Overview of Source Imaging

P. Danielewicz

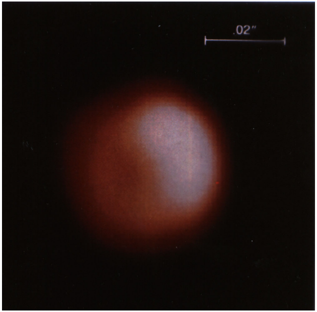
Natl Superconducting Cyclotron Lab, Michigan State U

Workshop on Particle Correlations and Femtoscopy,
Kromieriz, August 15-17 2005

Outline

Astronomy

Intensity/phase interferometry first used to assess sizes of astronomical objects. Astronomers have since moved to details:



red giant Betelgeuse

Can we do comparably well?

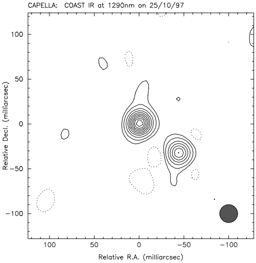


Figure 5.7: Reconstructed image of Capella, from data taken on 25 October 1997 at a wavelength of 1.3 µm. The contours are at -4, 4, 10, 20, 30, ..., 90% of the peak ux. The map has been restored with a circular beam for clarity.

binary star Capella, Monnier Rep Prog Phy 66(03)789

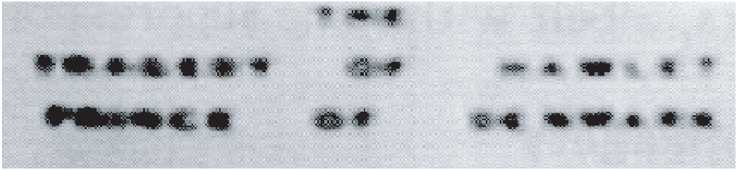
Imaging

Geometric information from imaging. General task:

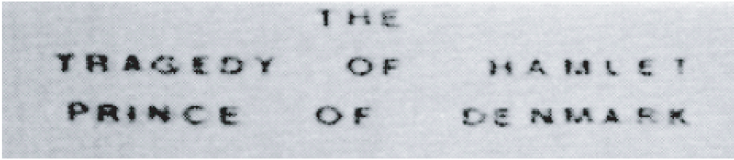
$$C(q) = \int dr\, K(q,r)\, S(r)$$

From data w/ errors, $C(q)$, determine the source $S(r)$.
Requires inversion of the kernel K.
Optical recognition: K - blurring function, max entropy method

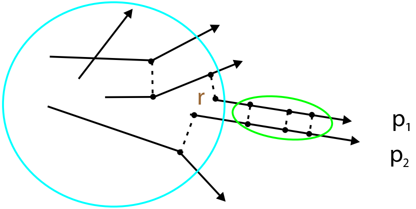
C:



S:



Factorization of Final-State Amplitude in Reactions



2-ptcle inclusive cross section at low $|\mathbf{p}_1 - \mathbf{p}_2|$

$$\frac{d\sigma}{d\mathbf{p}_1\, d\mathbf{p}_2} = \int \mathrm{d}\mathbf{r}\, S'_{\mathbf{P}}(\mathbf{r})\, |\Phi^{(-)}_{\mathbf{p}_1-\mathbf{p}_2}(\mathbf{r})|^2$$

data source 2-ptcle wf

S': distribution of emission points in 2-ptcle CM

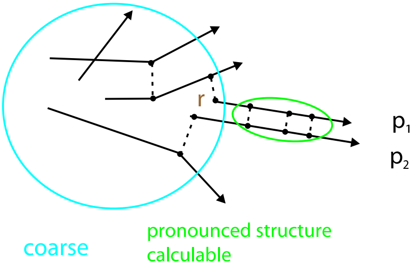
Normalizing with 1-ptcle cross sections yields correlation f:

$$C(\mathbf{p}_1 - \mathbf{p}_2) = \frac{\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{p}_1\, d\mathbf{p}_2}}{\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{p}_1}\,\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{p}_2}} = \int \mathrm{d}\mathbf{r}\, S_{\mathbf{P}}(\mathbf{r})\, |\Phi^{(-)}_{\mathbf{p}_1-\mathbf{p}_2}(\mathbf{r})|^2$$

Then the relative source is normalized to unity: $\int \mathrm{d}\mathbf{r}\, S_{\mathbf{P}}(\mathbf{r}) = 1$.

