Results on Lévy stable parametrizations of Bose-Einstein Correlations using L3 at LEP

Tamás Novák

Radboud Universiteit Nijmegen

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WPCF

Introduction

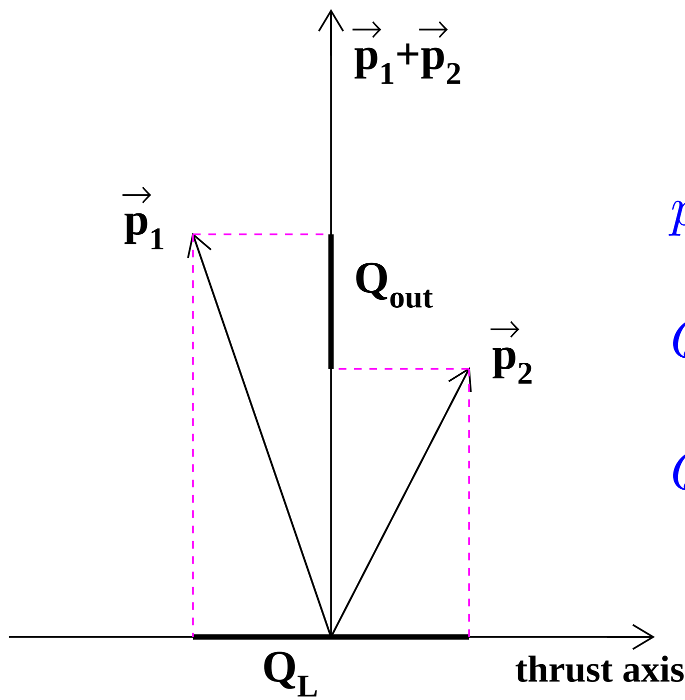
Data Sample

- Hadronic Z decays from 1994 using L3 detector ($\sqrt{s} \approx$ 91.2 GeV $\approx M_Z$)
- 'Standard' event and track selection ≈ 1 million events
- Concentrate on 2-jet events using Durham alogithm $y_{cut} = 0.006 \approx$ 13 million pairs
- Correct distribution bin-by-bin by MC: $\frac{N_{gen}}{N_{det}}$

Introduction

Use Longitudinal Center of Mass System (LCMS):

Boost each π-pair along thrust axis



$p_{L1} = p_{L2}$

Q_{out} is defined by $\vec{p}_1 + \vec{p}_2$

$Q_{\text{side}} \perp (Q_{\text{L}}, Q_{\text{out}})$

Advantages of LCMS:

$$Q^2 = Q_{\text{L}}^2 + Q_{\text{side}}^2 + Q_{\text{out}}^2 - (\Delta E)^2 = Q_{\text{L}}^2 + Q_{\text{side}}^2 + Q_{\text{out}}^2(1 - \beta^2)$$

where $\beta \equiv \frac{p_{\text{out}\,1} + p_{\text{out}\,2}}{E_1 + E_2}$. Thus the energy difference couples only to Q_{out} .

Introduction

- Two-particle Bose-Einstein correlation function:

$$R_2(p_1,p_2) = \frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)}$$

- To study only BEC, replace $\rho_1(p_1)\rho_1(p_2)$ by $\rho_0(p_1,p_2)$, the 2-particle density, if no BEC (reference sample → mixed events).

$$R_2(p_1,p_2) = \frac{\rho_2(p_1,p_2)}{\rho_0(p_1,p_2)}$$

- Since BEC only at small $Q^2 = -q^\mu q_\mu = -(p_1-p_2)^2$

$$R_2(Q) = \frac{\rho_2(Q)}{\rho_0(Q)}.$$

- Assuming incoherent particle production and spatial source density $f(x)$

$$\boxed{R_2(Q) = 1 + \lambda|\tilde{f}(Q)|^2},$$

where $\tilde{f}(Q) = \int \mathrm{d}x \exp(iQ\cdot x) f(x)$ is the Fourier transform of $f(x)$.

Bose-Einstein

Earlier studies at LEP: Elongation in the pion source (not spherical source)

$$\frac{R_{\text{out}}}{R_{\text{L}}} = 0.65 \pm 0.03 \pm^{0.06}_{0.09} \qquad \frac{R_{\text{side}}}{R_{\text{L}}} = 0.81 \pm 0.02 \pm^{0.03}_{0.19}$$

