Effects of Latt'QCD EoS and the Continuous Emission on v_2 and Some Other Observables

Y. Hama[1] R.P.G. Andrade[1] F. Grassi[1] O. Socolowski[2]
T. Kodama[3] B. Tavares[3] S.S. Padula[4]

[1]Instituto de Física, Universidade de São Paulo, Brazil

[2]Instituto Tecnológico da Aeronáutica, Brazil

[3]Instituto de Física, Universidade Federal do Rio de Janeiro, Brazil

[4]Instituto de Física Teórica, Universidade Estadual Paulista, Brazil

WPCF 2005, Kroměříž

Outline

Outline

1. The Basic Ingredients of Hydrodynamic Models
    - Equations of Motion - SPheRIO Code
    - Equations of State
    - Initial Conditions
    - Decoupling Procedure
2. Our Results
    - Pseudorapidity Distribution
    - Transverse-Momentum Distribution
    - Elliptic-Flow Parameter v_2
    - HBT Radii

Equations of Motion
SPheRIO code

- Energy-Momentum Conservation:

$$\partial_\nu T^{\mu\nu} = 0, \quad \text{where} \quad T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - p g^{\mu\nu};$$

- Baryon-Number Conservation:

$$\partial_\mu (n_B u^\mu) = 0;$$

- Other Quantum-Number Conservation ($n_S, n_{ch}, \ldots$):

$$\begin{aligned} \partial_\mu (n_S u^\mu) &= 0 \quad , \\ \cdots & \quad . \end{aligned}$$

Equations of Motion
SPheRIO code

- Energy-Momentum Conservation:

$$\partial_\nu T^{\mu\nu} = 0, \quad \text{where} \quad T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu};$$

- Baryon-Number Conservation:

$$\partial_\mu (n_B u^\mu) = 0;$$

- Other Quantum-Number Conservation ($n_S, n_{ch}, \ldots$):

$$\partial_\mu (n_S u^\mu) = 0 \quad , \quad \ldots \quad .$$

Equations of Motion
SPheRIO code

- Energy-Momentum Conservation:

$$\partial_\nu T^{\mu\nu} = 0, \quad \text{where} \quad T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu};$$

- Baryon-Number Conservation:

$$\partial_\mu (n_B u^\mu) = 0;$$

- Other Quantum-Number Conservation (n_S , n_{ch} , ...):

$$\partial_\mu (n_S u^\mu) = 0 \quad , \qquad \cdots \quad .$$

Equations of Motion
SPheRIO code

We solve these equations with

SPheRIO
(Smoothed Particle hydrodynamic evolution of Relativistic heavy IOn collisions) code,

[C.E. Aguiar, T. Kodama, T. Osada and Y. Hama, *J. Phys.* **G 27** (2001) 75]

based on SPH (Smoothed-Particle Hydrodynamics) algorithm.

[L.B. Lucy, *Astrophys. J.* **82** (1977) 1013;
R.A. Gingold and J.J. Monaghan, *Mon. Not. R. Astro. Soc.* **181** (1977) 375]

Equations of State

- EoS with 1st.-Order Phase Transition (1OPT EoS):
  - QGP phase: MIT Bag Model
  - Hadron phase: Resonance gas, with excluded-volume

[see, e.g., Y. Hama, T. Kodama and O. Socolowski, *Braz. J. Phys.* **35** (2005) 24]

- Lattice QCD Results:
  - Critical End Point in the Phase Diagram
- Phenomenological Parametrization of Lattice QCD EoS: We call this parametrization CP EoS.
- Comparisons of CP EoS with 1OPT EoS.

Equations of State

- EoS with 1st.-Order Phase Transition (1OPT EoS):
  - QGP phase: MIT Bag Model
  - Hadron phase: Resonance gas, with excluded-volume

[see, e.g., Y. Hama, T. Kodama and O. Socolowski, *Braz. J. Phys.* **35** (2005) 24]

- Lattice QCD Results:
  - Critical End Point in the Phase Diagram
- Phenomenological Parametrization of Lattice QCD EoS: We call this parametrization CP EoS.
- Comparisons of CP EoS with 1OPT EoS.

Equations of State

- EoS with 1st.-Order Phase Transition (1OPT EoS):
  - QGP phase: MIT Bag Model
  - Hadron phase: Resonance gas, with excluded-volume

[see, e.g., Y. Hama, T. Kodama and O. Socolowski, *Braz. J. Phys.* **35** (2005) 24]

- Lattice QCD Results:
  - Critical End Point in the Phase Diagram
- Phenomenological Parametrization of Lattice QCD EoS: We call this parametrization CP EoS.
- Comparisons of CP EoS with 1OPT EoS.

Equations of State

- EoS with 1st.-Order Phase Transition (1OPT EoS):
  - QGP phase: MIT Bag Model
  - Hadron phase: Resonance gas, with excluded-volume

  [see, e.g., Y. Hama, T. Kodama and O. Socolowski, *Braz. J. Phys.* **35** (2005) 24]

- Lattice QCD Results:
  - Critical End Point in the Phase Diagram

- Phenomenological Parametrization of Lattice QCD EoS: We call this parametrization CP EoS.

- Comparisons of CP EoS with 1OPT EoS.

Equations of State

- EoS with 1st.-Order Phase Transition (1OPT EoS):
  - QGP phase: MIT Bag Model
  - Hadron phase: Resonance gas, with excluded-volume

  [see, e.g., Y. Hama, T. Kodama and O. Socolowski, *Braz. J. Phys.* **35** (2005) 24]

- Lattice QCD Results:
  - Critical End Point in the Phase Diagram

- Phenomenological Parametrization of Lattice QCD EoS: We call this parametrization CP EoS.

- Comparisons of CP EoS with 1OPT EoS.

Equations of State

- EoS with 1st.-Order Phase Transition (1OPT EoS):
  - QGP phase: MIT Bag Model
  - Hadron phase: Resonance gas, with excluded-volume

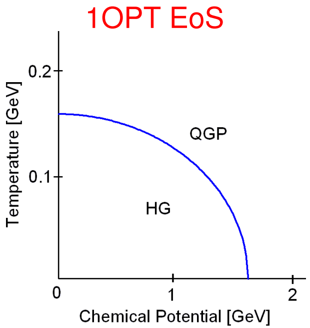
[see, e.g., Y. Hama, T. Kodama and O. Socolowski, *Braz. J. Phys.* **35** (2005) 24]

- Lattice QCD Results:
  - Critical End Point in the Phase Diagram

- Phenomenological Parametrization of Lattice QCD EoS: We call this parametrization CP EoS.

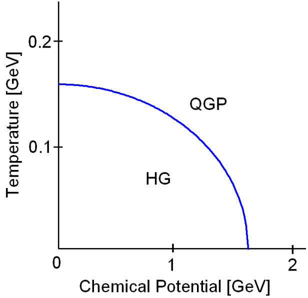
- Comparisons of CP EoS with 1OPT EoS.

Phse Diagram in (T, μ) Plane

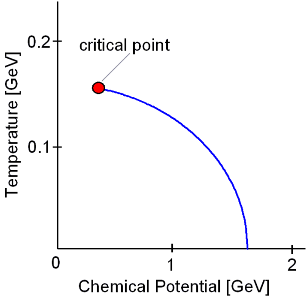


Phse Diagram in (T, μ) Plane

1OPT EoS

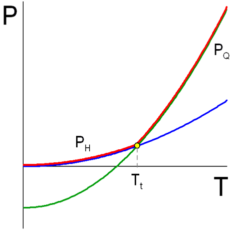


CP EoS



Parametrization of Lattice QCD EoS

In 1OPT EoS:

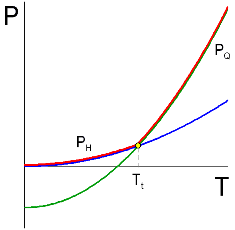


P is given by

$$(P - P_Q)(P - P_H) = 0\,.$$

Parametrization of Lattice QCD EoS

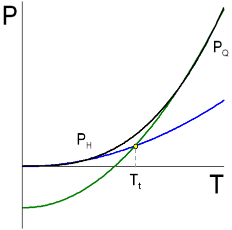
In 1OPT EoS:



P is given by

$$(P - P_Q)(P - P_H) = 0\,.$$

In CP EoS:



given μ, we parametrize P as

$$(P - P_Q)(P - P_H) = \delta(\mu)\,,$$

with $\delta(\mu) = \delta_0 \exp\left[-(\mu/\mu_c)^2\right]$.