Note: C may only give access to the density of relative emission points in 2-ptcle CM, integrated there over time

Factorization of Final-State Amplitude in Reactions



2-ptcle inclusive cross section at low $|\mathbf{p}_1 - \mathbf{p}_2|$

$$\frac{d\sigma}{d\mathbf{p}_1\, d\mathbf{p}_2} = \int \mathrm{d}\mathbf{r}\, S'_{\mathbf{P}}(\mathbf{r})\, |\Phi^{(-)}_{\mathbf{p}_1-\mathbf{p}_2}(\mathbf{r})|^2$$

data source 2-ptcle wf

S': distribution of emission points in 2-ptcle CM

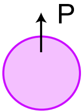
Normalizing with 1-ptcle cross sections yields correlation f:

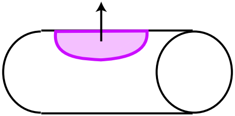
$$C(\mathbf{p}_1 - \mathbf{p}_2) = \frac{\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{p}_1\, d\mathbf{p}_2}}{\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{p}_1}\,\frac{1}{\sigma}\frac{d\sigma}{d\mathbf{p}_2}} = \int \mathrm{d}\mathbf{r}\, S_{\mathbf{P}}(\mathbf{r})\, |\Phi^{(-)}_{\mathbf{p}_1-\mathbf{p}_2}(\mathbf{r})|^2$$

Then the relative source is normalized to unity: $\int \mathrm{d}\mathbf{r}\, S_{\mathbf{P}}(\mathbf{r}) = 1$.

Note: C may only give access to the density of relative emission points in 2-ptcle CM, integrated there over time

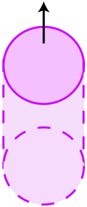
Expectations Regarding the Source

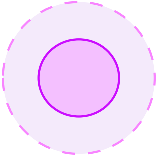


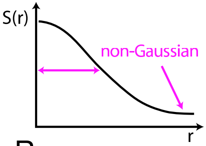


Instantaneous volume surface freeze-out.

Prolonged emission from source moving in pair cm.







Any determination of source characteristics from data, unaided by reaction theory, is an imaging.

Resonance emission from a source stationary in pair frame.

Integral Relation

Of interest is the deviation of correlation function from unity:

$$\mathcal{R}(\mathbf{q}) = C(\mathbf{q}) - 1 = \int \mathrm{d}\mathbf{r} \left(|\Phi_{\mathbf{q}}^{(-)}(\mathbf{r})|^2 - 1 \right) S(\mathbf{r}) \equiv \int \mathrm{d}\mathbf{r}\, K(\mathbf{q}, \mathbf{r})\, S(\mathbf{r})$$

Learning on S possible when $|\Phi_{\mathbf{q}}^{(-)}(\mathbf{r})|^2$ deviates from 1, either due to symmetrization or interaction within the pair.

The spin-averaged kernel K depends only on the relative angle between $\mathbf{q}$ and $\mathbf{r}$. This facilitates the angular decomposition. With

$$K(\mathbf{q}, \mathbf{r}) = \sum_{\ell} (2\ell + 1)\, K_\ell(q, r)\, P^\ell(\cos\theta)\,, \quad \text{and}$$

$$\mathcal{R}(\mathbf{q}) = \sqrt{4\pi} \sum_{\ell m} \mathcal{R}^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{q}})\,, \quad S(\mathbf{r}) = \sqrt{4\pi} \sum_{\ell m} S^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{r}})$$

we reduce the 3D relation to a set of 1D relations:

$$\mathcal{R}^{\ell m}(q) = 4\pi \int \mathrm{d}r\, r^2\, K_\ell(q, r)\, S^{\ell m}(r)$$

Integral Relation

Of interest is the deviation of correlation function from unity:

$$\mathcal{R}(\mathbf{q}) = C(\mathbf{q}) - 1 = \int \mathrm{d}\mathbf{r} \left(|\Phi^{(-)}_{\mathbf{q}}(\mathbf{r})|^2 - 1 \right) S(\mathbf{r}) \equiv \int \mathrm{d}\mathbf{r}\, K(\mathbf{q}, \mathbf{r})\, S(\mathbf{r})$$

Learning on S possible when $|\Phi^{(-)}_{\mathbf{q}}(\mathbf{r})|^2$ deviates from 1, either due to symmetrization or interaction within the pair.

The spin-averaged kernel K depends only on the relative angle between $\mathbf{q}$ and $\mathbf{r}$. This facilitates the angular decomposition. With

$$K(\mathbf{q}, \mathbf{r}) = \sum_{\ell} (2\ell + 1)\, K_{\ell}(q, r)\, P^{\ell}(\cos\theta)\,, \qquad \text{and}$$

$$\mathcal{R}(\mathbf{q}) = \sqrt{4\pi} \sum_{\ell m} \mathcal{R}^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{q}})\,, \qquad S(\mathbf{r}) = \sqrt{4\pi} \sum_{\ell m} S^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{r}})$$

we reduce the 3D relation to a set of 1D relations:

$$\mathcal{R}^{\ell m}(q) = 4\pi \int \mathrm{d}r\, r^2\, K_{\ell}(q, r)\, S^{\ell m}(r)$$

$\ell = 0$ & Pure Interference

Different multipolarities of deformation for the source and correlation functions are directly related to each other.