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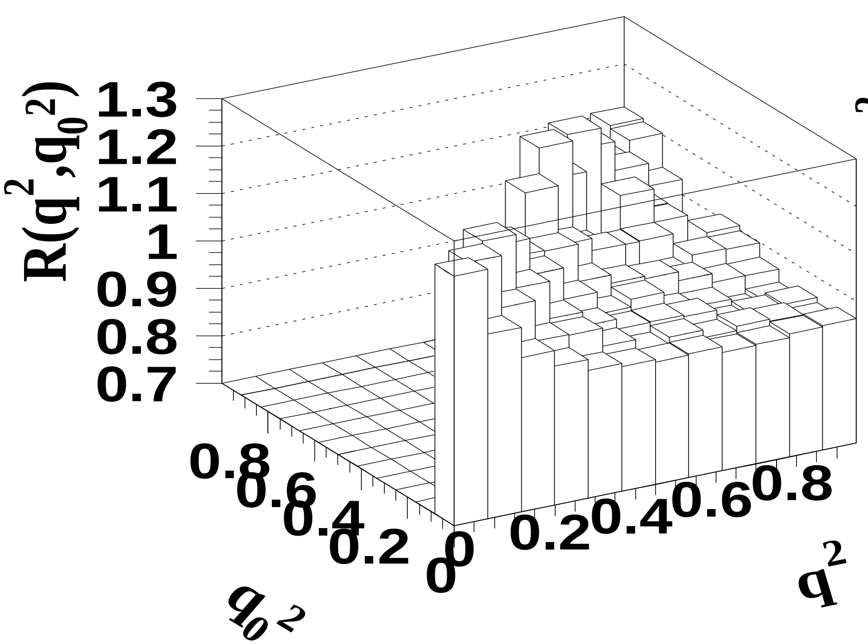
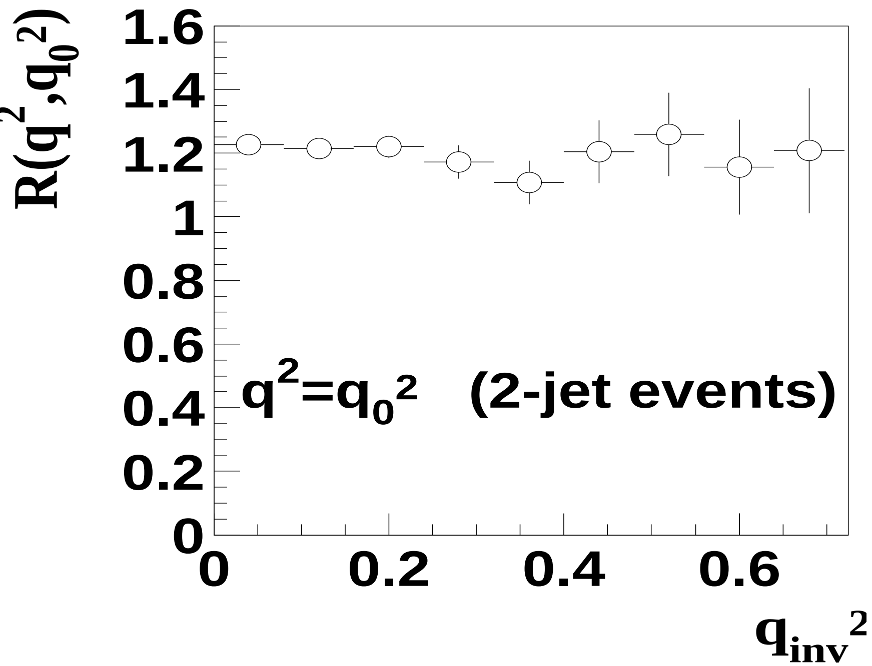
However, Q_{inv} (indicates spherical source) may also give a good fit



