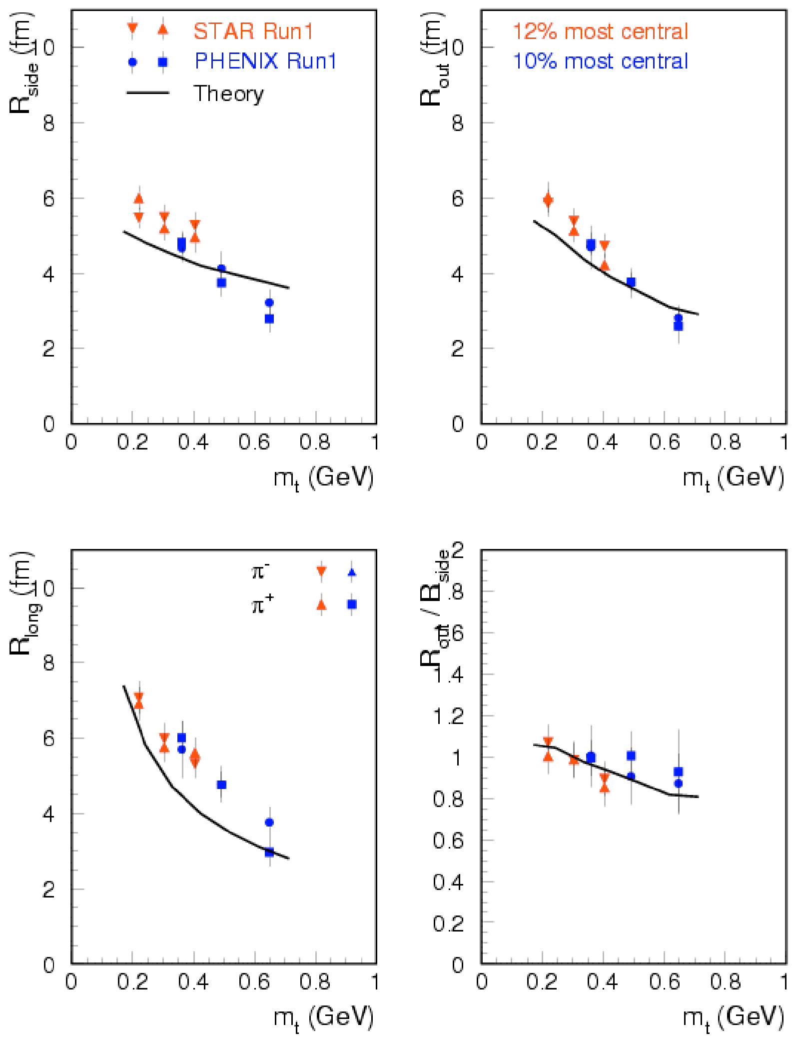
Comparison of results of models



Acceptable

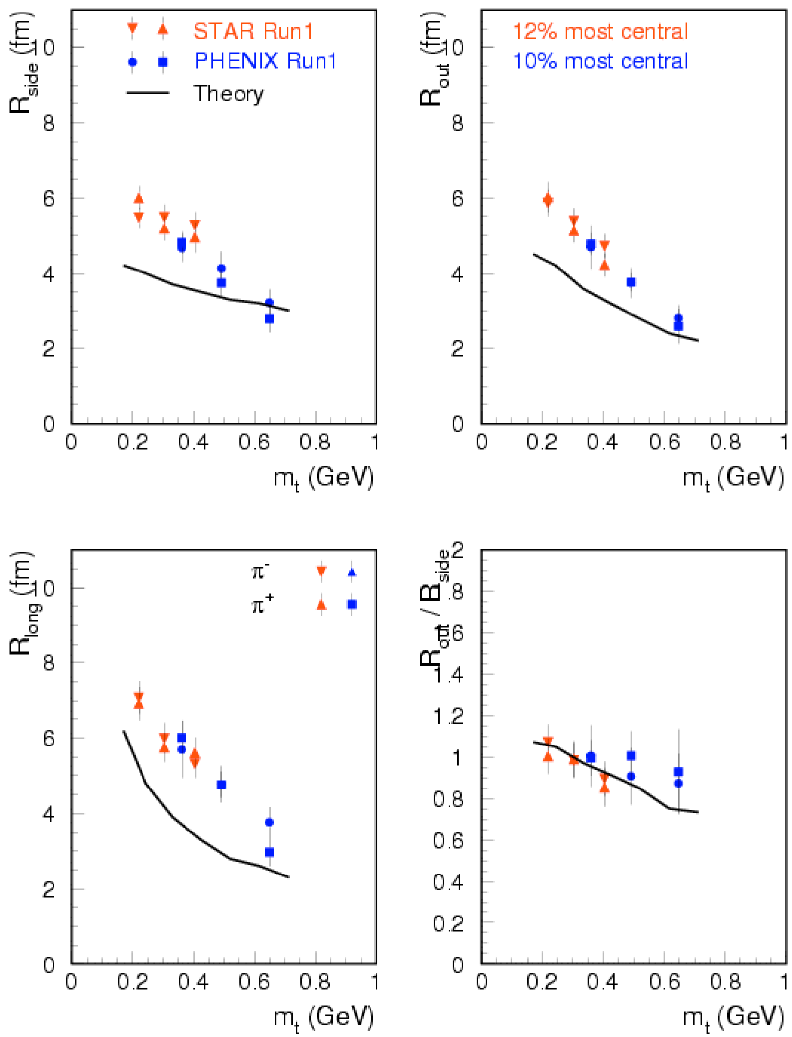
Comparison of results of models

nucl-ex/0307026-2 fits to 130 AGeV Au+Au



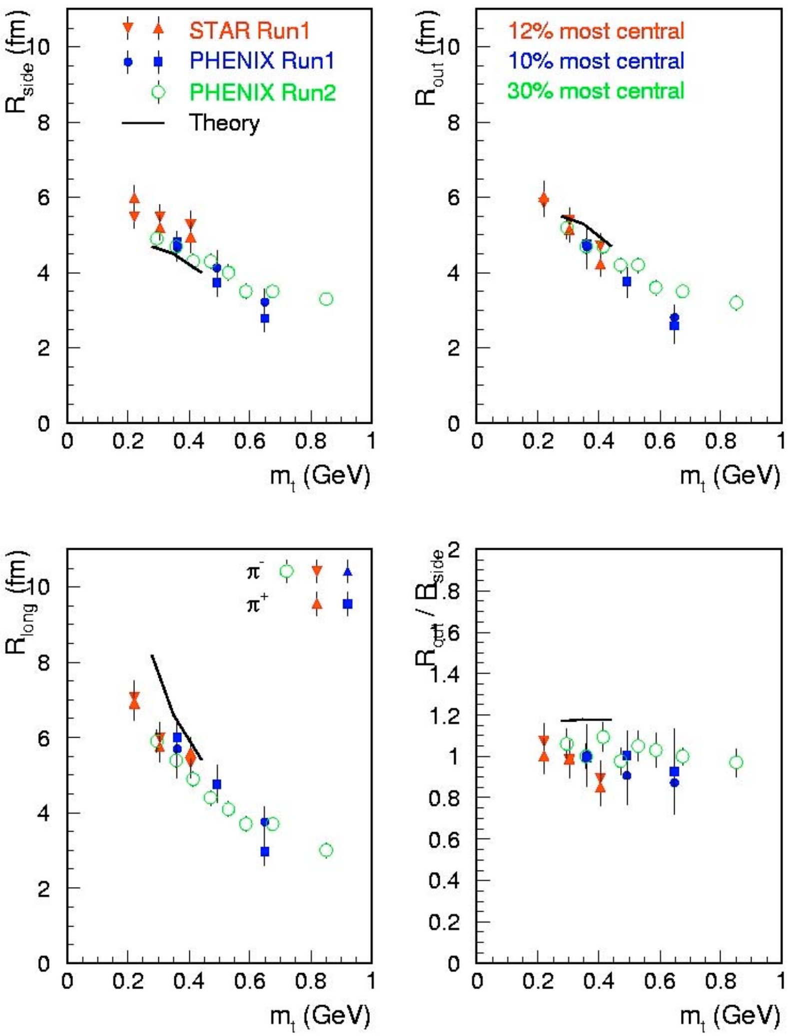
Comparison of results of models

nucl-ex/0307026-3 fits to 130 AGeV Au+Au



Comparison of results of models

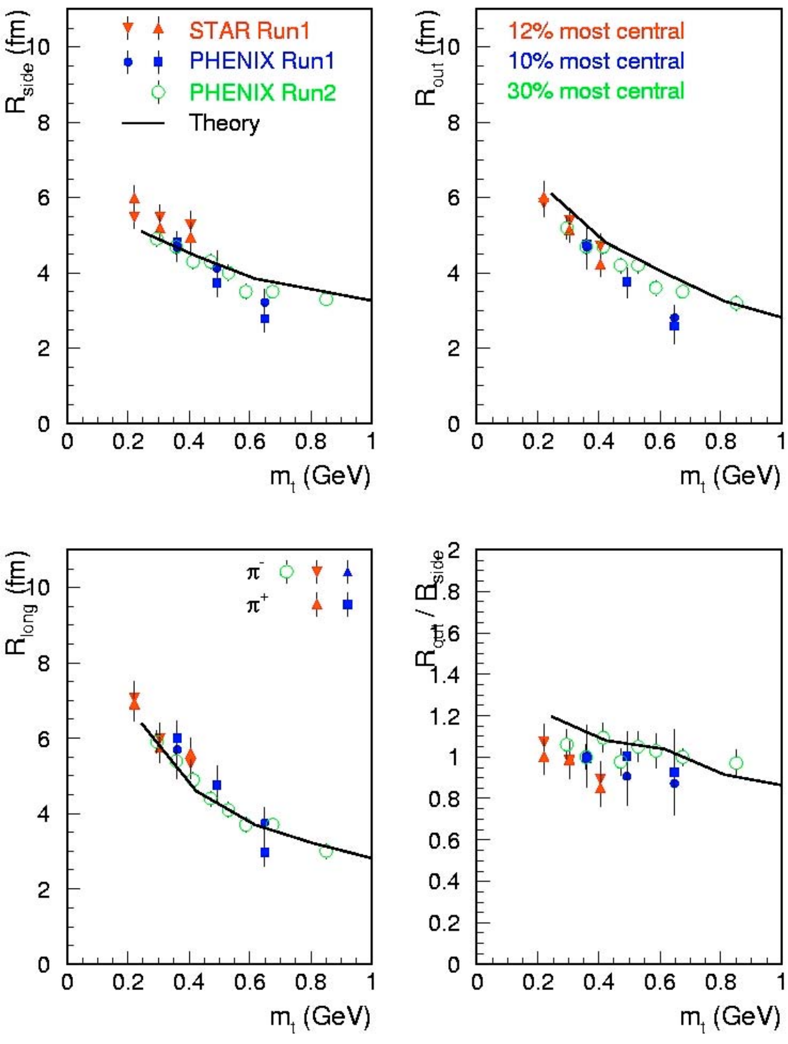
nucl-th/0205053 fits to 200 AGeV Au+Au



~Acceptable

Comparison of results of models

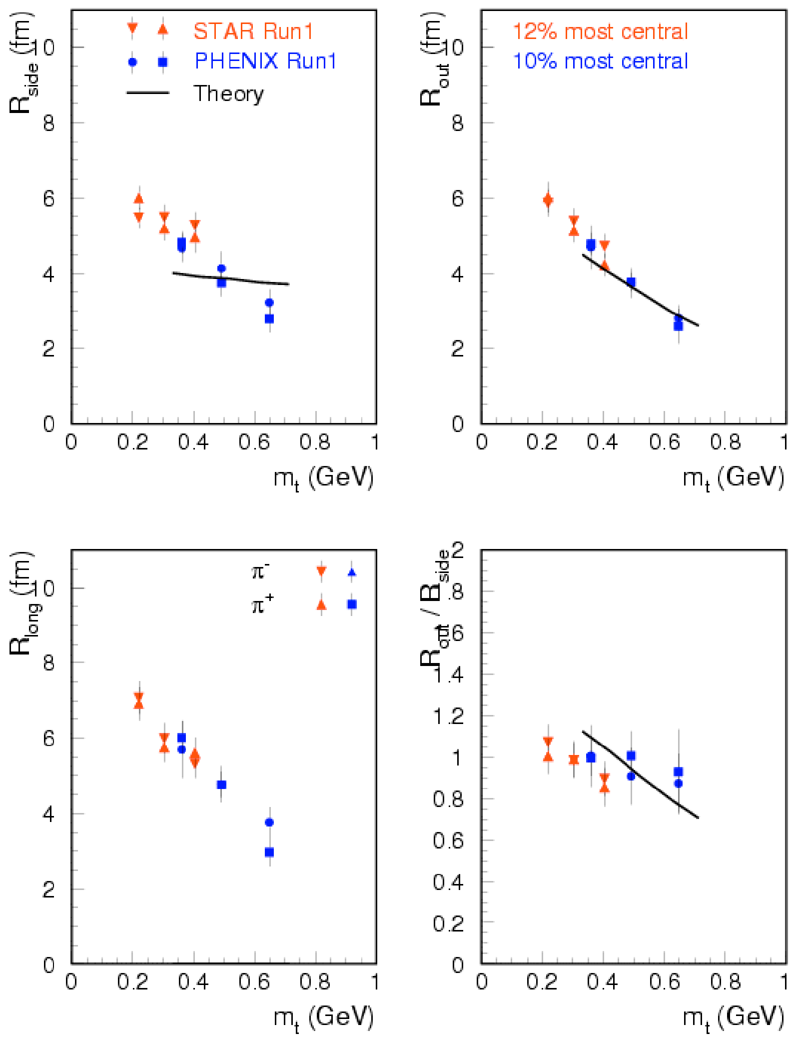
hep-ph/0404140 fits to 200 AGeV Au+Au



~Acceptable

Comparison of results of models

nucl-th/0212053-2 fits to 130 AGeV Au+Au



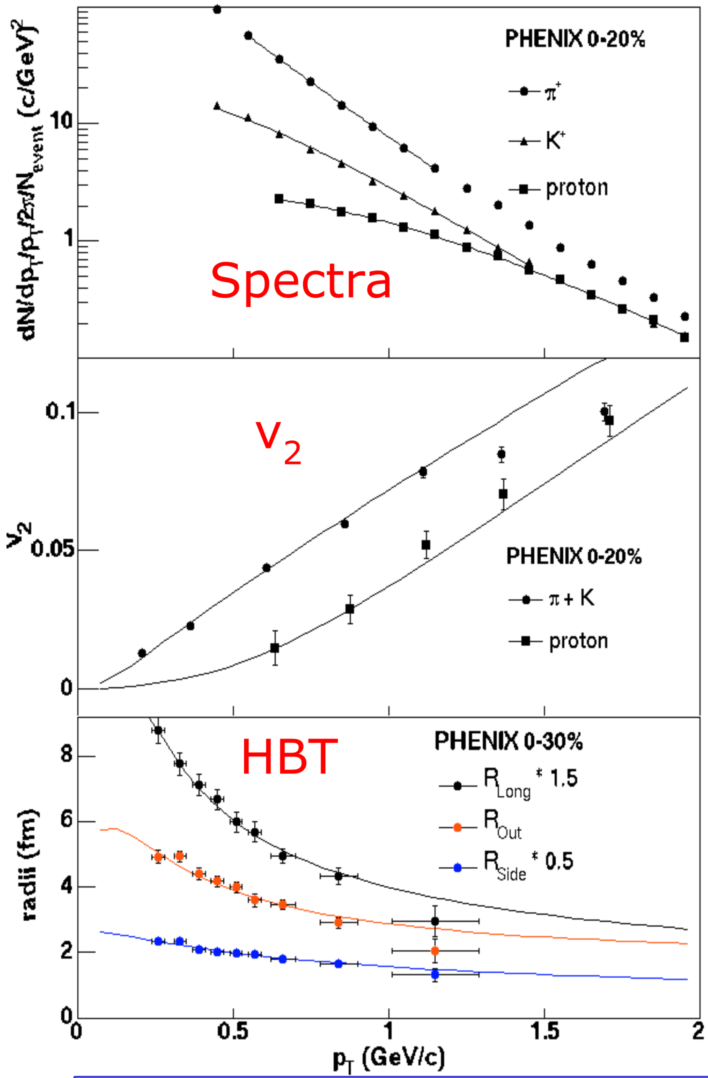
The HBT test

Less unpromising models: don't fail fitting Au+Au HBT data @ RHIC

- **nucl-th/0204054** Multiphase Transport model (AMPT)
  Z. Lin, C. M. Ko, S. Pal
- **nucl-th/0205053** Hadron cascade model
  T. Humanic
- **nucl-th/0207016** Buda-Lund hydro (hep-ph/9503494, 9509040)
  T. Cs. B. Lörstad, A. Ster et al.
  (nucl-th/0403074, /0402037, /0311102)
- **hep-ph/0209054** Cracow model (single freeze-out, thermal)
  W. Broniowski, A. Baran, W. Florkowski
- **nucl-ex/0307026** Blast wave model (Schnedermann, Heinz)