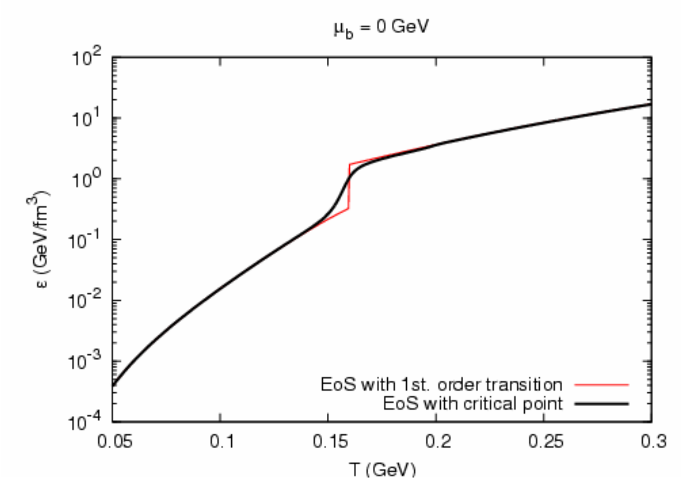
Parametrization of Lattice QCD EoS

Then,

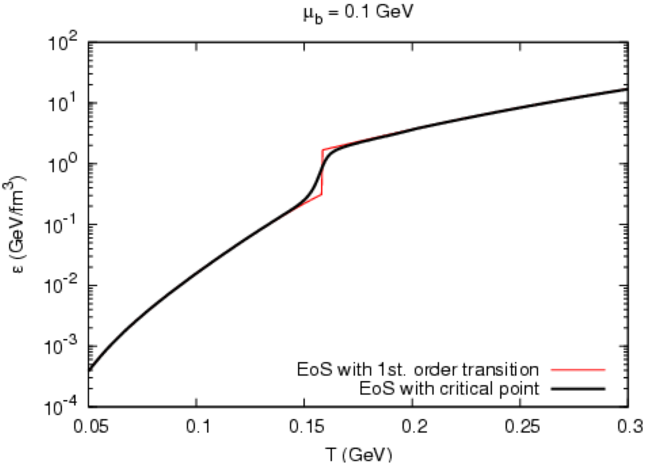
$$\begin{aligned}
P &= \lambda P_H + (1-\lambda)P_Q + \frac{2\delta}{\sqrt{(P_Q-P_H)^2+4\delta}} \\
s &= \lambda s_H + (1-\lambda)s_Q \\
n_B &= \lambda n_H + (1-\lambda)n_Q - \frac{2\left(\mu/\mu_c^2\right)\delta}{\sqrt{(P_Q-P_H)^2+4\delta}} \\
\epsilon &= \lambda \epsilon_H + (1-\lambda)\epsilon_Q - \frac{2\left[1+(\mu/\mu_c)^2\right]\delta}{\sqrt{(P_Q-P_H)^2+4\delta}}
\end{aligned}$$

where $\lambda \equiv \left[1-(P_Q-P_H)/\sqrt{(P_Q-P_H)^2+4\delta}\right]/2$.