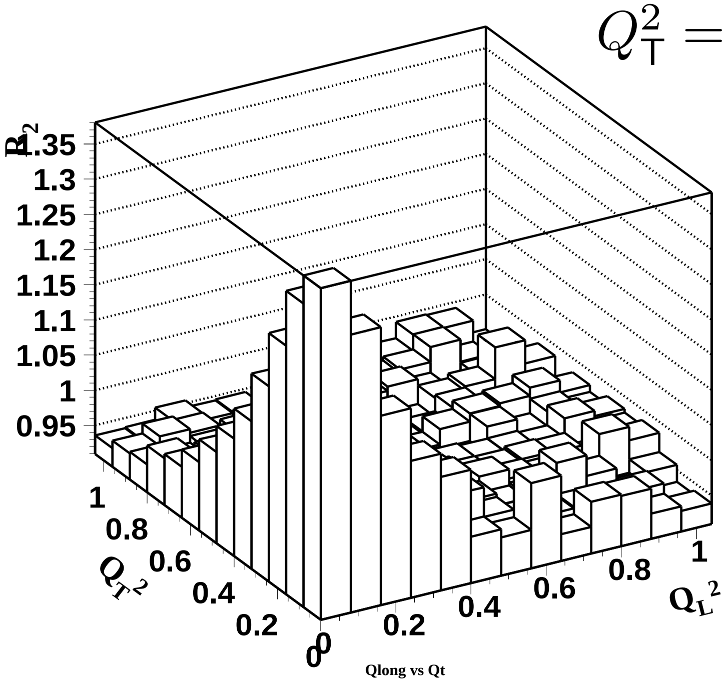


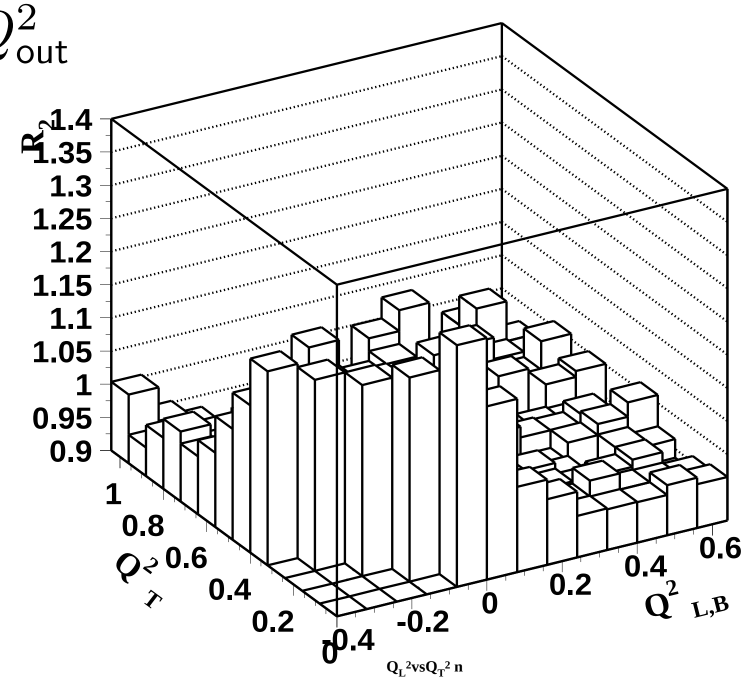
Bertsch-Pratt *vs.* Bowler

Bertsch-Pratt Parametrization
($Q_{\mathsf{L}}^2 = |p_{\mathsf{L1}} - p_{\mathsf{L2}}|^2$)

Bowler Parametrization
($Q_{\mathsf{L,B}}^2 = Q_{\mathsf{L}}^2 - (\Delta \mathsf{E})^2$)

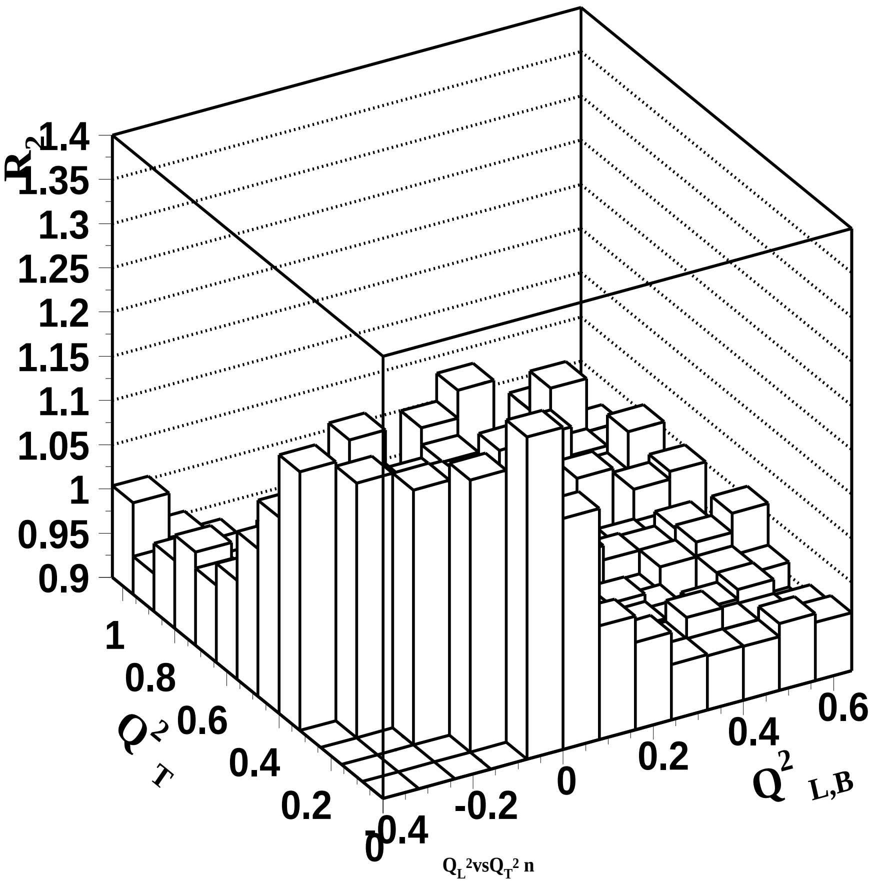
$$Q_{\mathsf{T}}^2 = Q_{\mathsf{side}}^2 + Q_{\mathsf{out}}^2$$