  M. A. Lisa, F. Retiere, PRC70, 044907 (2004)
- **hep-ph/0404140** Time dependent Duke hydro model
  T. Renk
- **nucl-th/0411031** Seattle model (quantum opacity)
  J. G. Cramer, G. A. Miller, J.M.S. Wu, J.-H. Yoon
- **nucl-th/0507057** Kiev-Nantes model
  Borysova, Sinyukov, Akkelin, Erazmus, Karpenko

-> More restrictive tests are needed: spectra, v2, HBT, dn/dy

Successfull models at RHIC. A: Blastwave



T=106 ± 1 MeV

$\langle\beta_{InPlane}\rangle = 0.571 \pm 0.004$ c

$\langle\beta_{OutOfPlane}\rangle = 0.540 \pm 0.004$ c

$R_{InPlane} = 11.1 \pm 0.2$ fm

$R_{OutOfPlane} = 12.1 \pm 0.2$ fm

Life time (τ) = 8.4 ± 0.2 fm/c

Emission duration = 1.9 ± 0.2 fm/c

χ^2/dof = 120 / 86

(Errors are statistical only, CL = 0.91 %)

For comparision:

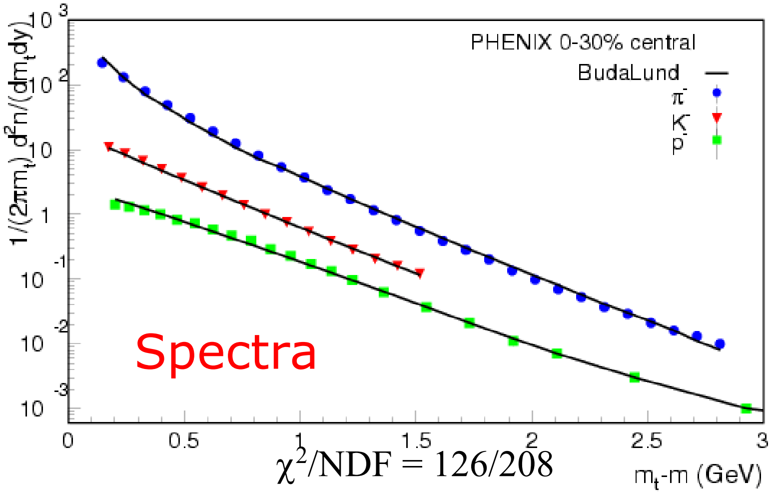
$\langle\beta_T\rangle = 0.555 \pm 0.003$ c

$R = \mathrm{sqrt}(R_{InPlane}R_{OutOfPlane}) = 11.6 \pm 0.3$ fm

F. Retiere, nucl-ex/0405024; F. Retiere and M. A. Lisa, nucl-th/0312024

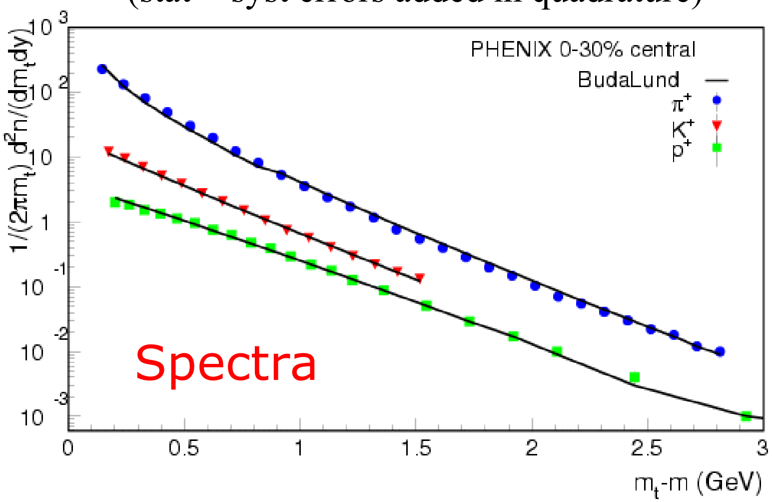
Successfull model 2: Buda-Lund

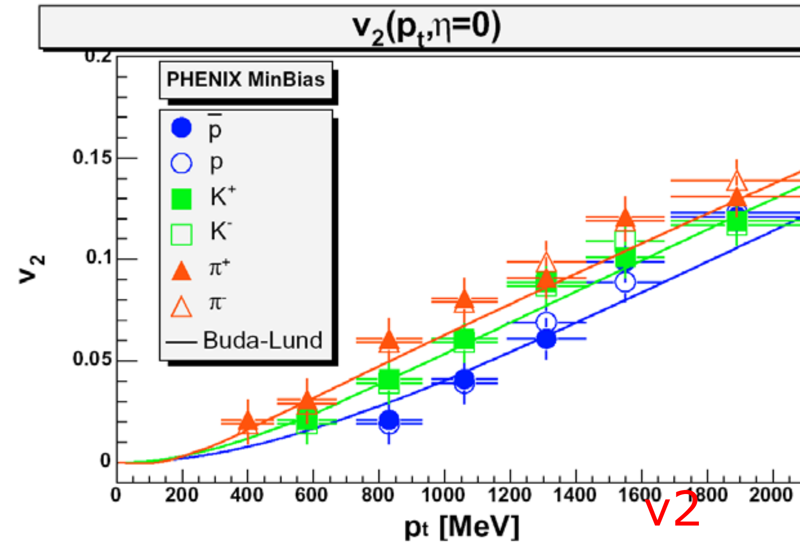
BudaLund v1.5 hydro fits to 200 AGeV Au+Au



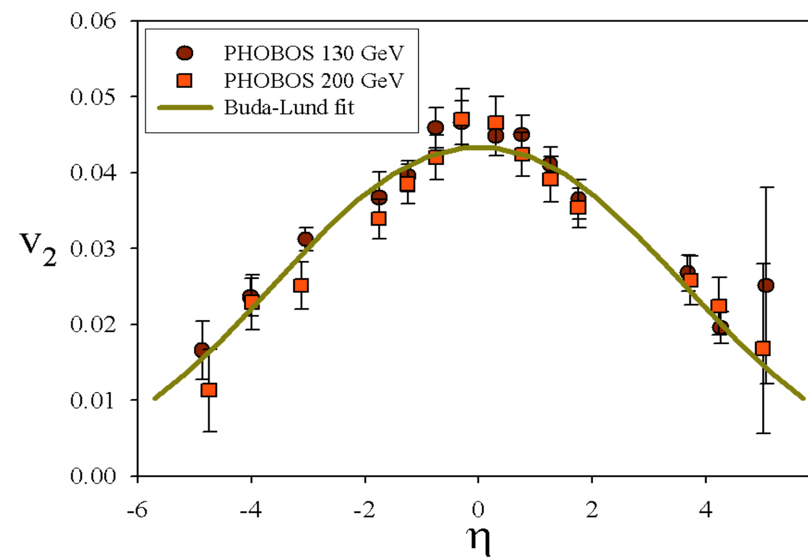
$\chi^2/NDF = 126/208$

(stat + syst errors added in quadrature)