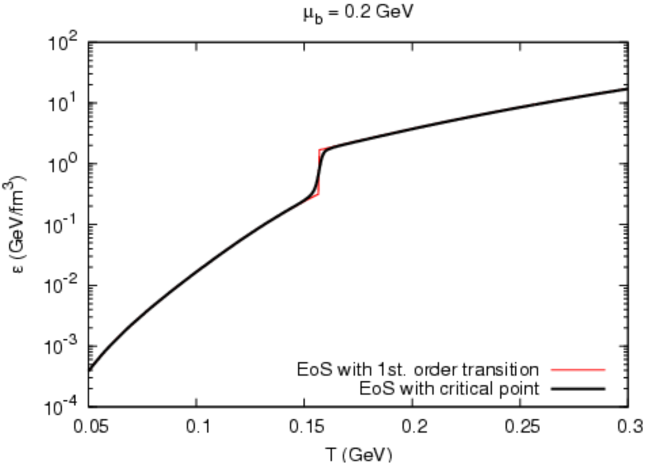
Equations of State: $\epsilon(T)$



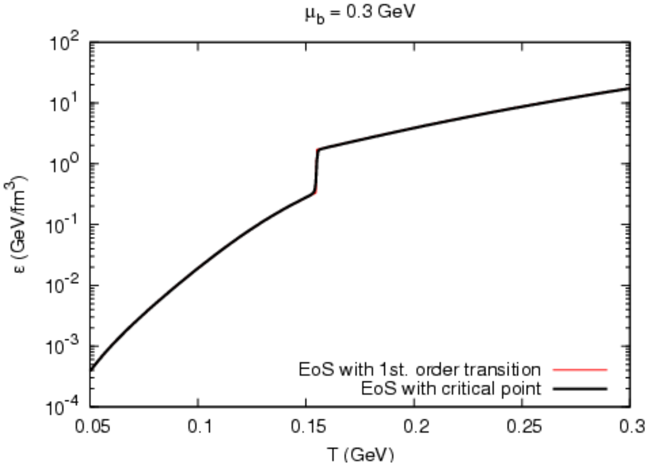
Equations of State: $\epsilon(T)$



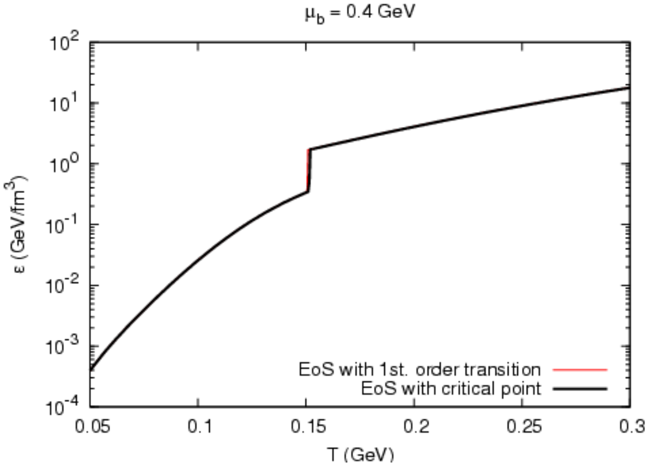
Equations of State: $\epsilon(T)$



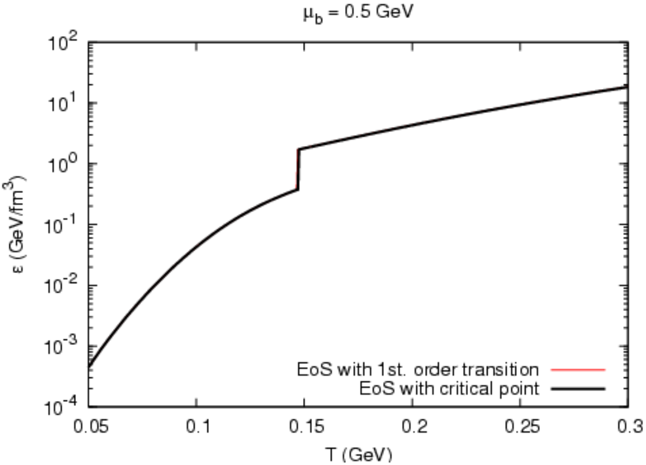
Equations of State: $\epsilon(T)$



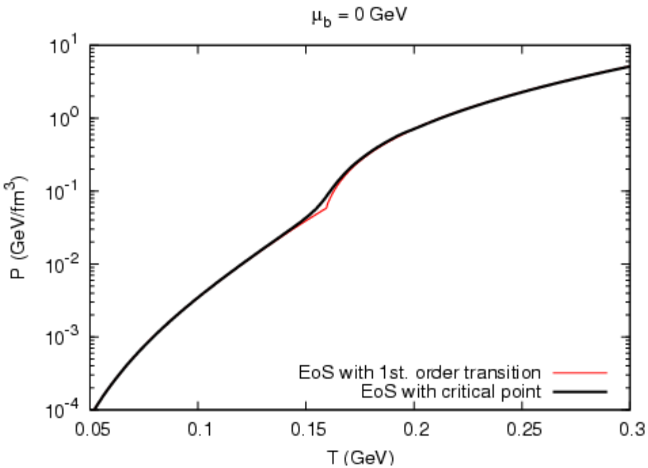
Equations of State: $\epsilon(T)$



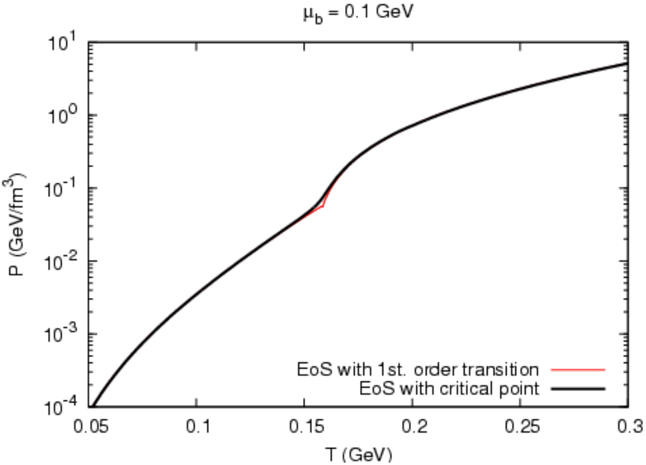
Equations of State: $\epsilon(T)$



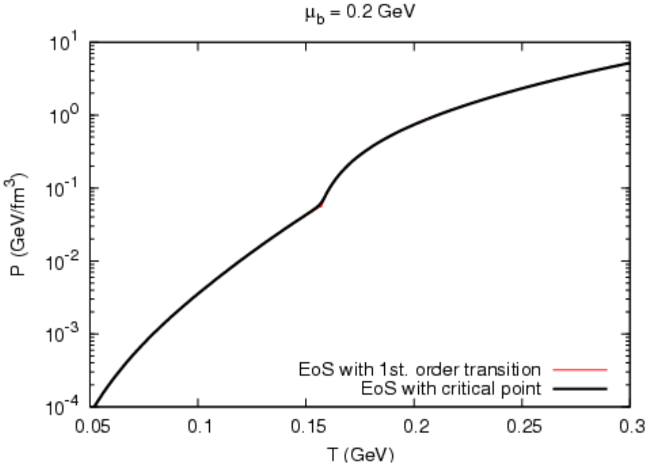
Equations of State: $p(T)$



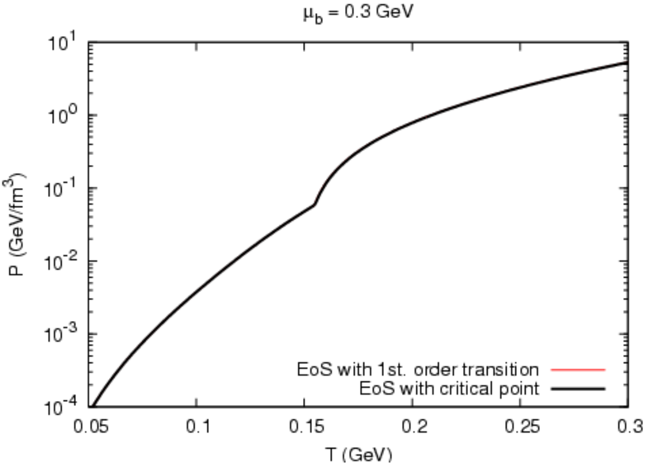
Equations of State: $p(T)$



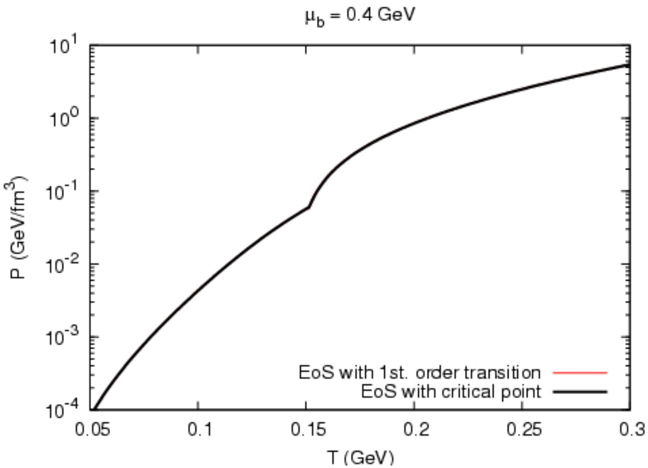
Equations of State: $p(T)$



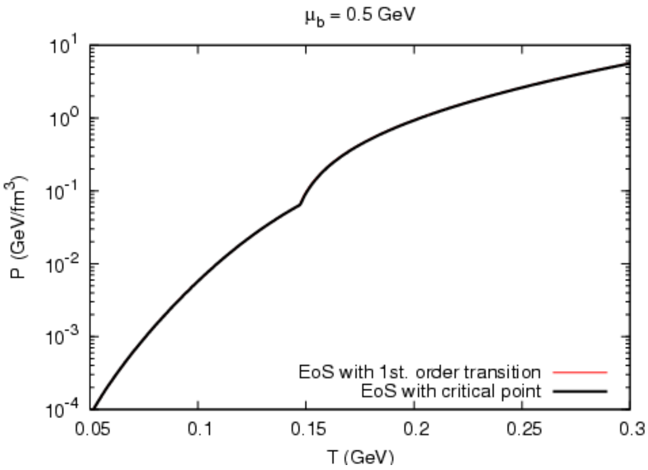