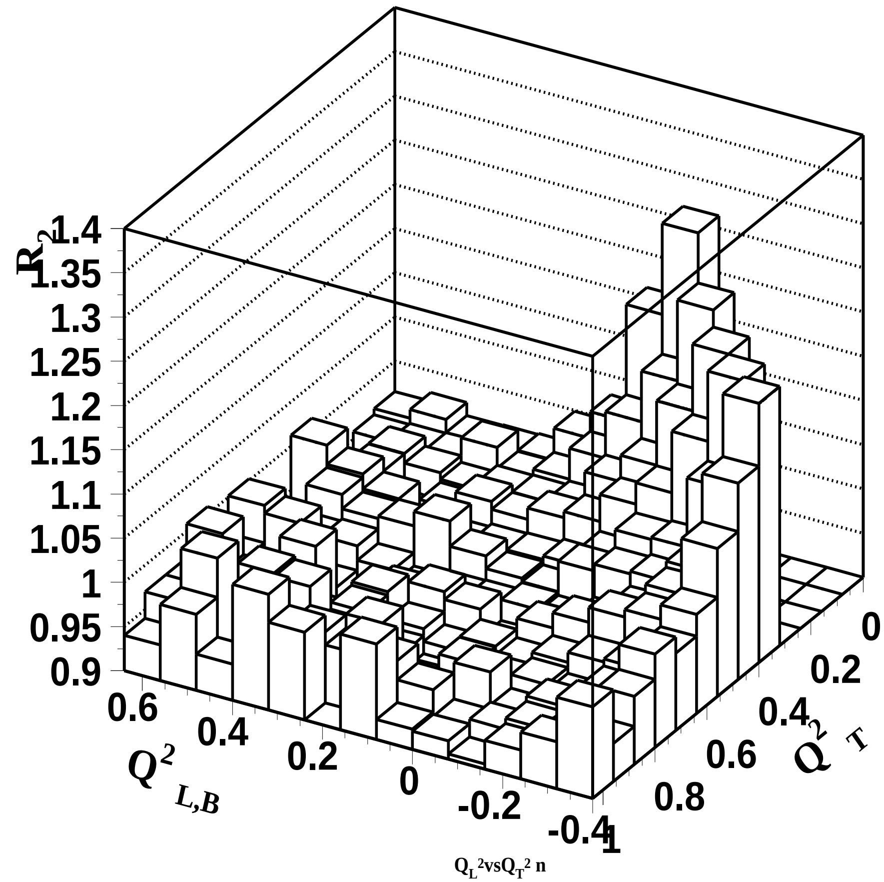




$$Q_{\mathsf{inv}}^2 = Q_{\mathsf{L}}^2 + Q_{\mathsf{side}}^2 + Q_{\mathsf{out}}^2(1 - \beta^2)$$

$$Q_{\mathsf{inv}}^2 = Q_{\mathsf{L,B}}^2 + Q_{\mathsf{side}}^2 + Q_{\mathsf{out}}^2$$


R2
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
1
0.8
0.6
0.4
0.2
0
Q2 T
-0.4
-0.2
0
0.2
0.4
0.6
Q2 L,B
QL2vsQT2 n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Q2 L,B
0
0.2
0.4
0.6
0.8
1
Q2 T
QL2vsQT2 n

Is Q_{inv} the right variable?