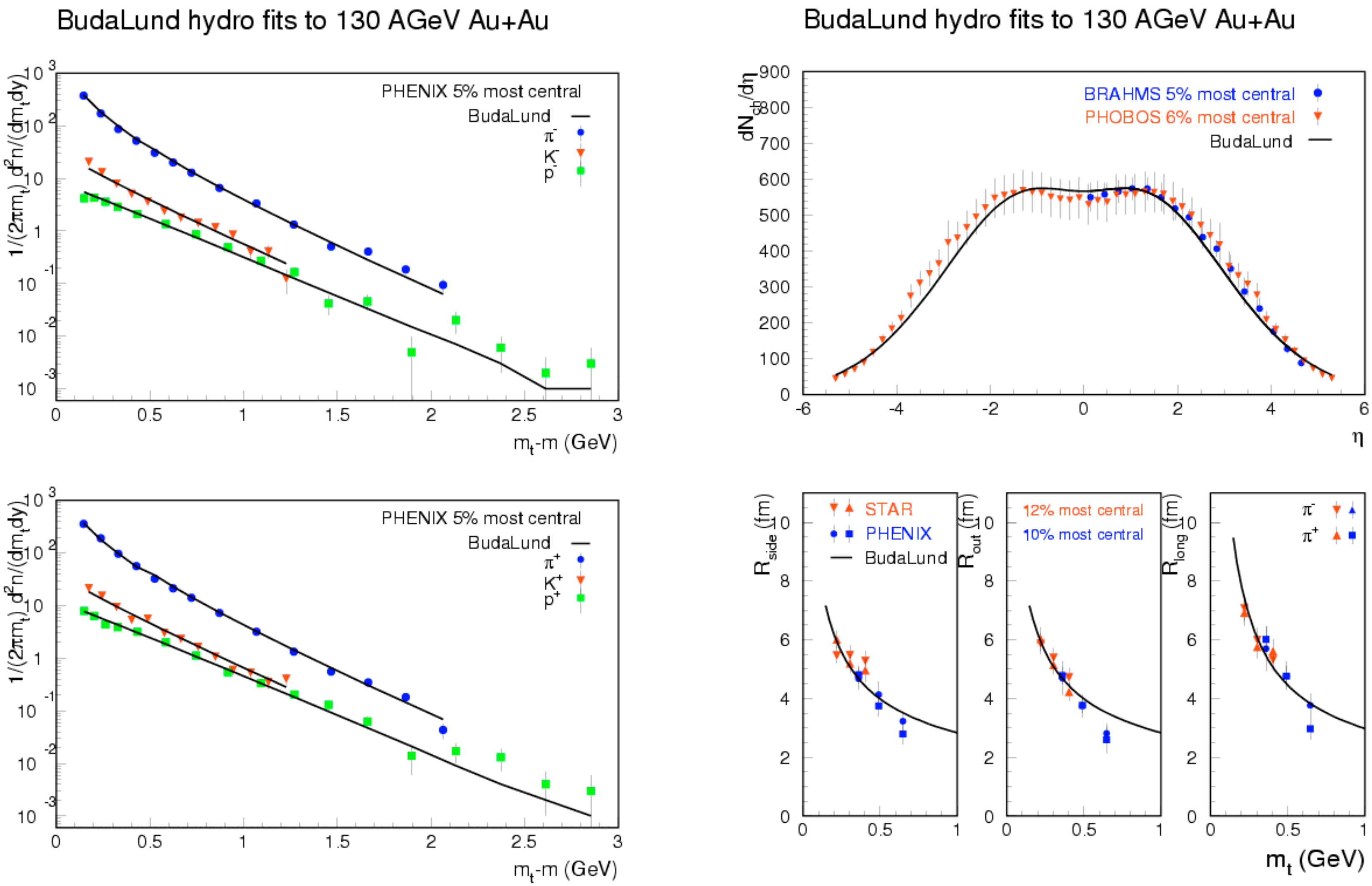


v2



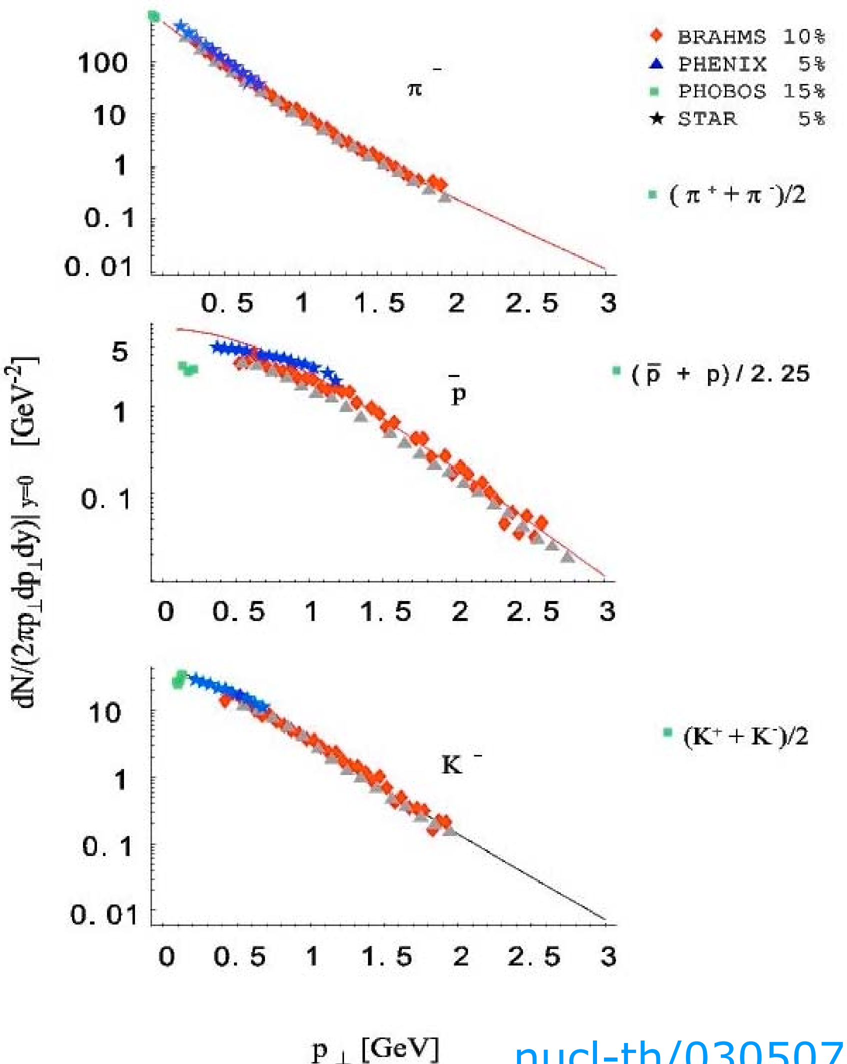
nucl-th/0311102, nucl-th/0207016, nucl-th/0403074

Successfull model 2: Buda-Lund

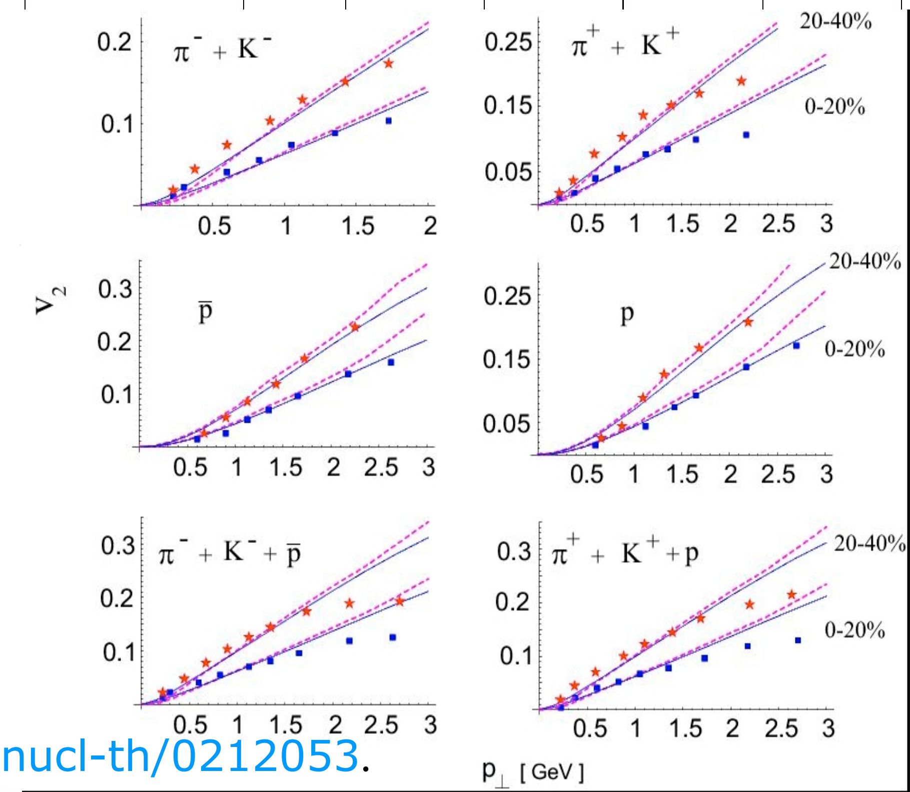


nucl-th/0311102, nucl-th/0207016

Successfull model 3: Cracow model



	c [%]	τ [fm]	ρ_{max} [fm]	$\langle\beta_\perp\rangle$	$\beta_\perp^{max}$
PHENIX	0 – 5	7.86 ± 0.38	7.15 ± 0.13	0.50 ± 0.02	0.67 ± 0.02
	20 – 30	6.14 ± 0.32	5.62 ± 0.11	0.50 ± 0.02	0.68 ± 0.03
	30 – 40	5.73 ± 0.16	4.95 ± 0.05	0.48 ± 0.01	0.65 ± 0.01
	40 – 50	4.75 ± 0.28	3.96 ± 0.09	0.47 ± 0.03	0.64 ± 0.03

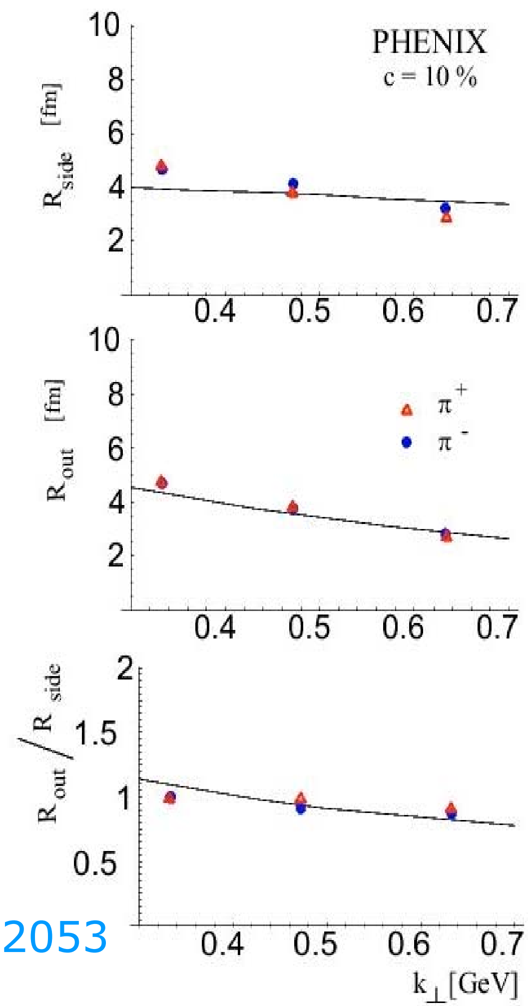


nucl-th/0305075, nucl-th/0212053.

Fig. 1. The transverse-momentum spectra of pions, protons, and kaons measured by BRAHMS (diamonds), PHENIX (triangles), PHOBOS (squares), and STAR (stars) for the most central events in the $Au + Au$ collisions at $\sqrt{s_{NN}} = 200$ GeV. The

V_2: A. Baran, PhD Thesis (Cracow)

Successfull model 3: Cracow model



nucl-th/0212053

FIGURE 6. Model predictions for the pionic R_{side} and R_{out} HBT correlation radii (top two panels), and their ratio (bottom panel), confronted with the PHENIX data $Au + Au$ data at $\sqrt{s_{NN}} = 130$ GeV and average centrality 10%. The quantity $k_\perp$ is the total momentum of the pion pair.