The $\ell = 0$ version:

$$\mathcal{R}^0(q) = 4\pi \int \mathrm{d}r\, r^2\, K_0(q,r)\, S^0(r)$$

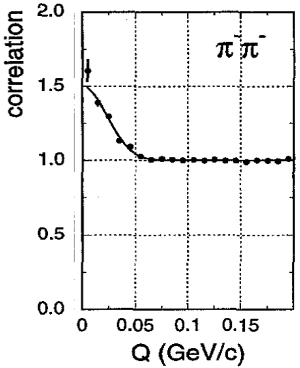
where $\mathcal{R}^0(q)$, K_0 and $S^0(r)$ – angle-averaged correlation, kernel and source, respectively.

For pure interference, π^0's or γ's, $\Phi^{(-)}_{\mathbf{q}}(\mathbf{r}) = \frac{1}{\sqrt{2}}\left(e^{i\mathbf{q}\cdot\mathbf{r}} + e^{-i\mathbf{q}\cdot\mathbf{r}}\right)$, the kernel $K = |\Phi|^2 - 1$ results from the interference term in $|\Phi|^2$ and the correlation-source relation is just the FT:

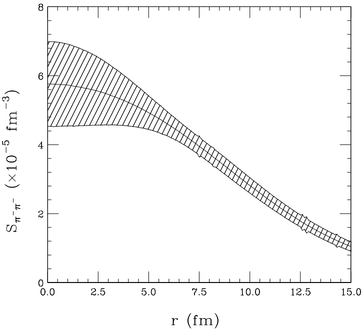
$$\mathcal{R}(\mathbf{q}) = \int d\mathbf{r} \cos(2\mathbf{qr})\, S(\mathbf{r}) \;\Rightarrow\; S(\mathbf{r}) = \frac{1}{\pi^3} \int d\mathbf{q} \cos(2\mathbf{qr})\, \mathcal{R}_{\mathbf{P}}(\mathbf{q})$$

$\ell = 0$ still an FT: $$\mathcal{R}^0(q) = \frac{2\pi}{q} \int dr\, r \sin(2qr) S^0(r)$$

Fourier-Transform of Correlation



Coulomb-corrected 1D π^--π^- correlation-function (Miskowiec *et al.*)



restored source: relative dstr of π^--π^- emission pts in central Au+Au at 10.8 GeV/c (E877)

Brown, PD PLB398(97)252

$S(r \to 0)$: entropy, freeze-out density (Brown, PD, Panitkin . . .)

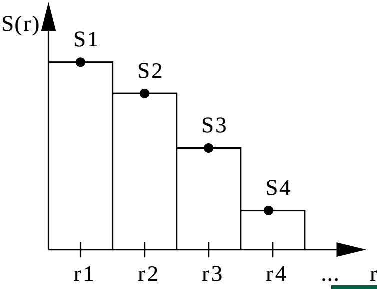
$S(0) \searrow \Leftrightarrow$ entropy $\nearrow$

Discretization & Algebraic Inversion

Source discretization w/ χ^2 fitting applies to any pair/multipolarity:

1. Discretize integral
$$\mathcal{R}_i = \sum_j 4\pi \, \Delta r \, r_j^2 \, K_0(q_i, r_j) \, S(r_j) \equiv \sum_j K_{ij} \, S_j$$
2. Vary $S(r_j)$ to minimize χ^2:
$$\chi^2 = \sum_i \frac{(\sum_j K_{ij} \, S_j - \mathcal{R}_i^{exp})^2}{\sigma_i^2}$$
3. S_j-derivative of χ^2 yields:
$$\sum_{ij} \frac{1}{\sigma_i^2} (K_{ij} \, S_j - \mathcal{R}_i^{exp}) \, K_{ij} = 0$$

with solution in a mtx form:
$$S = (K^\top K)^{-1} \, K^\top \, \mathcal{R}^{exp}$$