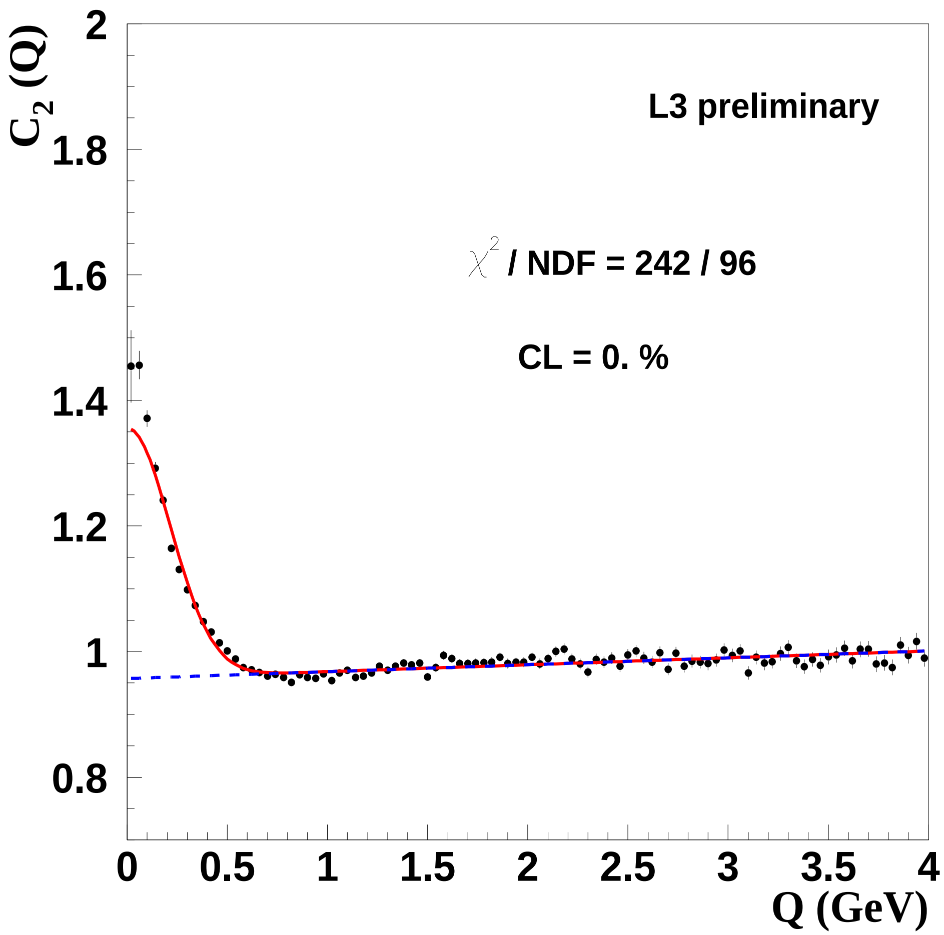
$$R_2(Q_{\mathsf{L,B}}, Q_{\mathsf{T}}) = \gamma\left[1 + \lambda \exp\left(-\left(R_{\mathsf{L,B}}^2 Q_{\mathsf{L,B}}^2 + R_{\mathsf{T}}^2 Q_{\mathsf{T}}^2\right)\right)\right]\left(1 + \delta Q_{\mathsf{L,B}}^2 + \epsilon Q_{\mathsf{T}}^2\right)$$

γ	λ	$R_{\mathsf{L,B}}$(fm)	R_{T}(fm)	χ^2/NDF	CL(%)
0.94±0.02	0.39±0.02	0.51±0.02	0.55±0.02	213/143	0.

Approximately Q_{inv} dependence, although the confidence level is low.

The shape is not Gaussian $\Longrightarrow$ Low CL.