Equations of State: $p(T)$



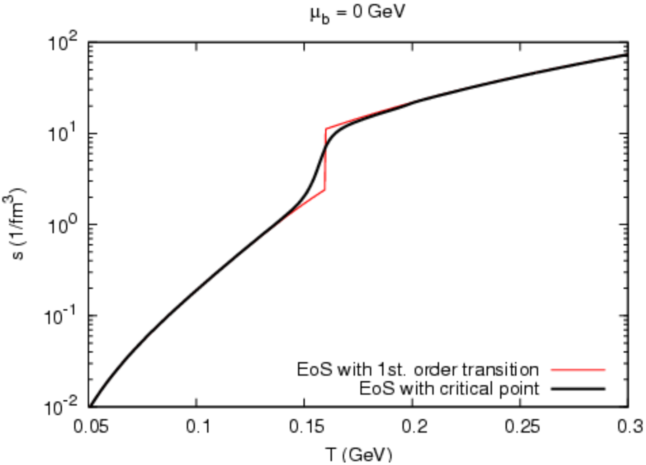
Equations of State: $p(T)$



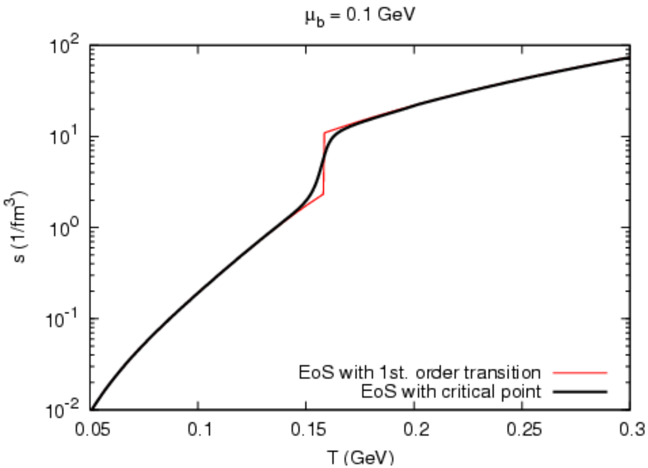
Equations of State: $p(T)$



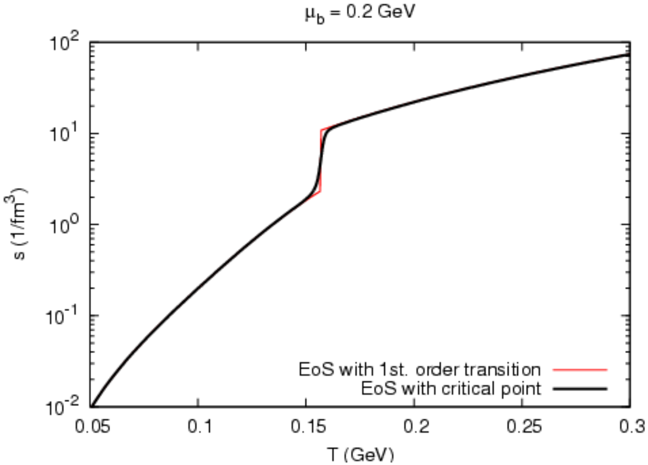
Equations of State: $s(T)$



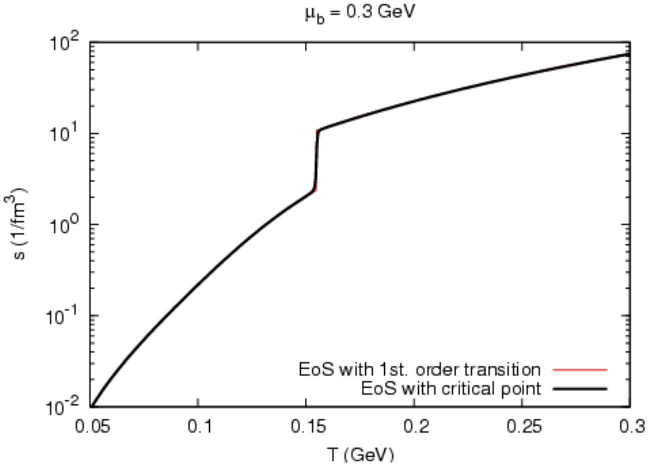
Equations of State: $s(T)$



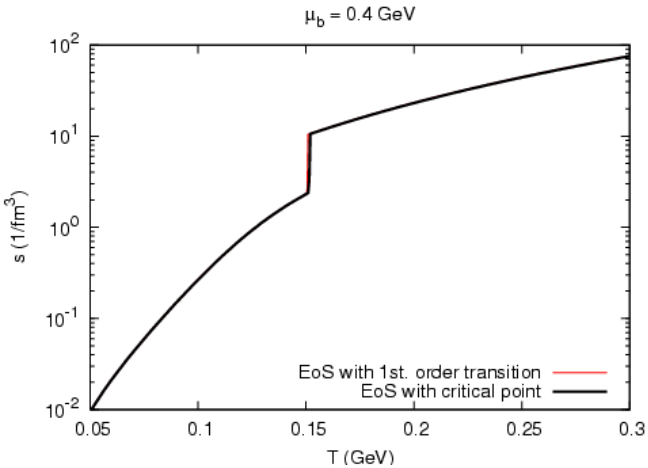
Equations of State: $s(T)$



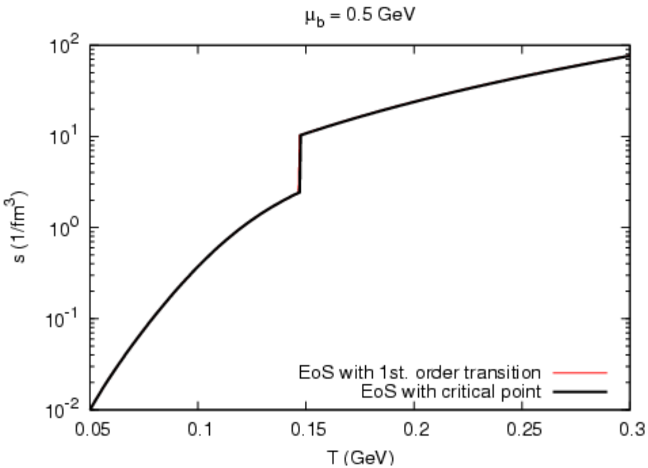
Equations of State: $s(T)$



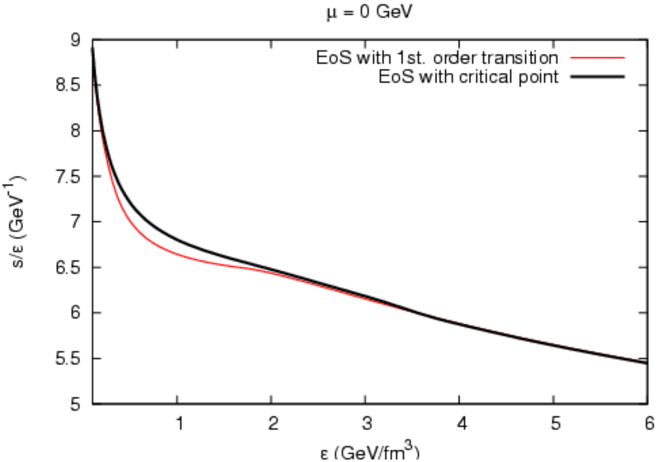
Equations of State: $s(T)$



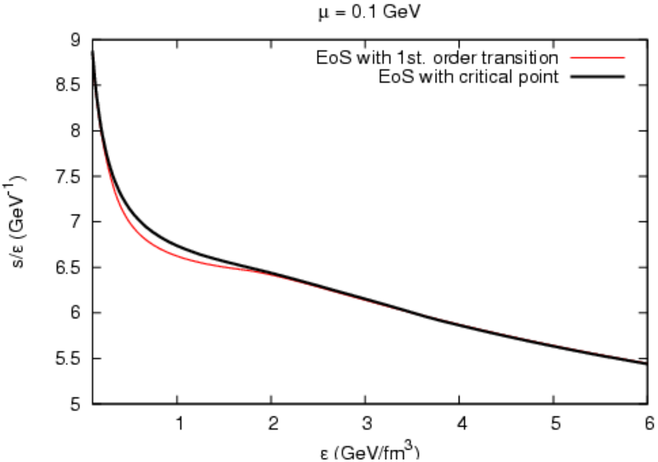
Equations of State: $s(T)$



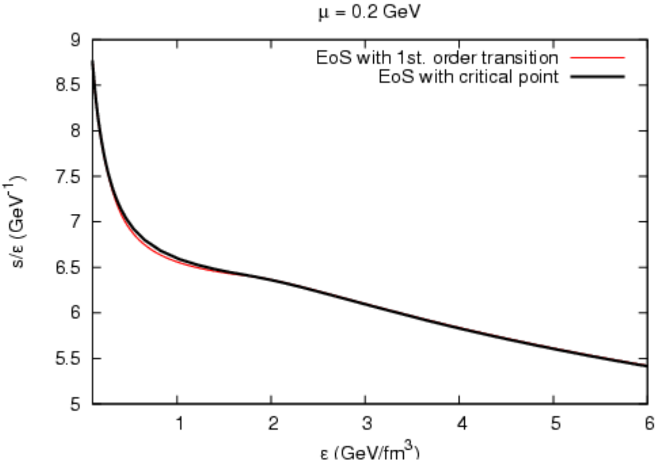
Equations of State: $\frac{s}{\epsilon}(\epsilon)$