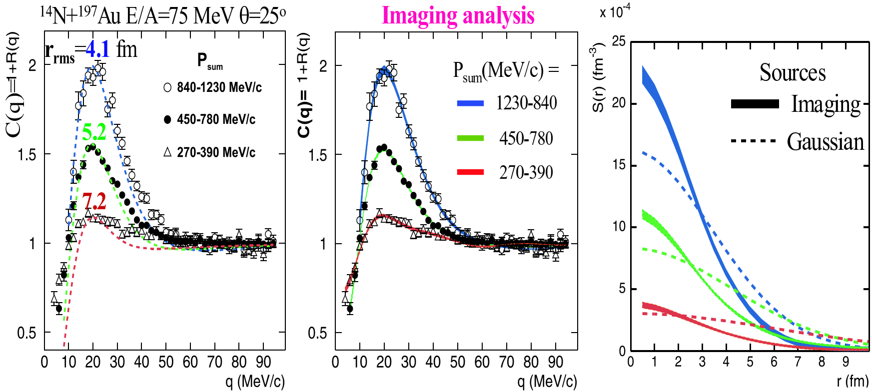


→ Bins adjusted to ensure inversion stability; a smoother basis.

Understanding of Angle-Averaged pp Correlations

Imaging shook up the interpretation of C_{pp}

Verde PRC65(02)054609

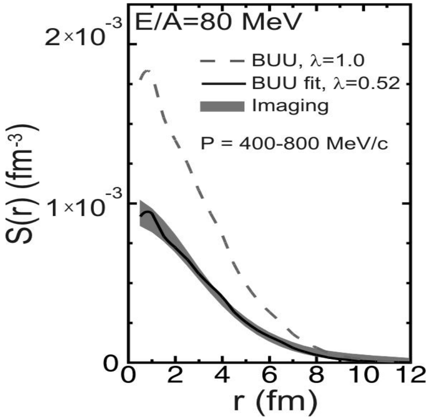


Gauss par: quickly changing radii. Imaging: quickly changing preequilibrium fraction, non-Gaussian source shapes!

Preequilibrium fraction: $\int^{r<R} \mathrm{d}\mathbf{r}\, S \equiv \lambda$

Imaged pp Source Compared to Transport

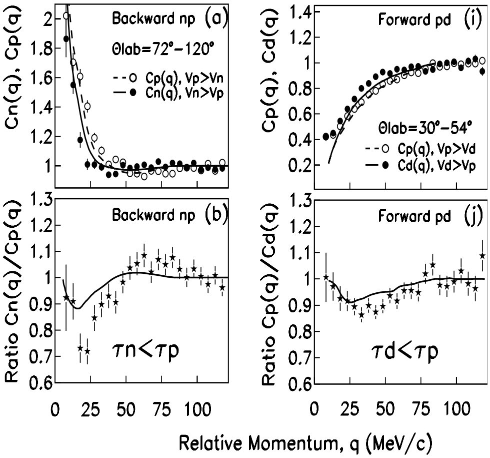
Verde PRC67(03)034606: Ar+Sc central collisions at 80 MeV/u, fast $400 < P_{tot} < 800$ MeV/c pairs



Nucleon-based transport reproduces correctly the *shape* of the preequilibrium source.

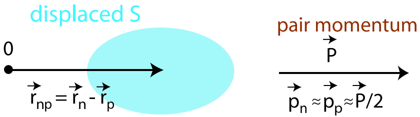
The transport *cannot* describe correctly the preequilibrium pair fraction.

Why Anisotropies: E.g. Time Difference in Emission



Anisotropic C, dependent on orientation of **q**

Attributable to anisotropic S:



if n emitted earlier than p

Ghetti *et al.*, PRL91(03)092701

Model fitted to data

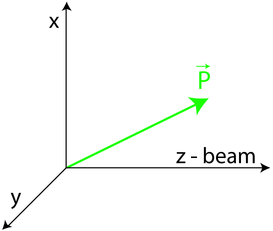
Systematic Treatment of Anisotropies??

As far as anisotropies are concerned, with

$$\mathcal{R}(\mathbf{q}) = \sqrt{4\pi} \sum_{\ell m} \mathcal{R}^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{q}}) \,, \qquad S(\mathbf{r}) = \sqrt{4\pi} \sum_{\ell m} S^{\ell m}(q)\, \mathrm{Y}^{\ell m}(\hat{\mathbf{r}})$$

we have

$$\mathcal{R}^{\ell m}(q) = 4\pi \int \mathrm{d}r\, r^2\, K_\ell(q,r)\, S^{\ell m}(r)$$



A set of 1D integral relations

Problem: Why turning real quantities, R & S, into imaginary, $R^{\ell m}$ & $S^{\ell m}$, of doubtful nature? Other basis than $\mathrm{Y}^{\ell m}$??

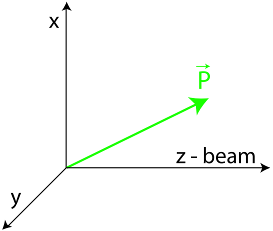
Systematic Treatment of Anisotropies??