Parametrization I

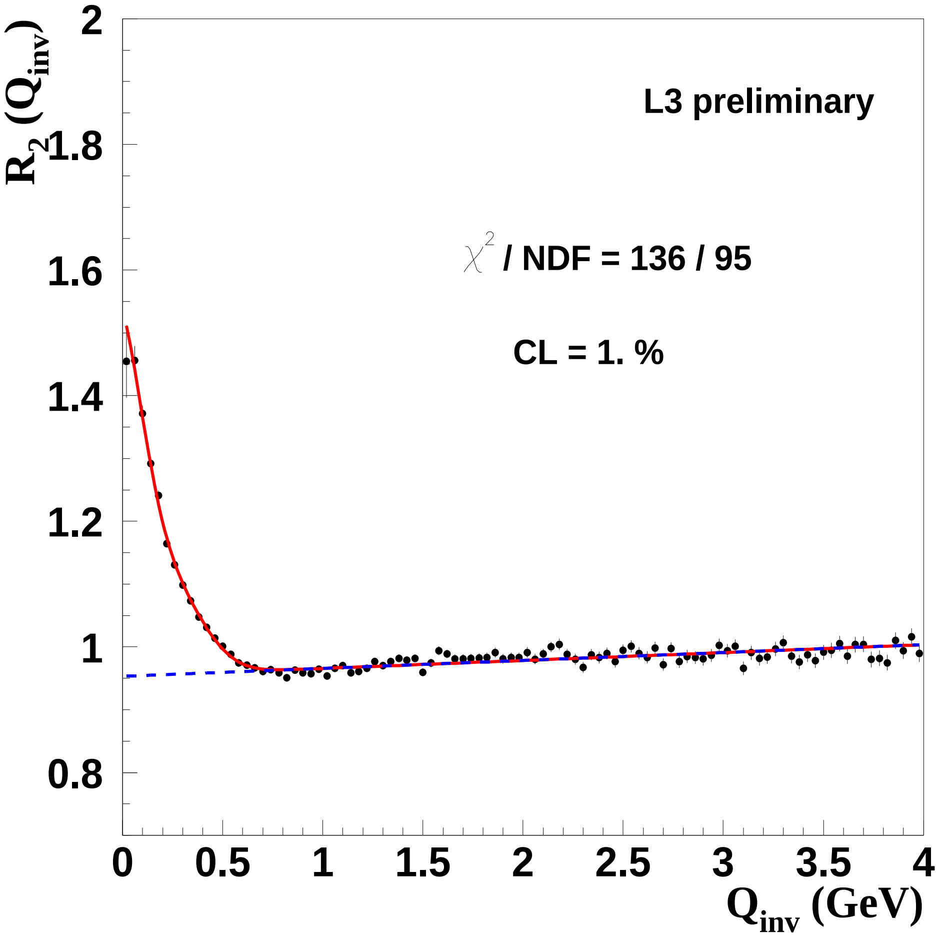


Assume Gaussian shape of the source:
(Central Limit Theorem $\Longrightarrow$ Gaussian)

$$R_2(Q) = \gamma\left[1 + \lambda \exp\left(-(RQ)^2\right)\right](1 + \delta Q)$$

Does not work! Let's try to expand it!

Parametrization II



Expand in terms of Hermite polynomials:

$$R_2(Q) = \gamma \left[1 + \lambda \exp\left(-(RQ)^2\right)\left(1 + \frac{\kappa}{3!} H_3(R_E Q)\right)\right](1 + \delta Q),$$

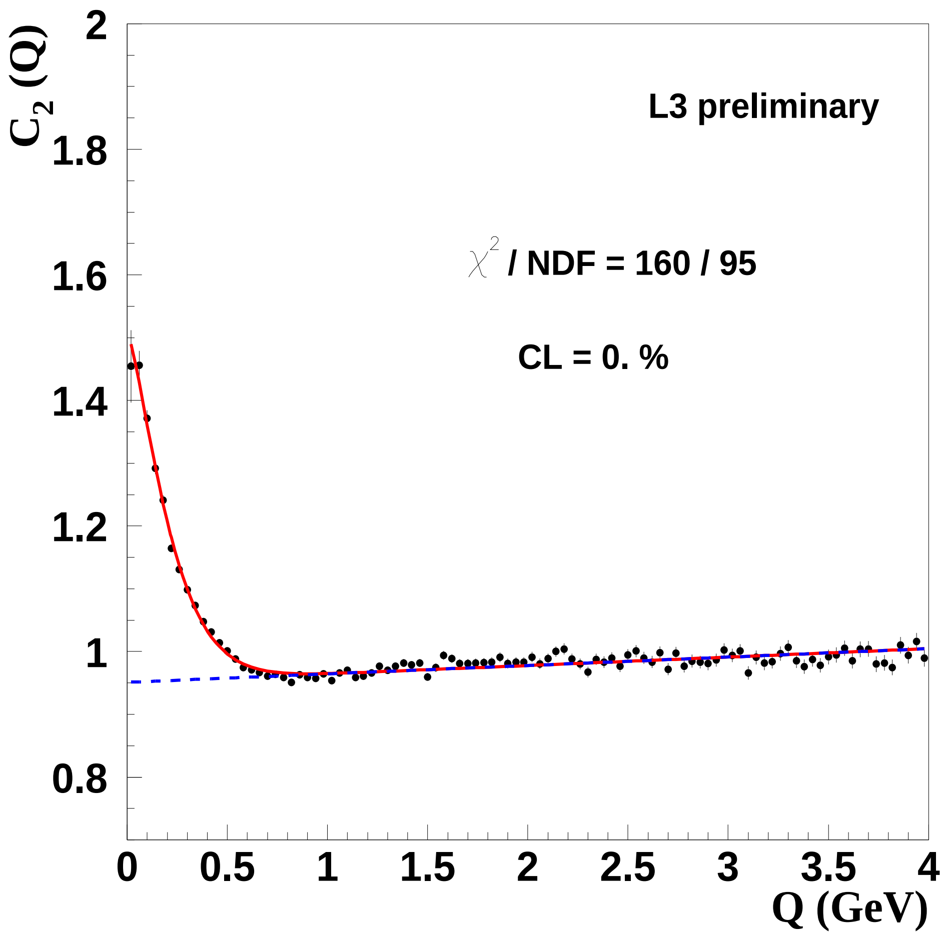
where

$$H_3(R_E Q) = (\sqrt{2} R_E Q)^3 - 3\sqrt{2} R_E Q$$

is the third Hermite polynomial.

Model independent ↔ Physical interpret.