Model features:
Thermal model included
(abundances driven
by T_{chem} and μ_B)
Assumes full Hubble flow
Sudden freeze-out
(at a constant proper-time)
Single freeze-out, $T_{chem} = T_{kin}$
Boost-invariance

Time dependent solution (!!)
of relativistic hydrodynamics,
as shown in nucl-th/0305059

Some common parts 1:

Idea: try to extract the flow profile in a unique way from all the three models. Focus on central collisions only.

Blastwave:

Bjorken + linear transverse flow : $u_{transv} = \sinh(\rho_0\, r)$

Buda-Lund: Bjorken + linear transverse flow (u propto r)

Cracow model: assumes Hubble flow, $u^\mu = x^\mu/\tau$

Idea: Characterize flow as

$$u_{long} = H_{long}\, x_{long}$$
$$u_{transv} = H_{transv}\, x_{transv}$$

Then compare the transverse or the longitudinal Hubble constants

Results:

Blastwave (a la Retiere):

$H_{long} = 1/\ \tau_0 = 0.12 \text{ +- } 0.01\ \ fm^{-1}$

$H_{tran} = 0.085 - 0.151\ fm^{-1}$

(r dependent, $H(r) = du_{trans}/dr = 0.085 \cosh(r\ \rho_0/R)$, $r <= R$, $\rho_0 = 0.98$)

Buda-Lund:

$H_{long} = 1/\ \tau_0 = 0.18 \text{ +- } 0.01\ fm^{-1}$

$H_{tran} = <u_t'>/R_s = 0.13 \text{ +- } 0.02\ fm^{-1}$

Cracow model:

$H_{long} = H_{tran} = 1/\ \tau_0 = 0.13 \text{ +- } 0.01\ fm^{-1}$

Approximate Hubble flow in all the three!

$H=0.13 \text{ +- } 0.02\ fm^{-1}$

Hubble flow not excluded by the data based on spectra, v_2 and HBT.

Additional similarities:

Blastwave: (Retiere QM2004) μ/T = const! (assumed)

Sudden freeze-out, neglect of resonances, T = 106 +- 1 MeV

Buda-Lund (Csanád et al, QM2004): $\mu(x)/T(x)$ = const! (found from fit)

Temperature profile, long lived resonances.

The average temperature T(surface) = T(center)/2 = 98 +- 7 MeV

Cracow model: μ/T = const! (assumed)

All resonances included, they decay but do not rescatter.

Before decays: T_{chem} = 165+-7 MeV (nucl-th/0106009)

After decays, $T_{eff}=T_{chem}$ - cca 30-40 MeV ~ 131+-10 MeV

Average temperature seems to be similar.

All the three models: thermal/statistical model abundances.

All the three models: successfully tested on CERN SPS data too.

Nonrelativistic hydrodynamics

- **Equations of nonrelativistic hydro:**

$$\begin{aligned}\partial_t n + \nabla(n\mathbf{v}) &= 0\\ \partial_t \mathbf{v} + (\mathbf{v}\nabla)\mathbf{v} &= -(\nabla p)/(mn)\\ \partial_t \epsilon + \nabla(\epsilon\mathbf{v}) &= -p\nabla\mathbf{v}\end{aligned}$$

- **Not closed, EoS needed:**

$$\begin{aligned}\epsilon &= \kappa p\\ p &= nT\end{aligned}$$

- **We use the following scaling variable:**

$$s = \frac{r_x^2}{X^2} + \frac{r_y^2}{Y^2} + \frac{r_z^2}{Z^2}$$

- **X, Y and Z are characteristic scales, depend on (proper-) time**

A nonrelativistic solution

- **A general group of scale-invariant solutions (hep-ph/0111139):**

$$
\begin{aligned}
n(t,\mathbf{r}) &= n_0 \frac{V_0}{V} \nu(s) \\
\mathbf{v}(t,\mathbf{r}) &= \left(\frac{\dot X}{X} r_x, \frac{\dot Y}{Y} r_y, \frac{\dot Z}{Z} r_z \right) \\
T(t,\mathbf{r}) &= T_0 \left(\frac{V_0}{V} \right)^{1/\kappa} \mathcal{T}(s) \\
\nu(s) &= \frac{1}{\mathcal{T}(s)} \exp\left(-\frac{T_i}{2T_0} \int_0^s \frac{du}{\mathcal{T}(u)} \right)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{r}' &= (r_x \frac{X_0}{X}, r_y \frac{Y_0}{Y}, r_z \frac{Z_0}{Z}) \\
n\,(t,\mathbf{r}) &= n(t_0,\mathbf{r}') \left(\frac{X_0 Y_0 Z_0}{XYZ} \right) \\
v_x\,(t,\mathbf{r}) &= v_x(t_0,\mathbf{r}') \frac{\dot X}{\dot X_0}, \ldots \\
T\,(t,\mathbf{r}) &= T(t_0,\mathbf{r}') \left(\frac{X_0 Y_0 Z_0}{XYZ} \right)^{1/\kappa}
\end{aligned}
$$

- **This is a solution, if the scales fulfill:**

$$
X\ddot X = Y\ddot Y = Z\ddot Z = \frac{T_i}{m} \left(\frac{V_0}{V} \right)^{1/\kappa}
$$

- **Temperature scaling function is arbitrary,**
  e.g. Constant temperature $\Rightarrow$ Gaussian density

Buda-Lund profiles:

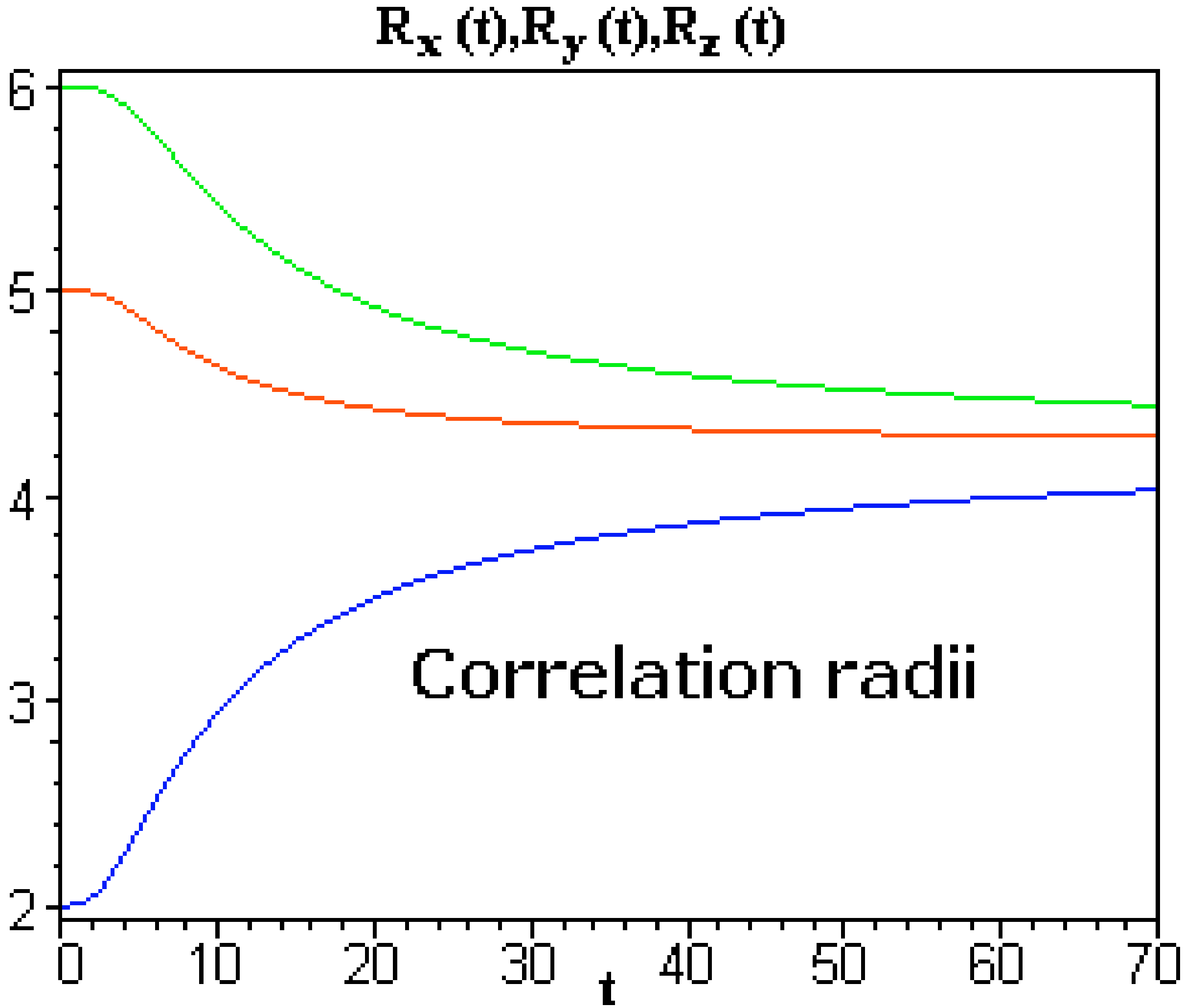
$$
\begin{aligned}
\mathcal{T}(s) &= \frac{1}{1+bs} \\
\nu(s) &= (1+bs) \exp\left[-\frac{T_i}{2T_0} (s + bs^2/2) \right]
\end{aligned}
$$

Zimányi-Bondorf-Garpman profiles:

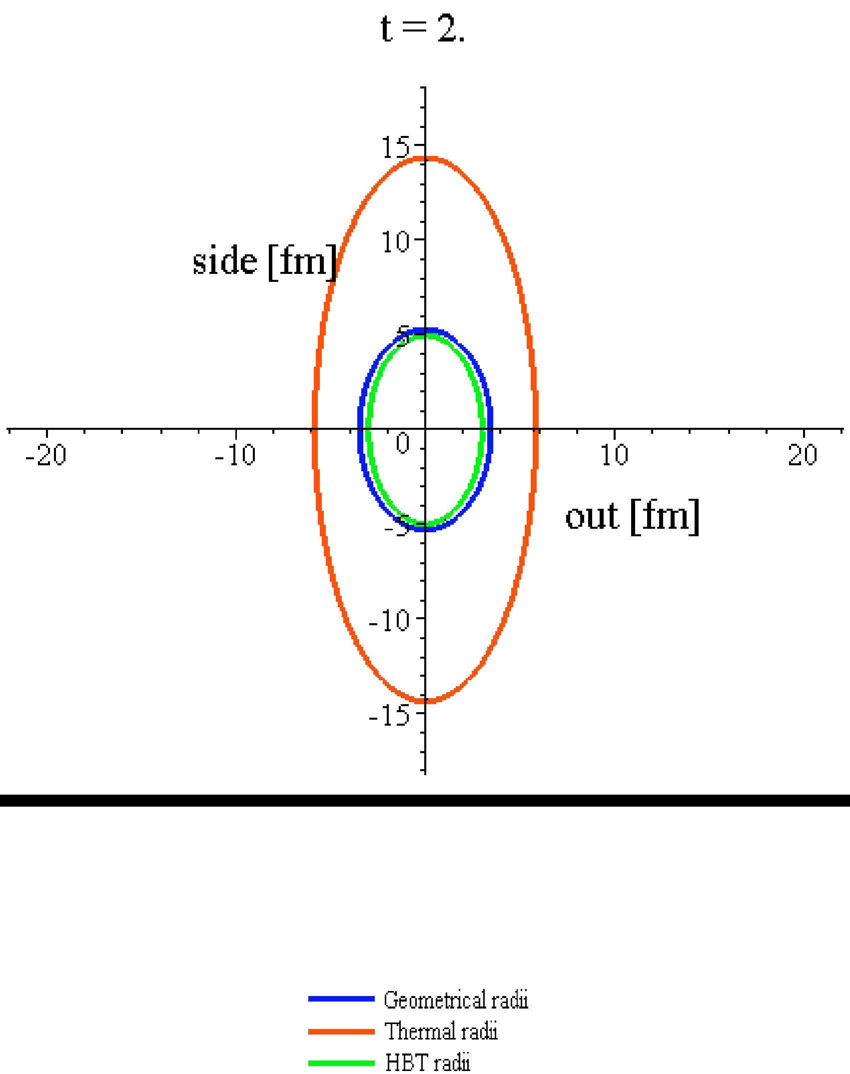
$$
\begin{aligned}
\mathcal{T}(s) &= (1-s)\,\Theta(1-s) \\
\nu(s) &= (1-s)^{\alpha}\,\Theta(1-s)
\end{aligned}
$$

Some numeric results from hydro

- **Propagate the hydro solution in time numerically:**



Geometrical & thermal & HBT radii



3d analytic hydro: exact time evolution (!!)

geometrical size (fugacity ~ const)

Thermal sizes (velocity ~ const)

HBT sizes (phase-space density ~ const)

HBT dominated by the smaller of the geometrical and thermal scales

nucl-th/9408022, hep-ph/9409327

hep-ph/9509213, hep-ph/9503494

HBT radii approach a const(t) (!!!)