As far as anisotropies are concerned, with

$$\mathcal{R}(\mathbf{q}) = \sqrt{4\pi} \sum_{\ell m} \mathcal{R}^{\ell m}(q) \, \mathrm{Y}^{\ell m}(\hat{\mathbf{q}}) \, , \qquad S(\mathbf{r}) = \sqrt{4\pi} \sum_{\ell m} S^{\ell m}(q) \, \mathrm{Y}^{\ell m}(\hat{\mathbf{r}})$$

we have

$$\mathcal{R}^{\ell m}(q) = 4\pi \int \mathrm{d}r \, r^2 \, K_\ell(q, r) \, S^{\ell m}(r)$$



A set of 1D integral relations

Problem: Why turning real quantities, R & S, into imaginary, $R^{\ell m}$ & $S^{\ell m}$, of doubtful nature? Other basis than $\mathrm{Y}^{\ell m}$??

Cartesian Basis

Take the direction vector: $\hat{n}_\alpha = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$

Rank-ℓ tensor product:

$$(\hat{n}^\ell)_{\alpha_1\ldots\alpha_\ell} \equiv \hat{n}_{\alpha_1}\,\hat{n}_{\alpha_1}\,\ldots\,\hat{n}_{\alpha_\ell} = \sum_{\ell'\leq\ell,m} c_{\ell' m}\,\mathrm{Y}^{\ell' m}$$

$\mathcal{D}^{(\ell,\ell)}$ projection operator that, within the space of rank-ℓ cartesian tensors, removes $\mathrm{Y}^{\ell' m}$ components with $\ell' < \ell$:

$$(\mathcal{D}\hat{n}^\ell)_{\alpha_1\ldots\alpha_\ell} = \sum_m c_{\ell m}\,\mathrm{Y}^{\ell m}$$

The components $\mathcal{D}\hat{n}^\ell$ are real and can be used to replace $\mathrm{Y}^{\ell m}$.

Low-ℓ Cartesian Harmonics

$$\begin{aligned}
\mathcal{D}\hat{n}^0 &= 1 \\
(\mathcal{D}\hat{n}^1)_\alpha &= \hat{n}_\alpha \\
(\mathcal{D}\hat{n}^2)_{\alpha_1\,\alpha_2} &= \hat{n}_{\alpha_1}\,\hat{n}_{\alpha_2} - \frac{1}{3}\delta_{\alpha_1\,\alpha_2} \\
(\mathcal{D}\hat{n}^3)_{\alpha_1\,\alpha_2\,\alpha_3} &= \hat{n}_{\alpha_1}\,\hat{n}_{\alpha_2}\,\hat{n}_{\alpha_3} - \frac{1}{5}(\delta_{\alpha_1\,\alpha_2}\,\hat{n}_{\alpha_3} + \delta_{\alpha_1\,\alpha_3}\,\hat{n}_{\alpha_2} + \delta_{\alpha_2\,\alpha_3}\,\hat{n}_{\alpha_1}) \\
(\mathcal{D}\hat{n}^4)_{\alpha_1\,\alpha_2\,\alpha_3\,\alpha_4} &= \hat{n}_{\alpha_1}\,\hat{n}_{\alpha_2}\,\hat{n}_{\alpha_3}\,\hat{n}_{\alpha_4} - \frac{1}{7}(\delta_{\alpha_1\,\alpha_2}\,\hat{n}_{\alpha_3}\,\hat{n}_{\alpha_4} + \ldots) \\
&\quad + \frac{1}{35}(\delta_{\alpha_1\,\alpha_2}\,\delta_{\alpha_3\,\alpha_4} + \ldots) \\
&\vdots
\end{aligned}$$

$\mathcal{D}$ can be called a detracing operator as

$$\sum_\alpha (\mathcal{D}\hat{n}^\ell)_{\alpha\,\alpha\,\alpha_3\ldots\alpha_\ell} = 0$$

Decomposition with Cartesian Harmonics

Completeness relation ($\mathcal{D} = \mathcal{D}^\top = \mathcal{D}^2$):

$$\delta(\Omega' - \Omega) = \frac{1}{4\pi} \sum_\ell \frac{(2\ell+1)!!}{\ell!} \sum_{\alpha_1 \ldots \alpha_\ell} (\mathcal{D}\hat{n}'^\ell)_{\alpha_1 \ldots \alpha_\ell} (\mathcal{D}\hat{n}^\ell)_{\alpha_1 \ldots \alpha_\ell}$$

$$= \frac{1}{4\pi} \sum_\ell \frac{(2\ell+1)!!}{\ell!} \sum_{\alpha_1 \ldots \alpha_\ell} (\mathcal{D}\hat{n}'^\ell)_{\alpha_1 \ldots \alpha_\ell} \hat{n}_{\alpha_1} \ldots \hat{n}_{\alpha_\ell}$$