Parametrization III



Generalization of CLT $\Longrightarrow$ Lévy

Symmetric Lévy parametrization

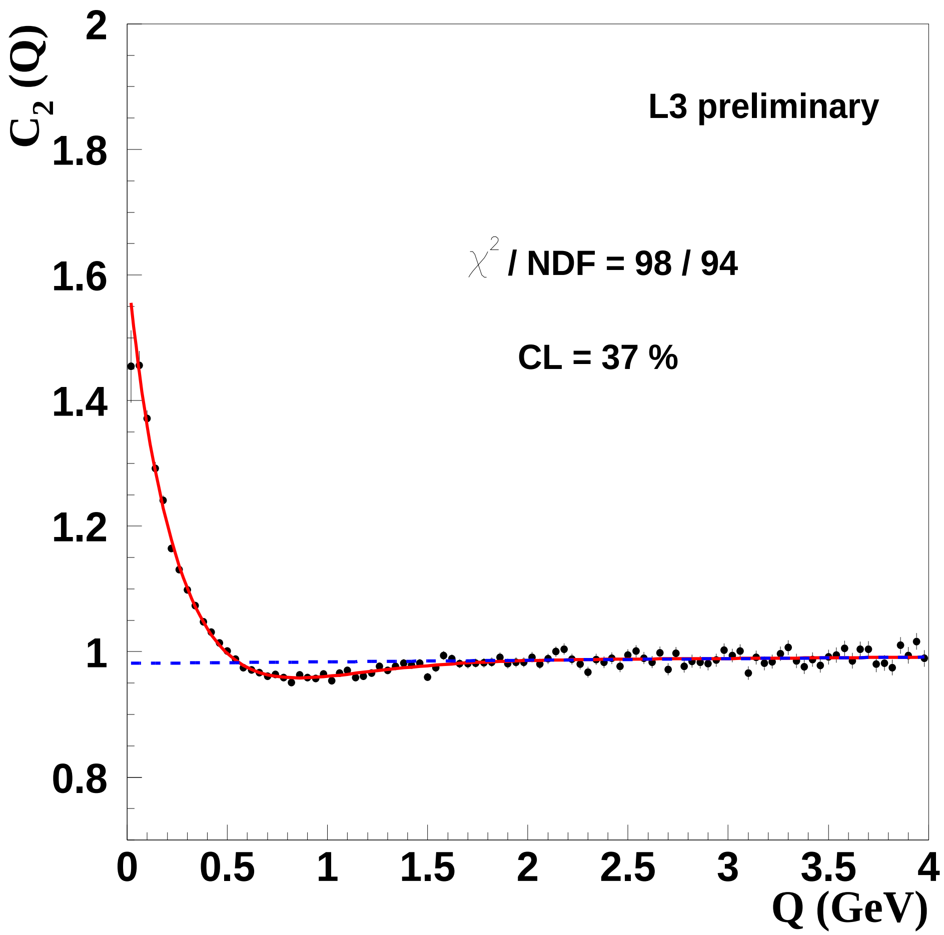
$$R_2(Q) = \gamma\,[1 + \lambda \exp\left(-(RQ)^{\alpha}\right)]\,(1 + \delta Q)$$

$\alpha = 1.3$ far from Gaussian.

The confidence level is still unacceptably low.

Go to Asymmetric Lévy distribution!

Parametrization IV



Asymmetric Lévy parametrization

Assumption: 1-sided Lévy stable distribution for time

($\tau_0 = 0$, no particle production before collision)

$$R_2(Q) = \gamma\left[1 + \lambda \cos\left[(R_a Q)^\alpha\right] \exp\left(-(RQ)^\alpha\right)\right](1 + \delta Q)$$

$$\alpha = 0.82 \pm 0.03$$

Good Confidence Level!

The τ - model I

Assumpition i): strong correlation between space-time and 4-momentum space:

$$\overline{x}^{\mu} = dk^{\mu},$$

where d is a constant of proportionality. For 2-jet events $d = \frac{\tau_l}{m_t}$.

Assumption ii): this correlation is narrower than the proper-time distribution.

Thus the emission function of the τ-model is

$$S(x,k) = \int_0^{\infty} \mathrm{d}\tau H(\tau) \delta_{\Delta}(x - dk) N_1(k),$$

hence $\int \mathrm{d}x^4 S(x,k) = N_1(k)$ is indeed the single particle spectra.

The τ - model II

Yano-Koonin formula:

$$\rho_2(k_1, k_2) = \int \mathrm{d}^4x_1\mathrm{d}^4x_2 S(x_1, k_1)S(x_2, k_2)\left(1 + \cos\left[(k_1 - k_2)(x_1 - x_2)\right]\right).$$

For any choice of d, one gets $(k_1 - k_2)(\bar{x}_1 - \bar{x}_2) = -0.5(d_1 + d_2)Q^2_{\mathsf{inv}}$.