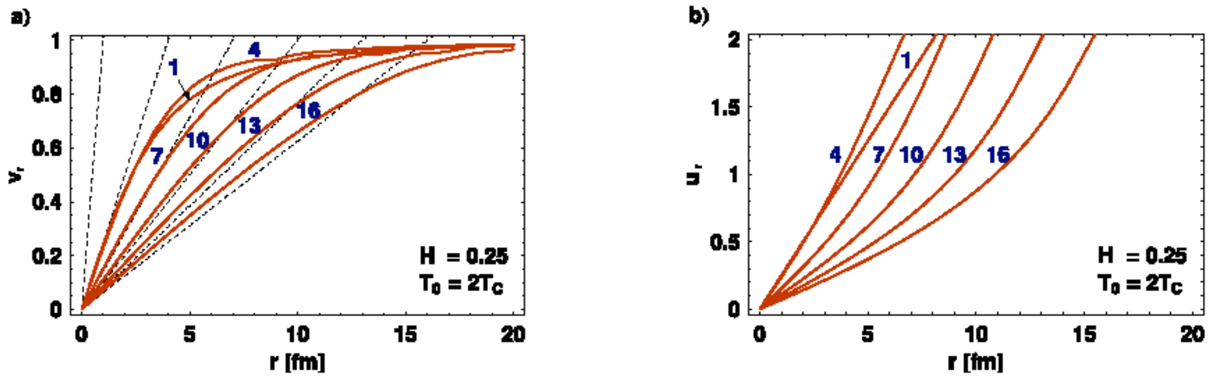
HBT volume -> spherical

HBT radii -> thermal, constant lengths!!

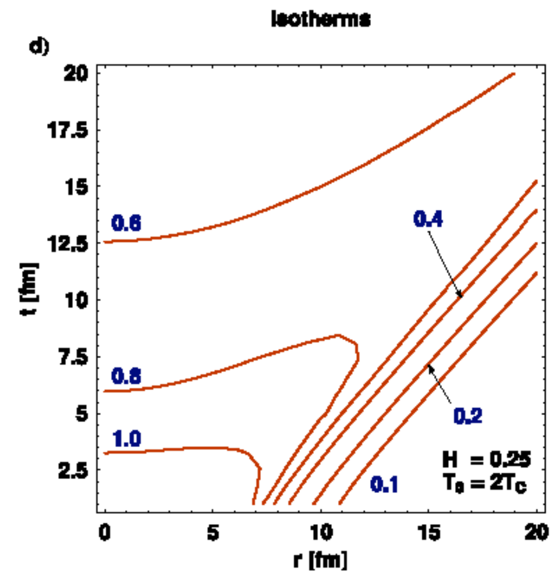
hep-ph/0108067, nucl-th/0206051

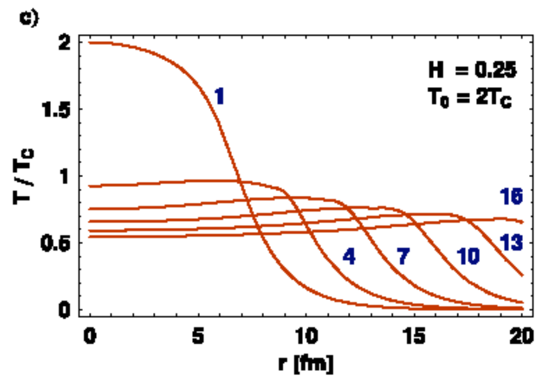
<-- Thanks to Máté Csanád for animation

Some num. rel. hydro solutions



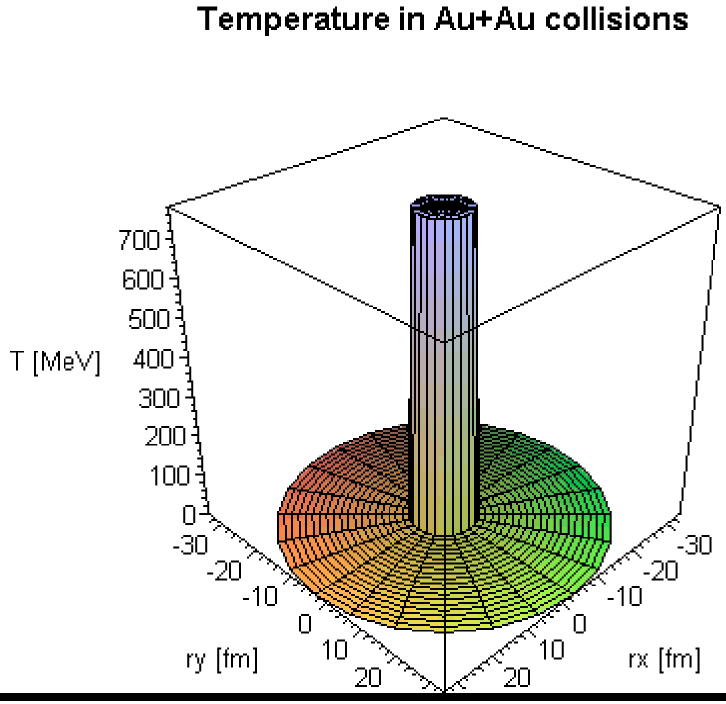
M. Chojnacki, W. Florkowski, T. Cs,
nucl-th/0410036
lattice QCD EOS (μ_B=0)
T_0(r) ~ initial entropy (Glauber)
H_0 ~ initial Hubble flow

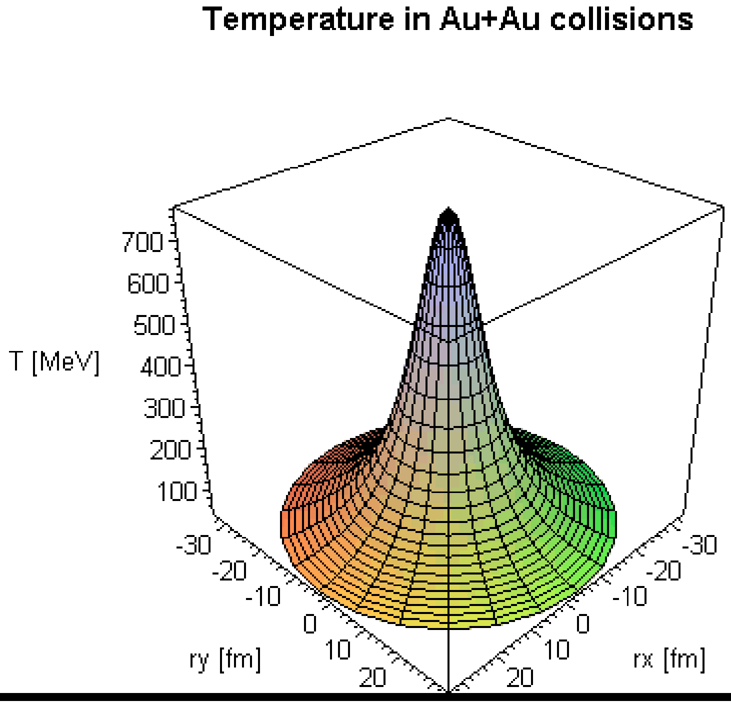




Support the quick development of the Hubble flow
and the Blast-wave, Buda-Lund and Cracow etc models

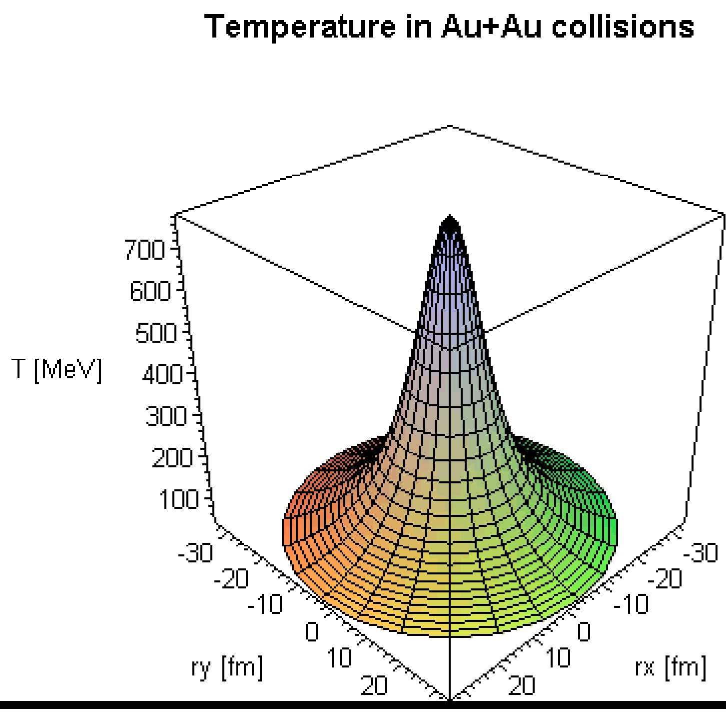
Time dependence,all 3 models

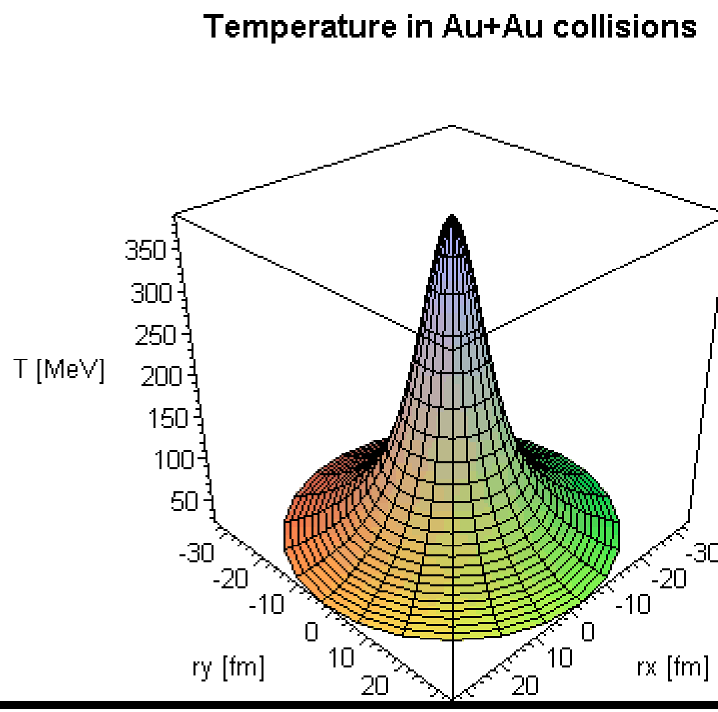




Blastwave or Cracow model type of cooling vs Buda-Lund type of cooling, $c_s^2 = 2/3$, half freeze-out time (animated)
http://csanad.web.elte.hu/phys/3danim/