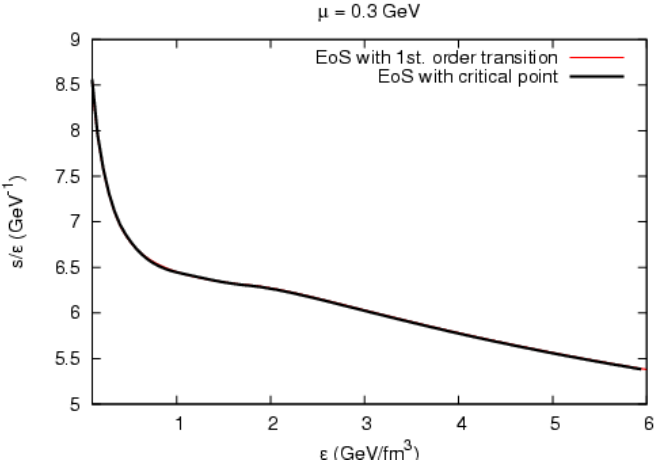
Equations of State: $\frac{s}{\epsilon}(\epsilon)$



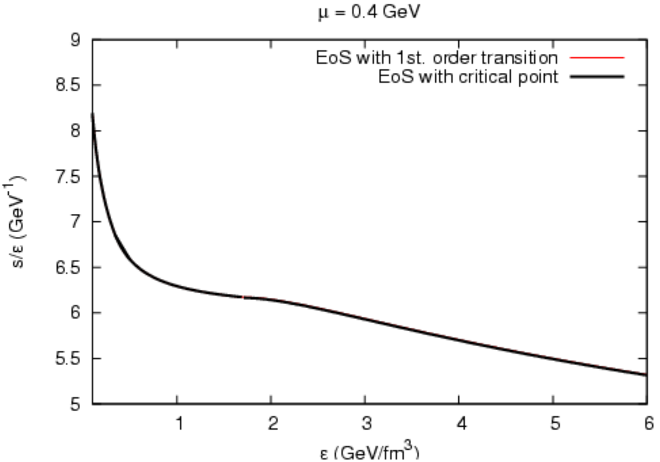
Equations of State: $\frac{s}{\epsilon}(\epsilon)$



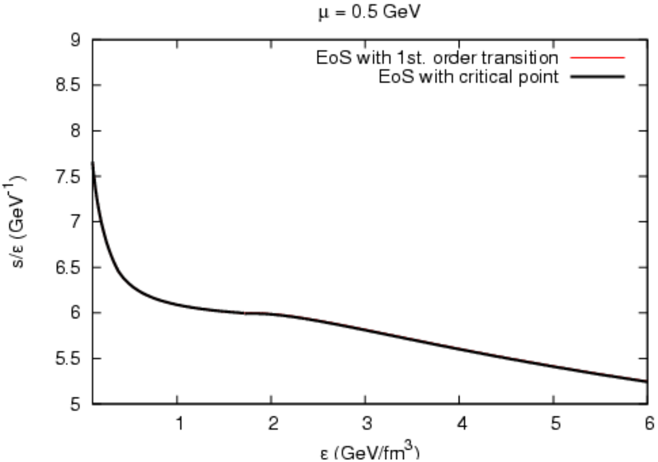
Equations of State: $\frac{s}{\epsilon}(\epsilon)$



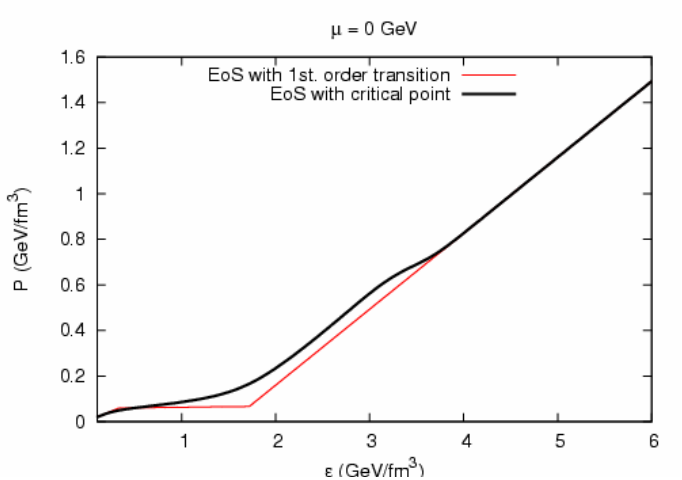
Equations of State: $\frac{s}{\epsilon}(\epsilon)$



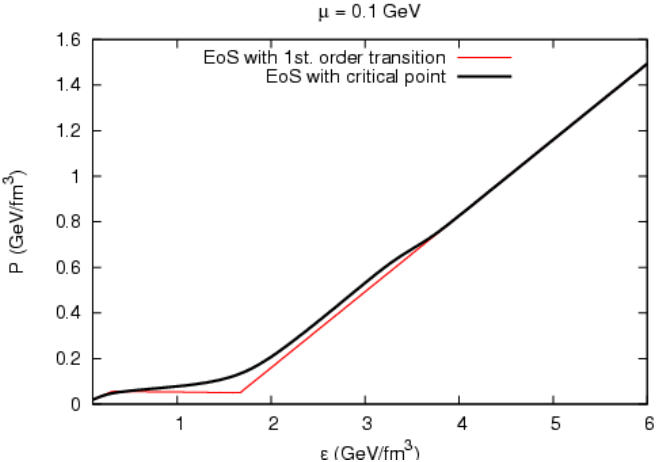
Equations of State: $\frac{s}{\epsilon}(\epsilon)$