In consequence

$$\mathcal{R}(\mathbf{q}) = \int d\Omega' \, \delta(\Omega' - \Omega) \, \mathcal{R}(\mathbf{q}') = \sum_\ell \sum_{\alpha_1 \ldots \alpha_\ell} \mathcal{R}^{(\ell)}_{\alpha_1 \ldots \alpha_\ell}(q) \, \hat{q}_{\alpha_1} \ldots \hat{q}_{\alpha_\ell}$$

where coefficients are angular moments

$$\mathcal{R}^{(\ell)}_{\alpha_1 \ldots \alpha_\ell}(q) = \frac{(2\ell+1)!!}{\ell!} \int \frac{d\Omega_\mathbf{q}}{4\pi} \mathcal{R}(\mathbf{q}) \, (\mathcal{D}\hat{q}^\ell)_{\alpha_1 \ldots \alpha_\ell}$$

Decomposition with Cartesian Harmonics

Completeness relation ($\mathcal{D} = \mathcal{D}^\top = \mathcal{D}^2$):

$$\begin{aligned}\delta(\Omega' - \Omega) &= \frac{1}{4\pi}\sum_\ell \frac{(2\ell+1)!!}{\ell!} \sum_{\alpha_1\ldots\alpha_\ell} (\mathcal{D}\hat{n}'^\ell)_{\alpha_1\ldots\alpha_\ell} (\mathcal{D}\hat{n}^\ell)_{\alpha_1\ldots\alpha_\ell} \\ &= \frac{1}{4\pi}\sum_\ell \frac{(2\ell+1)!!}{\ell!} \sum_{\alpha_1\ldots\alpha_\ell} (\mathcal{D}\hat{n}'^\ell)_{\alpha_1\ldots\alpha_\ell} \hat{n}_{\alpha_1} \ldots \hat{n}_{\alpha_\ell}\end{aligned}$$

In consequence

$$\mathcal{R}(\mathbf{q}) = \int \mathrm{d}\Omega' \, \delta(\Omega' - \Omega) \, \mathcal{R}(\mathbf{q}') = \sum_\ell \sum_{\alpha_1\ldots\alpha_\ell} \mathcal{R}^{(\ell)}_{\alpha_1\ldots\alpha_\ell}(q) \, \hat{q}_{\alpha_1} \ldots \hat{q}_{\alpha_\ell}$$

where coefficients are angular moments

$$\mathcal{R}^{(\ell)}_{\alpha_1\ldots\alpha_\ell}(q) = \frac{(2\ell+1)!!}{\ell!} \int \frac{\mathrm{d}\Omega_{\mathbf{q}}}{4\pi} \mathcal{R}(\mathbf{q}) \, (\mathcal{D}\hat{q}^\ell)_{\alpha_1\ldots\alpha_\ell}$$

Consequences

Cartesian coefficients for $\mathcal{R}$ & S directly related to each other:

$$\mathcal{R}^{(\ell)}_{\alpha_1 \cdots \alpha_\ell}(q) = 4\pi \int \mathrm{d}r\, r^2\, K_\ell(q,r)\, \mathcal{S}^{(\ell)}_{\alpha_1 \cdots \alpha_\ell}(r)$$

For weak anisotropies, only lowest-ℓ matter:

$$\mathcal{R}(\mathbf{q}) = \mathcal{R}^{(0)}(q) + \sum_\alpha \mathcal{R}^{(1)}_\alpha(q)\, \hat{q}_\alpha + \sum_{\alpha_1\, \alpha_2} \mathcal{R}^{(2)}_{\alpha_1 \alpha_2}(q)\, \hat{q}_{\alpha_1}\, \hat{q}_{\alpha_2} + \ldots$$

$\mathcal{R}^{(0)}$ - angle-averaged correlation

$\mathcal{R}^{(1)}_\alpha \equiv R^{(1)}\, e^{(1)}_\alpha$ - dipole distortion, magnitude + direction vector

$\mathcal{R}^{(2)}_{\alpha\beta}(q) = R^{(2)}_1\, e^{(2)}_{1\alpha}\, e^{(2)}_{1\beta} + R^{(2)}_3\, e^{(2)}_{3\alpha}\, e^{(2)}_{3\beta} - \left(R^{(2)}_1 + R^{(2)}_3\right)\, e^{(2)}_{2\alpha}\, e^{(2)}_{2\beta}$
- quadrupole distortion, 2 magnitude values + 3 orthogonal direction vectors