Sensitivity to the Equation of State





$c_s^2 = 2/3$ $c_s^2 = 1/3$

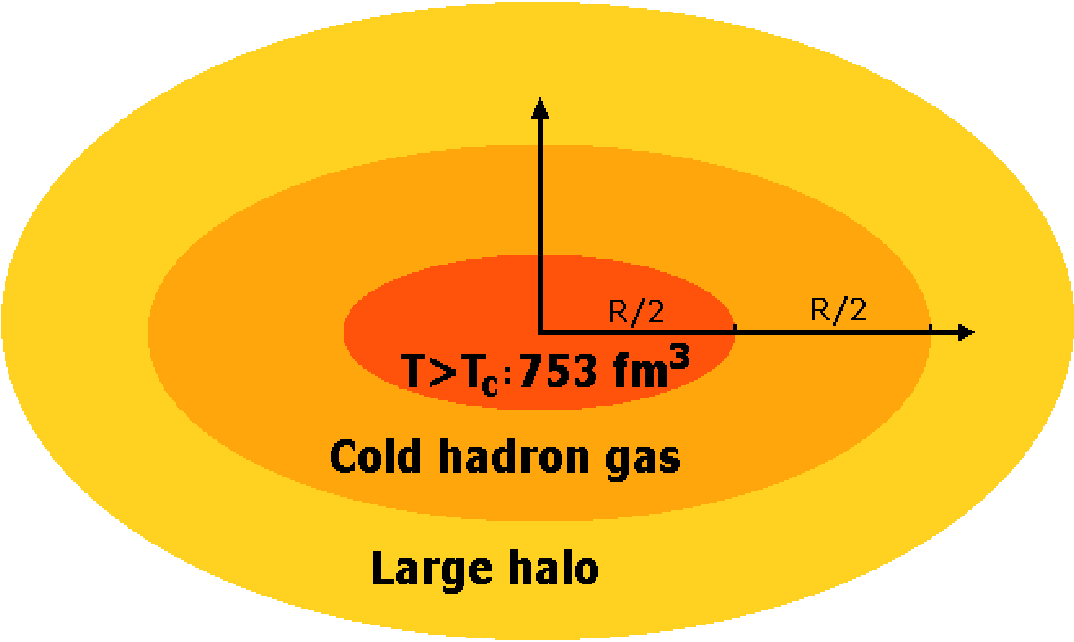
Different initial conditions, using two different equation of state but exactly the same hadronic final state possible. (!!)

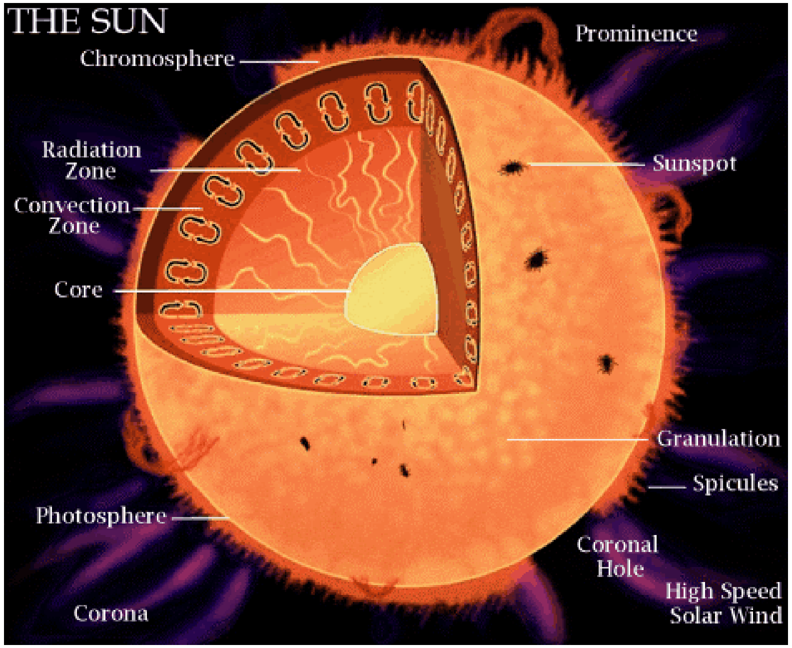
This is an exact, analytic result in hydro(!!).

A useful analogy

Fireball at RHIC ⇔ our Sun

• Core	⇔	Sun
• Halo	⇔	Solar wind
• $T_{0,RHIC}$ ~ 210 MeV	⇔	$T_{0,SUN}$ ~ 16 million K
• $T_{surface,RHIC}$ ~ 100 MeV	⇔	$T_{surface,SUN}$ ~6000 K





A solution of the "RHIC HBT puzzle"

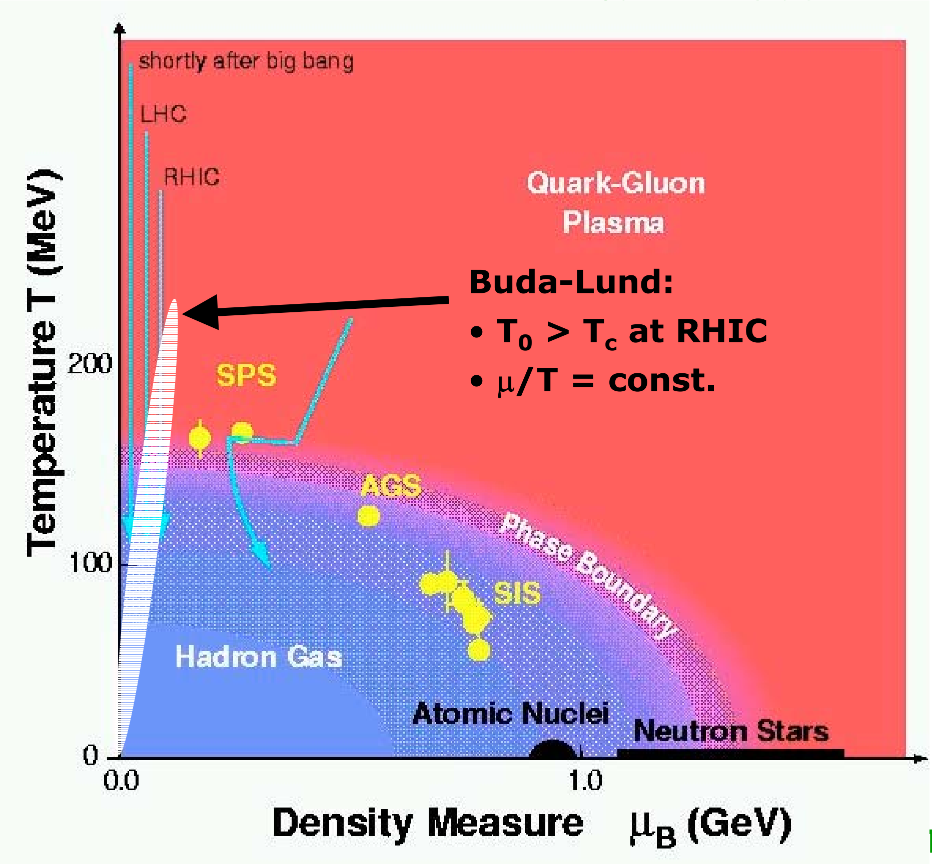
Many (~ 50) models fail.

This is not a "puzzle", but their trouble.

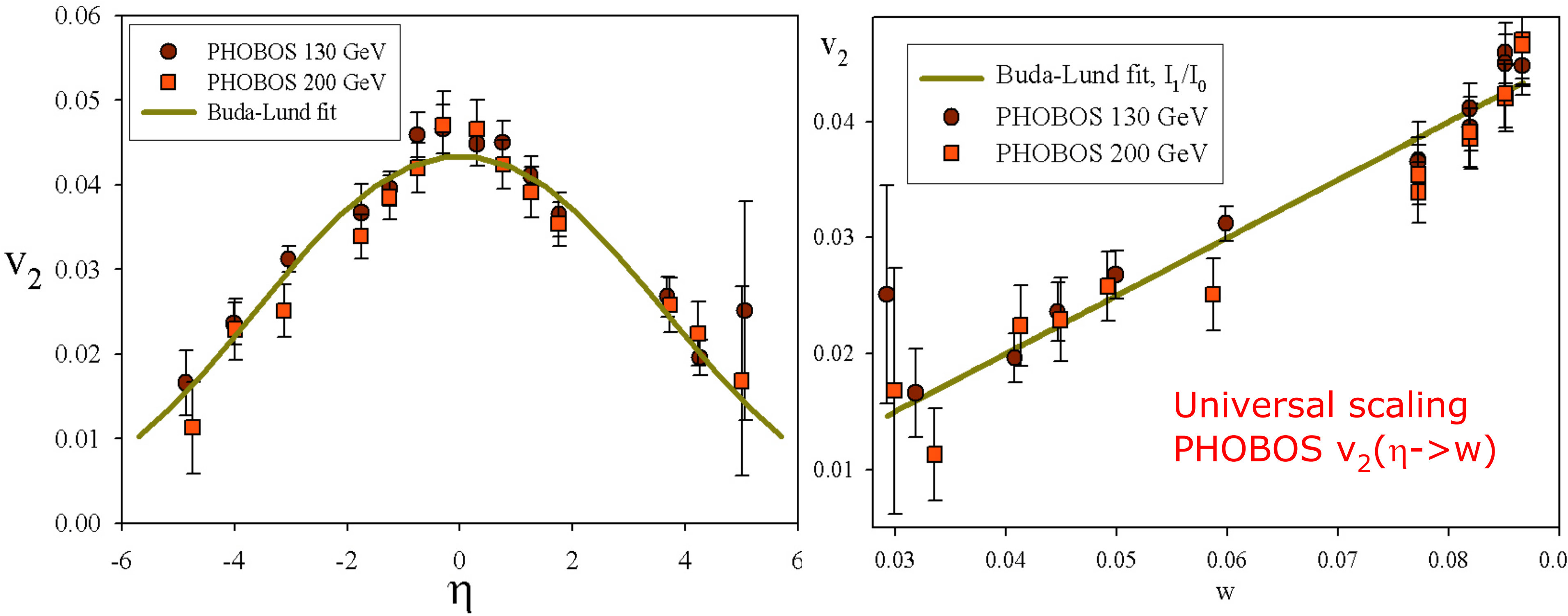
Presently 8 models pass the HBT test at RHIC.

Our answer: hot center, a fireball heated from inside

If we assume a hot center, $R_{out} \sim R_{side} \sim R_{long}$



Confirmation



see nucl-th/0310040 and nucl-th/0403074,
R. Lacey@QM2005/ISMD 2005
A. Ster @ QM2005.

Femptoscopy signals of various QGPs

strong 1st order
2nd order
cross-over
supercooled QGP (scQGP)

Strong 1st order phase transition

(Pratt, Bertsch, Rischke, Gyulassy) Rout >> Rside

2nd order QGP -> hadron (Critical End Point)