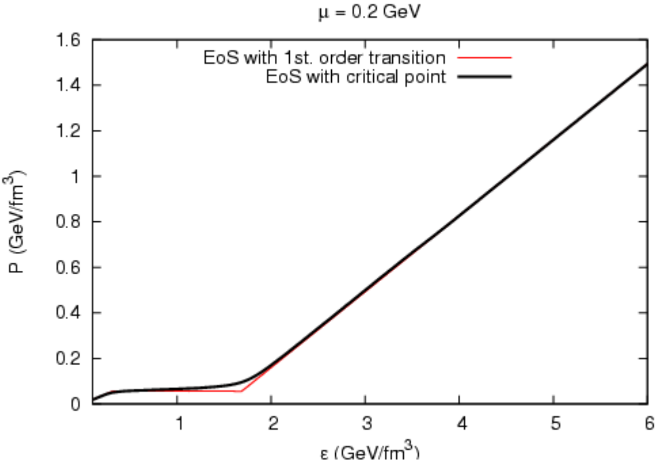
Equations of State: $p(\epsilon)$



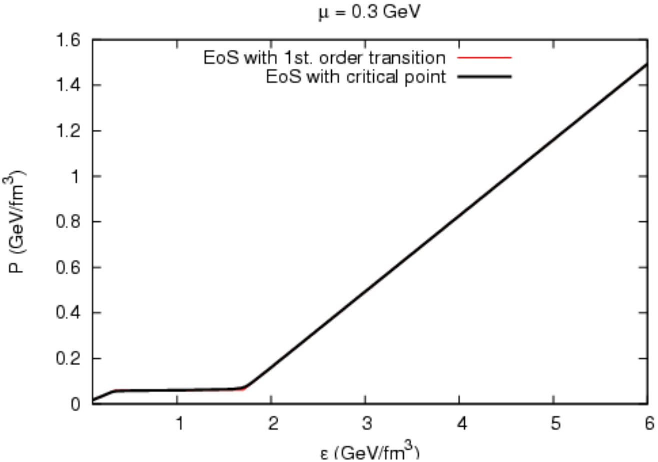
Equations of State: $p(\epsilon)$



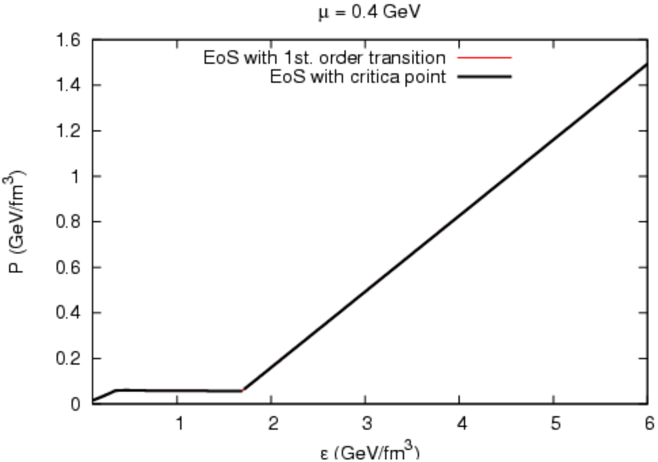
Equations of State: $p(\epsilon)$



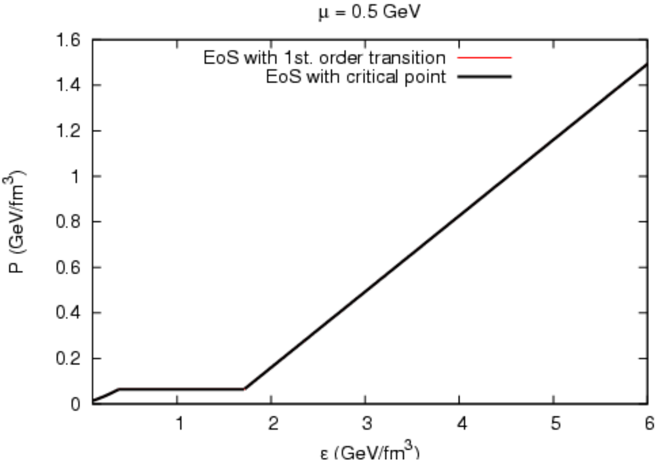
Equations of State: $p(\epsilon)$



Equations of State: $p(\epsilon)$



Equations of State: $p(\epsilon)$



Some Preliminary Conclusions

- For the same initial conditions, expressed by some
  - Energy-density distribution,
  - Velocity distribution and
  - Baryon-number distribution,

  the multiplicity is larger for CP EoS, as compared with the one for 1OPT EoS.

- The acceleration is larger in the transition (crossover) region for CP EoS.

Some Preliminary Conclusions

- For the same initial conditions, expressed by some
  - Energy-density distribution,
  - Velocity distribution and
  - Baryon-number distribution,

  the multiplicity is larger for CP EoS, as compared with the one for 1OPT EoS.
- The acceleration is larger in the transition (crossover) region for CP EoS.

Some Preliminary Conclusions

- For the same initial conditions, expressed by some
  - Energy-density distribution,
  - Velocity distribution and
  - Baryon-number distribution,

  the multiplicity is larger for CP EoS, as compared with the one for 1OPT EoS.
- The acceleration is larger in the transition (crossover) region for CP EoS.

Initial Conditions (IC)

- In usual hydrodynamic approach, one assumes some highly symmetric and smooth IC.
- However, our systems are small, so large fluctuatios are expected in real collisions.
- Many simulators, based on microscopic models, *e.g.*
  - HIJING,
  - VNI,
  - URASiMA,
  - NeXuS,
  - $\cdots$,

  show such event-by-event fluctuations.

Initial Conditions (IC)

- In usual hydrodynamic approach, one assumes some highly symmetric and smooth IC.
- However, our systems are small, so large fluctuatios are expected in real collisions.
- Many simulators, based on microscopic models, *e.g.*
  - HIJING,
  - VNI,
  - URASiMA,
  - NeXuS,
  - $\cdots$,

  show such event-by-event fluctuations.

Initial Conditions (IC)

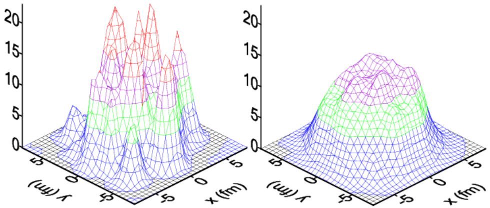
- In usual hydrodynamic approach, one assumes some highly symmetric and smooth IC.
- However, our systems are small, so large fluctuatios are expected in real collisions.
- Many simulators, based on microscopic models, *e.g.*
  - HIJING,
  - VNI,
  - URASiMA,
  - NeXuS,
  - $\cdots$,

  show such event-by-event fluctuations.

Initial Conditions:
Event-by-Event Fluctuating vs. Smooth Averaged

Energy density for central Au+Au collisions at 130 GeV, given by NeXus simulator,* at mid-rapidity

One random event   Average over 30 events



* [H.J. Drescher, F.M. Liu, S. Ostrapchenko, T. Pierog and K. Werner, *Phys. Rev.* **C65** (2002) 054902.]

Decoupling Procedure

- Often one assumes decoupling on a sharply defined hypersurface. We call this Sudden Freeze Out (FO).
- However, our systems are small, so partcles may escape from a layer with thickness comparable with the systems sizes.
- We introduce, at each space-time point x^μ, a certain momentum-dependent escaping probability

$$\mathcal{P}(x,k) = \exp\left[-\int_\tau^\infty \rho(x')\,\sigma v\,\mathrm{d}\tau'\right].$$

This is the Continuous Emission Model (CE).*

* [F. Grassi, Y. Hama and T. Kodama, *Phys. Lett.* **B355** (1996) 9]

Decoupling Procedure

- Often one assumes decoupling on a sharply defined hypersurface. We call this Sudden Freeze Out (FO).
- However, our systems are small, so partcles may escape from a layer with thickness comparable with the systems sizes.
- We introduce, at each space-time point x^μ, a certain momentum-dependent escaping probability

$$\mathcal{P}(x,k)=\exp\left[-\int_\tau^\infty \rho(x')\,\sigma v\,\mathrm{d}\tau'\right].$$

This is the Continuous Emission Model (CE).*

* [F. Grassi, Y. Hama and T. Kodama, *Phys. Lett.* **B355** (1996) 9]

Decoupling Procedure

- Often one assumes decoupling on a sharply defined hypersurface. We call this Sudden Freeze Out (FO).
- However, our systems are small, so partcles may escape from a layer with thickness comparable with the systems sizes.
- We introduce, at each space-time point x^μ, a certain momentum-dependent escaping probability

$$\mathcal{P}(x,k) = \exp\left[-\int_\tau^\infty \rho(x')\,\sigma v\,\mathrm{d}\tau'\right].$$

This is the Continuous Emission Model (CE).*

* [F. Grassi, Y. Hama and T. Kodama, *Phys. Lett.* **B355** (1996) 9]

Decoupling Procedure

In order to make the computation practicable,

- we take $\mathcal{P}$ on the average, *i.e.*, $\mathcal{P}(x,k) \to \langle \mathcal{P}(x,k) \rangle \equiv \mathcal{P}(x)$ and
- we approximate linearly the density $\rho(x') = \alpha\, s(x')$.