Consequences

Cartesian coefficients for $\mathcal{R}$ & S directly related to each other:

$$\mathcal{R}^{(\ell)}_{\alpha_1\cdots\alpha_\ell}(q) = 4\pi \int \mathrm{d}r\, r^2\, K_\ell(q,r)\, \mathcal{S}^{(\ell)}_{\alpha_1\cdots\alpha_\ell}(r)$$

For weak anisotropies, only lowest-ℓ matter:

$$\mathcal{R}(\mathbf{q}) = \mathcal{R}^{(0)}(q) + \sum_\alpha \mathcal{R}^{(1)}_\alpha(q)\, \hat{q}_\alpha + \sum_{\alpha_1\,\alpha_2} \mathcal{R}^{(2)}_{\alpha_1\alpha_2}(q)\, \hat{q}_{\alpha_1}\, \hat{q}_{\alpha_2} + \ldots$$

$\mathcal{R}^{(0)}$ - angle-averaged correlation

$\mathcal{R}^{(1)}_\alpha \equiv R^{(1)}\, e^{(1)}_\alpha$ - dipole distortion, magnitude + direction vector

$\mathcal{R}^{(2)}_{\alpha\beta}(q) = R^{(2)}_1\, e^{(2)}_{1\alpha}\, e^{(2)}_{1\beta} + R^{(2)}_3\, e^{(2)}_{3\alpha}\, e^{(2)}_{3\beta} - \left(R^{(2)}_1 + R^{(2)}_3\right)\, e^{(2)}_{2\alpha}\, e^{(2)}_{2\beta}$
- quadrupole distortion, 2 magnitude values + 3 orthogonal direction vectors

Consequences

Cartesian coefficients for $\mathcal{R}$ & S directly related to each other:

$$\mathcal{R}^{(\ell)}_{\alpha_1 \cdots \alpha_\ell}(q) = 4\pi \int \mathrm{d}r \, r^2 \, K_\ell(q,r) \, \mathcal{S}^{(\ell)}_{\alpha_1 \cdots \alpha_\ell}(r)$$

For weak anisotropies, only lowest-ℓ matter:

$$\mathcal{R}(\mathbf{q}) = \mathcal{R}^{(0)}(q) + \sum_{\alpha} \mathcal{R}^{(1)}_{\alpha}(q) \, \hat{q}_\alpha + \sum_{\alpha_1 \, \alpha_2} \mathcal{R}^{(2)}_{\alpha_1 \alpha_2}(q) \, \hat{q}_{\alpha_1} \, \hat{q}_{\alpha_2} + \ldots$$

$\mathcal{R}^{(0)}$ - angle-averaged correlation

$\mathcal{R}^{(1)}_{\alpha} \equiv R^{(1)} \, e^{(1)}_{\alpha}$ - dipole distortion, magnitude + direction vector

$\mathcal{R}^{(2)}_{\alpha\beta}(q) = R^{(2)}_1 \, e^{(2)}_{1\alpha} \, e^{(2)}_{1\beta} + R^{(2)}_3 \, e^{(2)}_{3\alpha} \, e^{(2)}_{3\beta} - \left(R^{(2)}_1 + R^{(2)}_3 \right) \, e^{(2)}_{2\alpha} \, e^{(2)}_{2\beta}$
- quadrupole distortion, 2 magnitude values + 3 orthogonal direction vectors

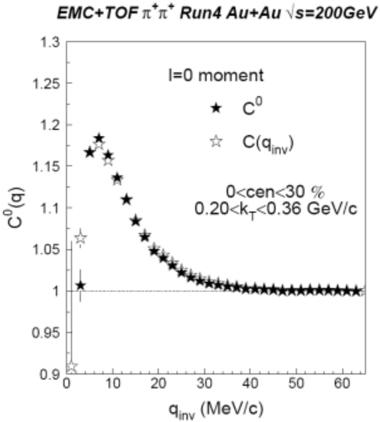
Source & Correlation Symmetries

$\pi^- - \pi^-$ or $\pi^+ - \pi^+$:

- Identical ptcles: $\vec{r} \rightarrow -\vec{r} \Leftrightarrow$ even-ℓ only
- Midrapidity: $z \rightarrow -z \Leftrightarrow$ even-z moments only
- Reaction-plane averaging: $y \rightarrow -y \Leftrightarrow$ even-y moments only
- In the end, also: $x \rightarrow -x \Leftrightarrow$ even-x moments only

- $\ell = 0$
- $\ell = 2$: x^2, y^2, z^2 (only 2 independent)
- $\ell = 4$: x^4, y^4, z^4, $x^2\,y^2$, $x^2\,z^2$, $y^2\,z^2$ (only 3 independent)

$\ell = 0$ Angle-Averaged Correlation



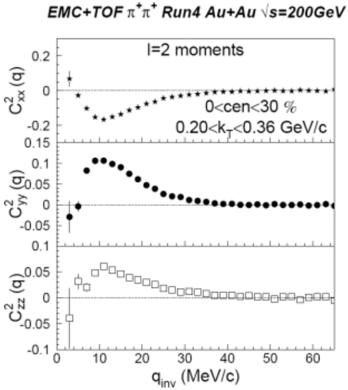
Analysis in the pair CM.