(T. Cs, S. Hegyi, T. Novák, W.A. Zajc) α(Lévy) decreases to 0.5 near to critical sqrt(sNN)

cross-over QGP -> hadrons (Lattice QCD, Buda-Lund)

hadrons appear from a region of $T_0 > T_c$

supercooledQGP: a) hadron flash b) Rout~Rside~Rlong

c) strangeness enhancement d) no ϕ mass shift

scQGP = supercooled QGP (1994) is not inconsistent with RHIC in 2005

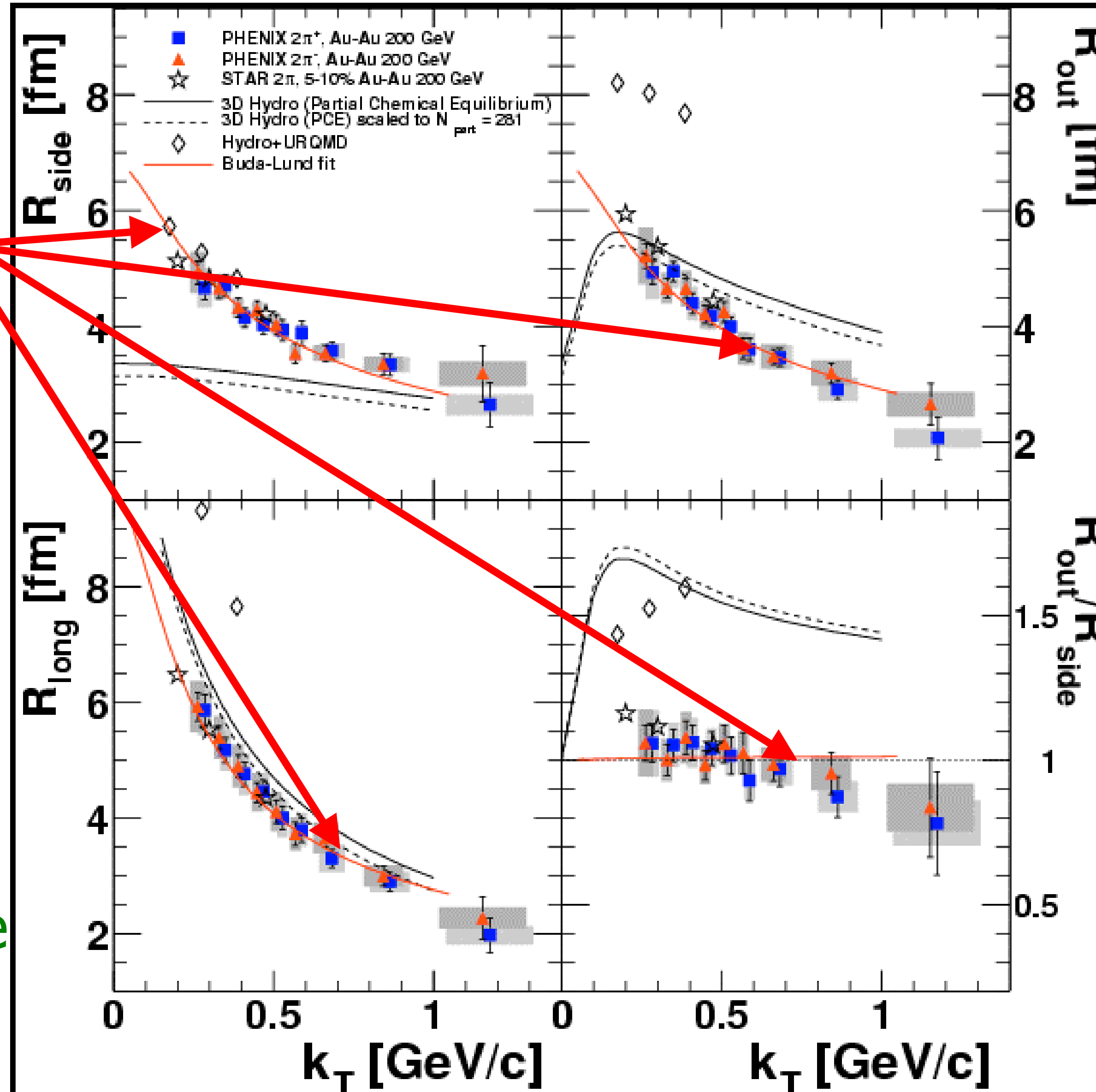
Recommendation: Focussing on the SUCCESSFUL models

Check out their predictions (e.g. mt scaling of radii for kaons, protons)

Femptoscopy signal of supercooled QGP

Buda-Lund hydrodynamic fit indicates sudden hadronization
- a hint for a previous supercooled QGP.

Hadrons with $T>T_c$ escape- a hint also for for cross-over type transition



R. Tagore: Playthings

Child, how happy you are sitting in the dust,
playing with a broken twig all the morning.
I smile at your play
with that little bit of a broken twig.
I am busy with my accounts,
adding up figures by the hour.
Perhaps you glance at me and think,
"What a stupid game to spoil your morning with!"
Child, I have forgotten
the art of being absorbed in sticks and mud-pies.
I seek out costly playthings,
and gather lumps of gold and silver.
With whatever you find you create your glad games,
I spend both my time and my strength over things I never can obtain.
In my frail canoe I struggle to cross the sea of desire,
and forget that I too am playing a game.

Thank you for your attention

Some backup slides follow
for more details see the talks of
András Ster and
Máté Csanád at this metting and at QM 2005

The generalized Buda-Lund model

- The original model was for axial symmetry only, central coll.

- In the most general hydrodynamical form:

  'Inspired by' nonrelativistic 3d hydrodynamical solutions:

$$S_c(x,p)d^4x = \frac{g}{(2\pi)^3} \frac{p^\mu d^4\Sigma_\mu(x)}{\exp\left(\frac{p^\nu u_\nu(x)}{T(x)} - \frac{\mu(x)}{T(x)}\right) + s_q}$$

- Have to assume special shapes:
  - Generalized Cooper-Frye prefactor:

$$p^\mu d^4\Sigma_\mu(x) = p^\mu u_\mu(x) H(\tau) d^4x \qquad H(\tau) = \frac{1}{(2\pi\Delta\tau^2)^{1/2}} \exp\left(-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right)$$

  - Four-velocity distribution:

$$u^\mu = (\gamma, \sinh\eta_x, \sinh\eta_y, \sinh\eta_z)$$

  - Temperature:

$$\frac{1}{T(x)} = \frac{1}{T_0}\left(1 + \frac{T_0 - T_s}{T_s}\, s\right)\left(1 + \frac{T_0 - T_e}{T_e}\, \frac{(\tau-\tau_0)^2}{2\Delta\tau^2}\right)$$

  - Fugacity:

$$\frac{\mu(x)}{T(x)} = \frac{\mu_0}{T_0} - s$$

Some analytic results

- **Distribution widths**

$$\frac{1}{R_{i,i}^2} = \frac{B(x_s,p)}{B(x_s,p)+s_q}\left(\frac{1}{X_i^2}+\frac{1}{R_{T,i}^2}\right)$$

$$\frac{1}{R_{T,i}^2} = \frac{m_t}{T_0}\left(\frac{a^2}{X_i^2}+\frac{\dot{X}_i^2}{X_i^2}\right) \qquad a^2 = \frac{T_0-T_s}{T_s} = \left\langle \frac{\Delta T}{T} \right\rangle_r$$

- **Slopes, effective temperatures**

$$T_{\mathit{eff}} = \frac{1}{2}\left(\frac{1}{T_{*,x}}+\frac{1}{T_{*,y}}\right) \qquad T_{*,i} = T_0 + m_t \dot{X}_i^2 \frac{T_0}{T_0+m_t a^2}$$

- **Flow coefficients**

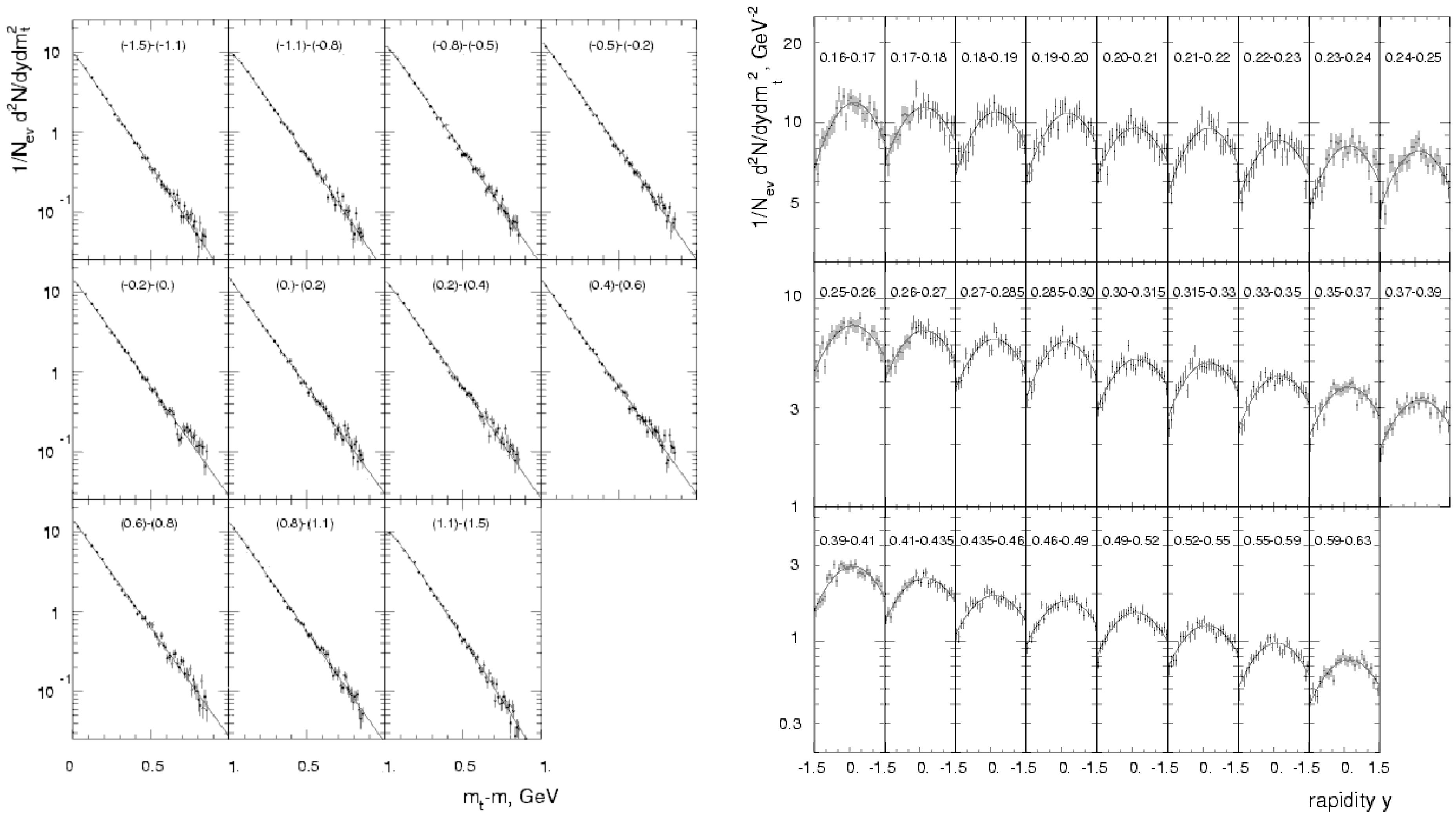
$$v_{2n} = \frac{I_n(w)}{I_0(w)}$$

$$v_{2n+1} = 0$$

$$w = \frac{p_t^2}{4\overline{m}_t}\left(\frac{1}{T_{*,y}}-\frac{1}{T_{*,x}}\right)$$

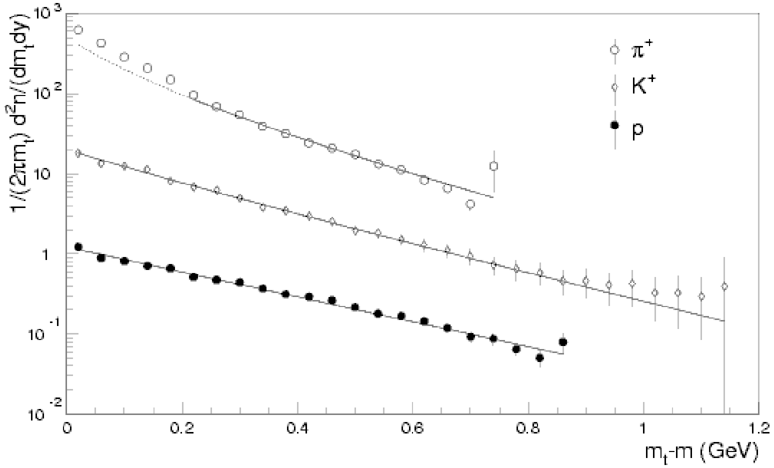
$$\overline{m}_t = m_t \cosh(\eta_s - y)$$