Thus,

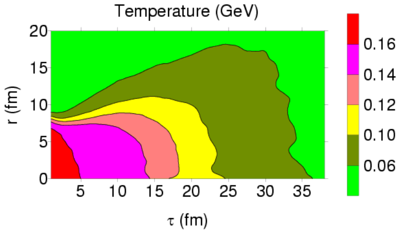
$$\mathcal{P}(x,k) \to \mathcal{P}(x) = \exp\left(-\kappa\,\frac{s^2}{|\mathrm{d}s/\mathrm{d}\tau|}\right),$$

where $\kappa = 0.5\,\alpha\,\langle \sigma v \rangle$ is estimated to be 0.3 , corresponding to $\langle \sigma v \rangle \approx 2\ \mathrm{fm}^2$.

Sudden Freezeout vs. Continuous Emission

Central Au+Au collisions at 130 *A* GeV

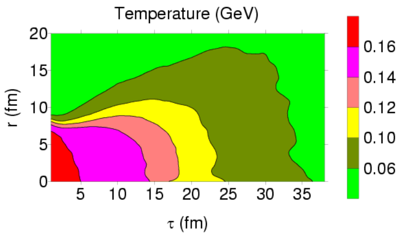
FO



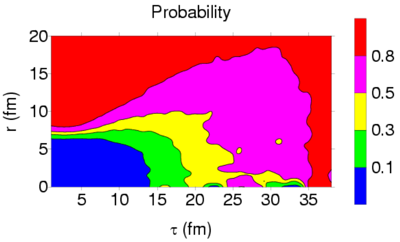
Sudden Freezeout vs. Continuous Emission

Central Au+Au collisions at 130 A GeV

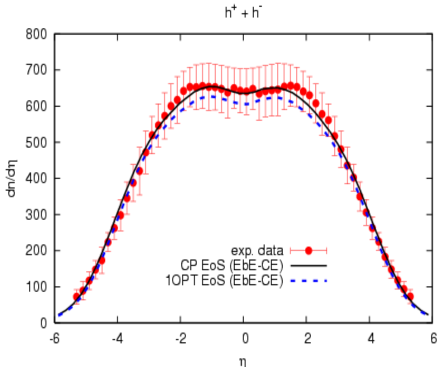
FO



CE



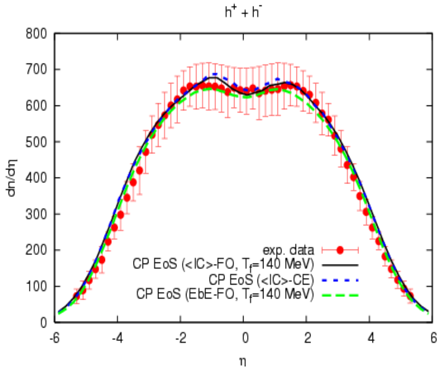
η distribution



Data: PHOBOS Collaboration, B.B.Back *et al.*, *Phys. Rev.* **C65** (2002) 054902. (Central)

- The equations of state with critical end point (CP EoS) increase the multiplicity in the mid-rapidity region, as compared with those with first-order phase transition (1OPT EoS).

η distribution

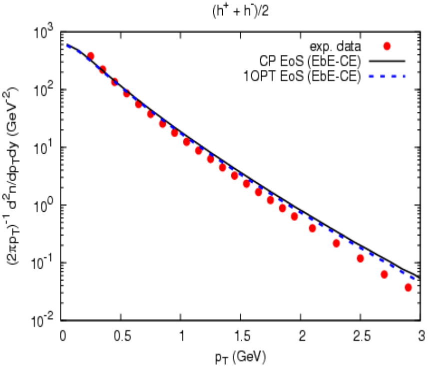


Data: PHOBOS Collaboration, B.B.Back *et al.*, *Phys.Rev.***C65**(2002) 054902. (Central)

- The use of the averaged initial conditions (<IC>) gives larger multiplicity as compared to event-by-event (EbE) ones.*
- As far as η distribution is concerned, the difference betwen the continuous emission (CE) and the sudden freezeout (FO) is small.

* [Y. Hama, T. Kodama and O. Socolowski, *Braz. J. Phys.* **35** (2005) 24]

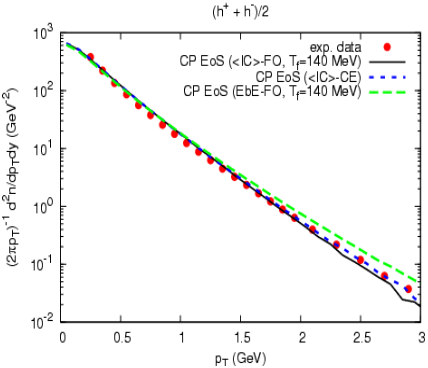
p_T distribution



Data: PHOBOS Collaboration, B.B.Back *et al.*, *Phys. Rev. Lett.* **93** (2004) 052303.
(Central collisions)

- The p_T distribution given by CP EoS is slightly flatter, as compared with the one predicted by 1OPT oS. However, the differece is small.

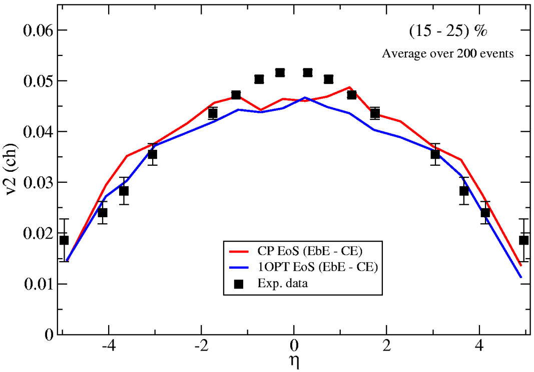
p_T distribution



Data: PHOBOS Collaboration, B.B.Back *et al.*, *Phys. Rev. Lett.* **93** (2004) 052303. (Central collisions)

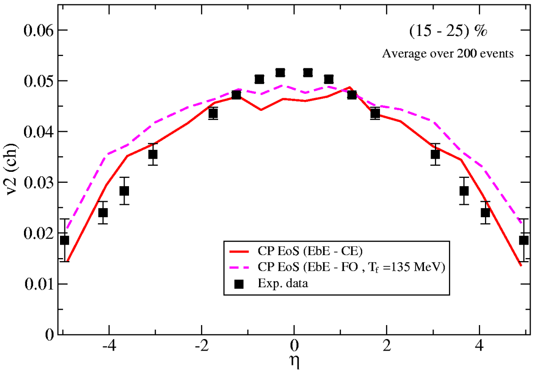
- The averaged initial conditions (<IC>) give steeper distribution as compared to the event-by-event (EbE) ones. In the latter, the expansion is more violent.
- The difference betwen the continuous emission (CE) and the sudden freezeout (FO) is small. However, the interpretations are different.