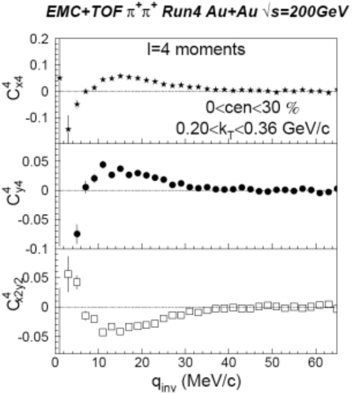
Paul Chung

different order of averaging...

Coulomb final-state interactions treated as an essential tool, not a nuisance!

$\ell = 2$ & $\ell = 4$ Correlations





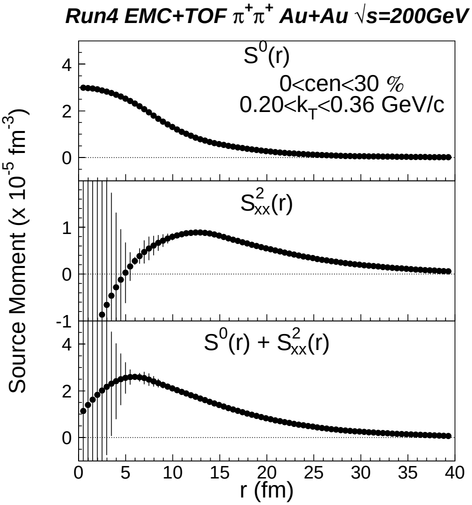
$C^2_{x^2} + C^2_{y^2} + C^2_{z^2} = 0$

Only independent $\ell = 4$ functions shown

Note: C in the x-direction is $C = C^0 + C^2_{x^2} + C^4_{x^4} + \ldots$, etc.

Full 3D info in terms of few 1D plots!

Imaged Source Moments



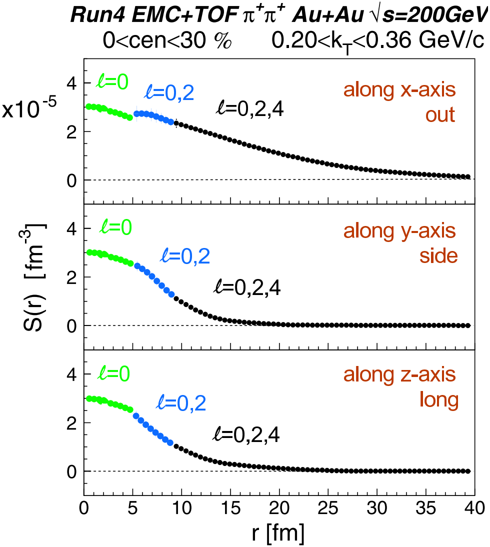
Moments contributing to x-direction

Source enhancement in x-direction compared to average.

At low r poor resolution for high ℓ.

Here, same source basis used for different ℓ - optimally should be different.

Imaged Source Along Different Directions



$$S = S^0 + S^2_{x^2} + S^4_{x^4} + \dots$$
$$\ell = 0 \quad 2 \quad 4$$

Imaged source dramatically extended in the outward direction in the pair CM, indicating a prolonged emission in that frame!

$R_{out} = 15.5$ fm
$R_{side} = 4.2$ fm
$R_{long} = 4.7$ fm

Final Points

- Correlations at low relative velocities yield access to source spatial characteristics in the pair CM! Temporal information in the pair CM is not accessible.
- Coulomb final-state interactions should be employed for source-determination and not be patched up.
- Source shape imaging is feasible: PD & Pratt, PLB618(05)60, D. Brown et al., nucl-th/0507015.
- Identical pion data (Paul Chung) demonstrate dramatically prolonged pion emission in the pair CM.
- How to interpret the prolonged emission?? Instantaneous freeze-out in the LCM violates causality. Lorentz effects clearly important. Hydro models need to make assumptions about the freeze-out surface...
- The focus should be the accessible, i.e. pair -CM info, and not an info patched-up with auxiliary considerations.