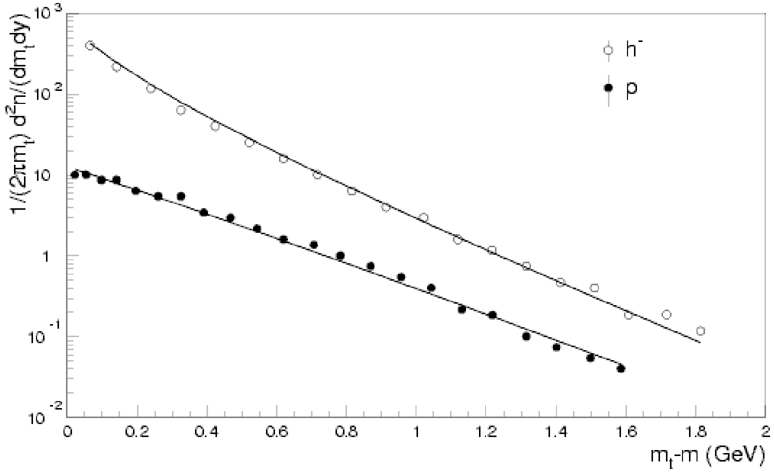
Buda-Lund fits to NA22 h + p data

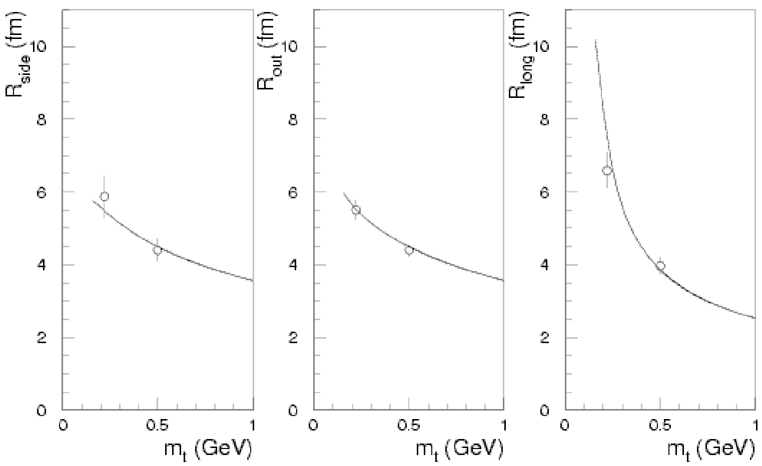


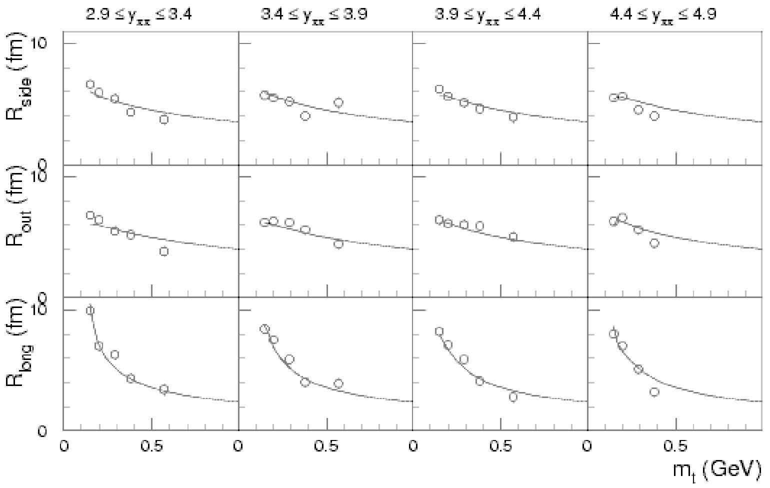
N. M. Agababyan et al, EHS/NA22 , PLB 422 (1998) 395
T. Csörgő, hep-ph/0001233, Heavy Ion Phys. 15 (2002) 1-80

Buda-Lund fits to NA44/49 data



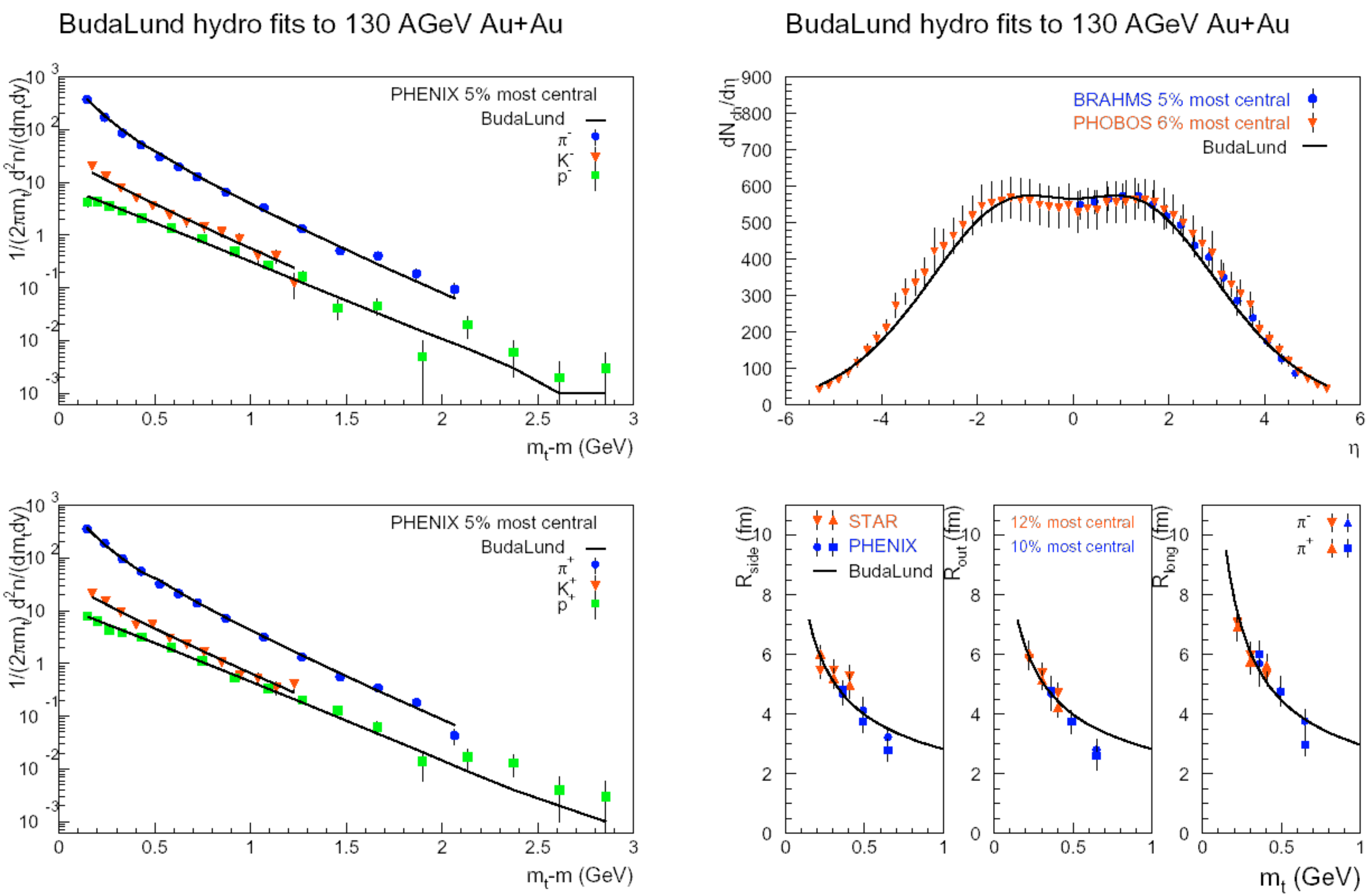






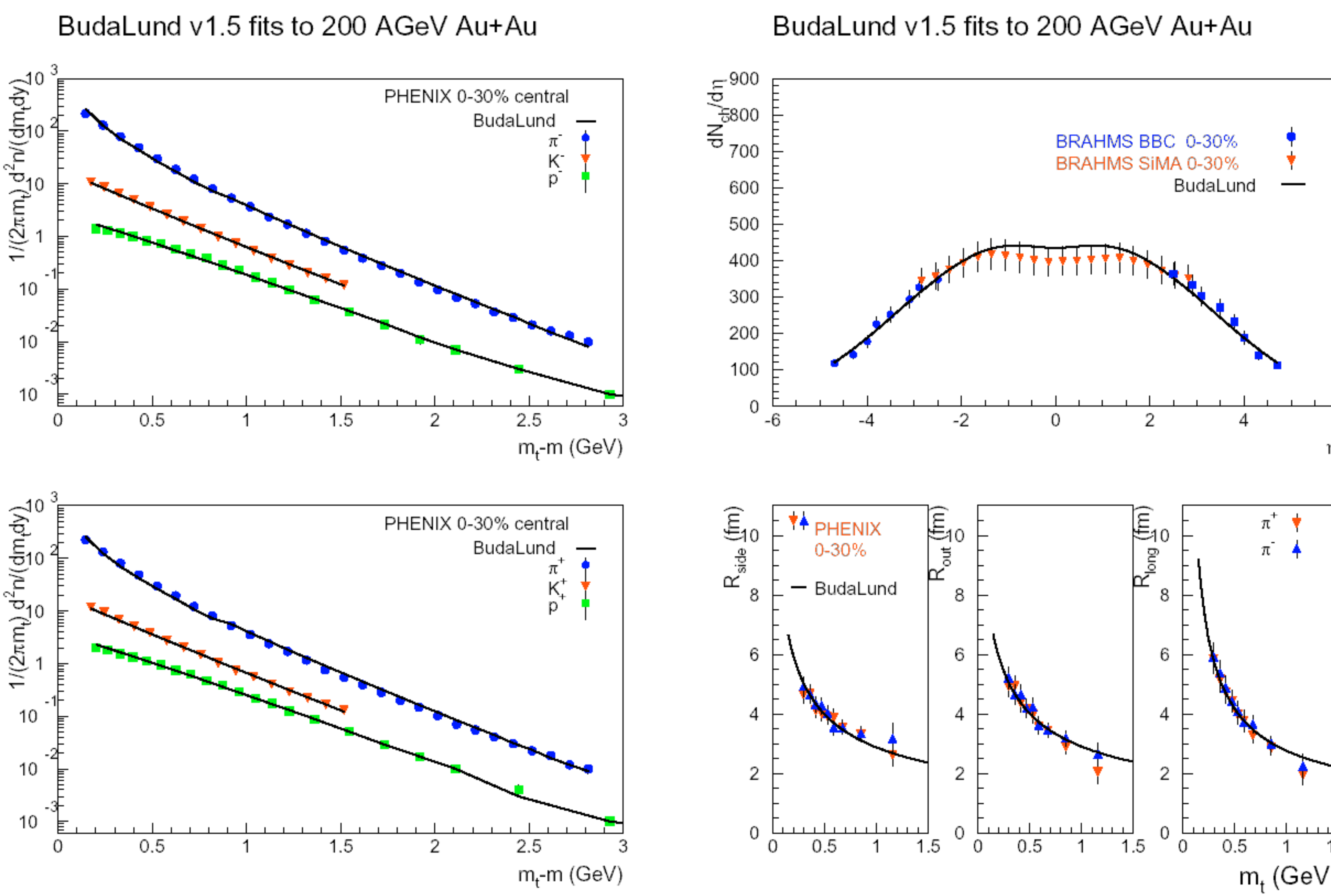
A. Ster, T. Cs, B. Lörstad, hep-ph/9907338

BudaLund fits to 130 GeV RHIC data



M. Csanád, T. Csörgő, B. Lörstad, A. Ster, nucl-th/0311102, ISMD03

BudaLund fits to 200 GeV RHIC data



M. Csanád, T. Csörgő, B. Lörstad, A. Ster, nucl-th/0403074, QM04

Fit results, comparing RHIC and SPS

BL at RHIC	Au+Au, 200 GeV	Au+Au, 130 GeV
T_0 [MeV]	196 $\pm$ 13	214 $\pm$ 7
T_e [MeV]	117 $\pm$ 12	102 $\pm$ 11
T_s [fm]	$T_0/2$ fixed	$T_0/2$ fixed
R_s [fm]	12.4 $\pm$ 1.6	8.6 $\pm$ 0.4

BL at SPS	Pb+Pb, NA44	Pb+Pb, NA49	h+p, NA22
T_0 [MeV]	145 $\pm$ 3	134 $\pm$ 3	140 $\pm$ 3
T_e [MeV]	76 $\pm$ 87	115 $\pm$ 5	-
T_s [fm]	134 $\pm$ 11	125 $\pm$ 5	82 $\pm$ 8

$$T_0(\mathrm{RHIC}) > T_c \text{ by } 5\sigma$$

$$T_0(\mathrm{RHIC}) > T_0(\mathrm{SPS}) \text{ by } 5\sigma$$

$$\frac{T_0 - T_s}{T_s} \simeq 1 \text{ at RHIC}$$

$$\frac{T_0 - T_s}{T_s} \ll 1 \text{ at SPS Pb+Pb}$$