$v_2(\eta)$



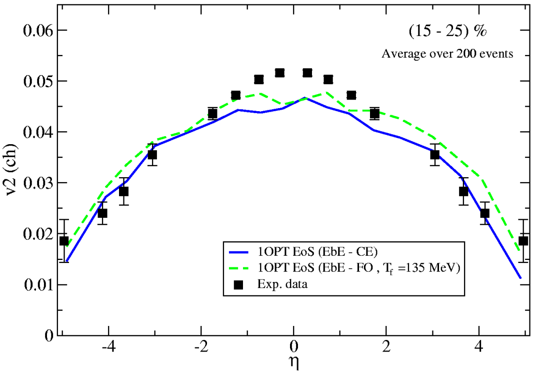
Data: PHOBOS Collaboration, B.B. Back *et al.*, nucl-ex/0407012

$v_2(\eta)$



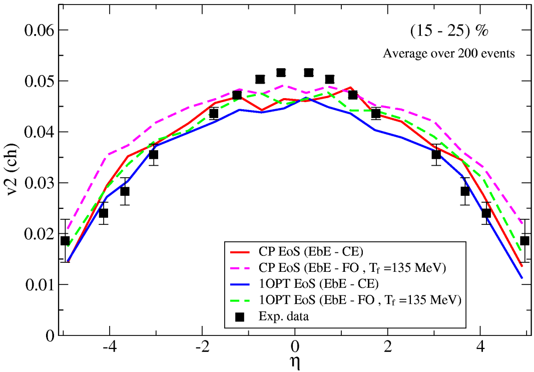
Data: PHOBOS Collaboration, B.B. Back *et al.*, nucl-ex/0407012

$v_2(\eta)$



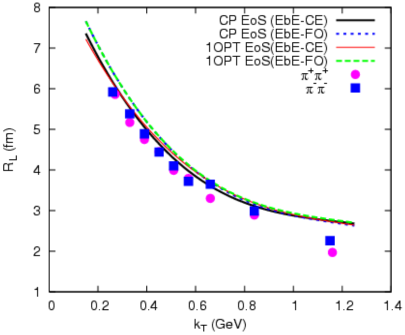
Data: PHOBOS Collaboration, B.B. Back *et al.*, nucl-ex/0407012

$v_2(\eta)$



Data: PHOBOS Collaboration, B.B. Back *et al.*, nucl-ex/0407012

HBT Radii
R_L (Event by Event)

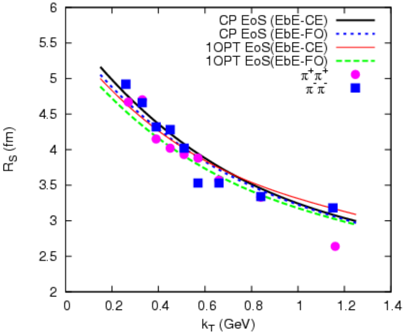


Data: PHENIX Collaboration, S.S. Adler *et al.*, *Phys. Rev. Lett.* **93** (2004) 152302.

- Practically, there is no difference between the results of CP EoS and 1OPT EoS.
- The data are well reproduced.
- The difference between Sudden Freezeout (FO) and the Continuous Emission (CE) is small.

HBT Radii

R_s (Event by Event)

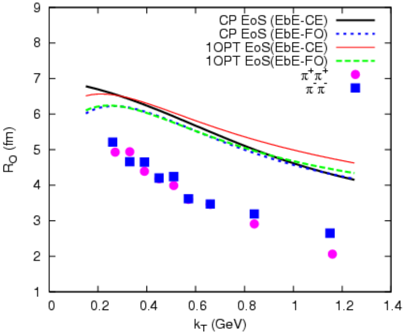


Data: PHENIX Collaboration, S.S. Adler *et al.*, *Phys. Rev. Lett.* **93** (2004) 152302.

- k_T dependence predicted by CP EoS is steeper than the one given by 1OPT EoS.
- The data are more or less well reproduced, but the agreement is the best for CP EoS and with CE.

HBT Radii

R_o (Event by Event)

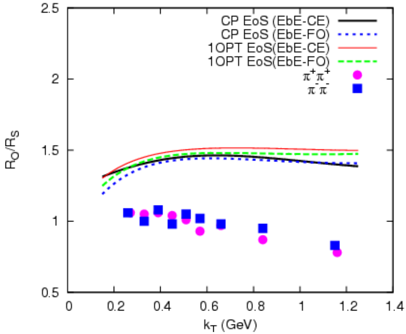


Data: PHENIX Collaboration, S.S. Adler *et al.*, *Phys. Rev. Lett.* **93** (2004) 152302.

- k_T dependence predicted by CP EoS is steeper than the one given by 1OPT EoS.
- Quantitatively, the data are poorely reproduced.
- But, k_T dependence is best reproduced by CP EoS and with CE.

HBT Radii

R_o/R_s (Event by Event)



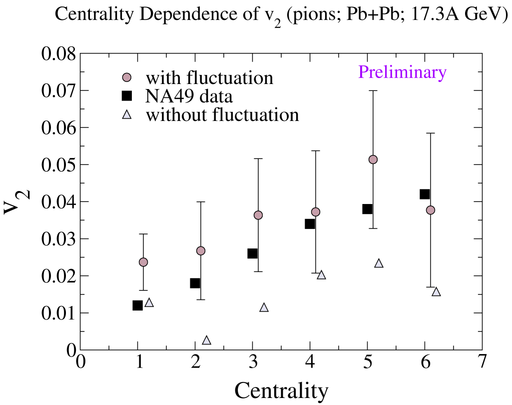
Data: PHENIX Collaboration, S.S. Adler *et al.*,

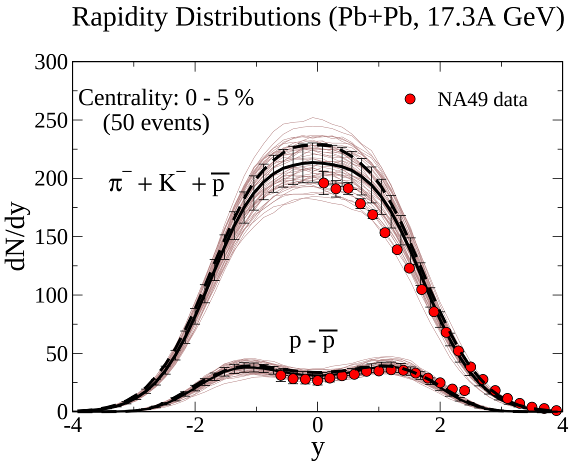
Phys. Rev. Lett. **93** (2004) 152302.

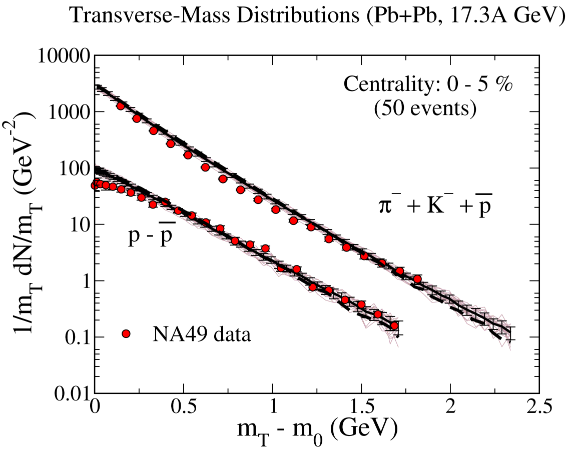
Summary

- The multiplicity becomes larger for CP EoS in the mid-rapidity.
- The p_T distribution becomes flatter. However, the differece is small.
- Larger v_2 . Continuous Emission makes the η distribution narrower.
- HBT radii slightly closer to data.

- Outlook
  - The effect of the emission on the interacting component has not been taken into account. Probably it makes R_o smaller.


Centrality Dependence of v2 (pions; Pb+Pb; 17.3A GeV)
with fluctuation
NA49 data
without fluctuation
Preliminary
v2
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
1
2
3
4
5
6
7
Centrality


Rapidity Distributions (Pb+Pb, 17.3A GeV)
Centrality: 0 - 5 %
(50 events)
NA49 data
π⁻ + K⁻ + p̄
p - p̄
dN/dy
y
300
250
200
150
100
50
0
-4
-2
0
2
4


Transverse-Mass Distributions (Pb+Pb, 17.3A GeV)
Centrality: 0 - 5 %
(50 events)
$\pi^- + K^- + \bar{p}$
$p - \bar{p}$
NA49 data
$1/m_T\, dN/m_T\, (GeV^{-2})$
$m_T - m_0$ (GeV)
10000
1000
100
10
1
0.1
0.01
0
0.5
1
1.5
2
2.5