Conditions i) and ii) $\Longrightarrow$

$$R_2(k_1, k_2) \approx 1 + \lambda \mathrm{Re}\left[\widetilde{H}\left(\frac{Q^2_{\mathsf{inv}}d_1}{2}\right)\widetilde{H}\left(\frac{Q^2_{\mathsf{inv}}d_2}{2}\right)\right] \approx 1 + \lambda \mathrm{Re}\widetilde{H}^2\left(\frac{Q^2_{\mathsf{inv}}}{2\bar{m}_{\mathsf{T}}}\right),$$

where $\widetilde{H}(\omega) = \int \mathrm{d}\tau H(\tau)\exp(i\omega\tau)$ is a Fourier-transform of $H(\tau)$.

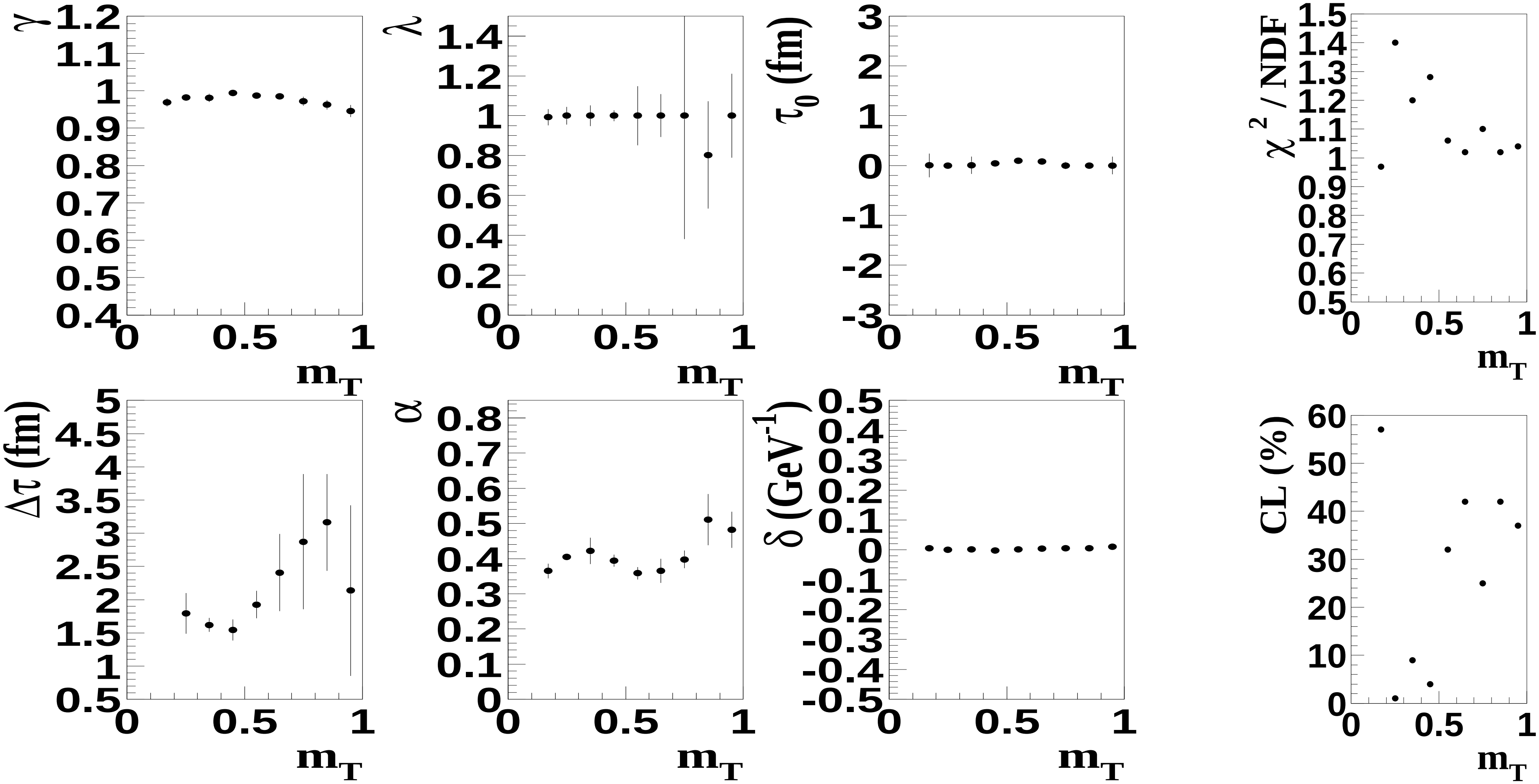
The τ - model III

Assumpition iii): source density function factorizes. $S(r,z,t) = I(r)G(\eta)H(\tau)$, where $\tau = \sqrt{t^2 - z^2}$ and $\eta = \frac{1}{2}\log\frac{t+z}{t-z}$. This assumption is not inconsistent with the L3 data.

Assumption iv): $H(\tau)$ is an asymmetric Lévy distribution $(= 0$ for $\tau < \tau_0)$.

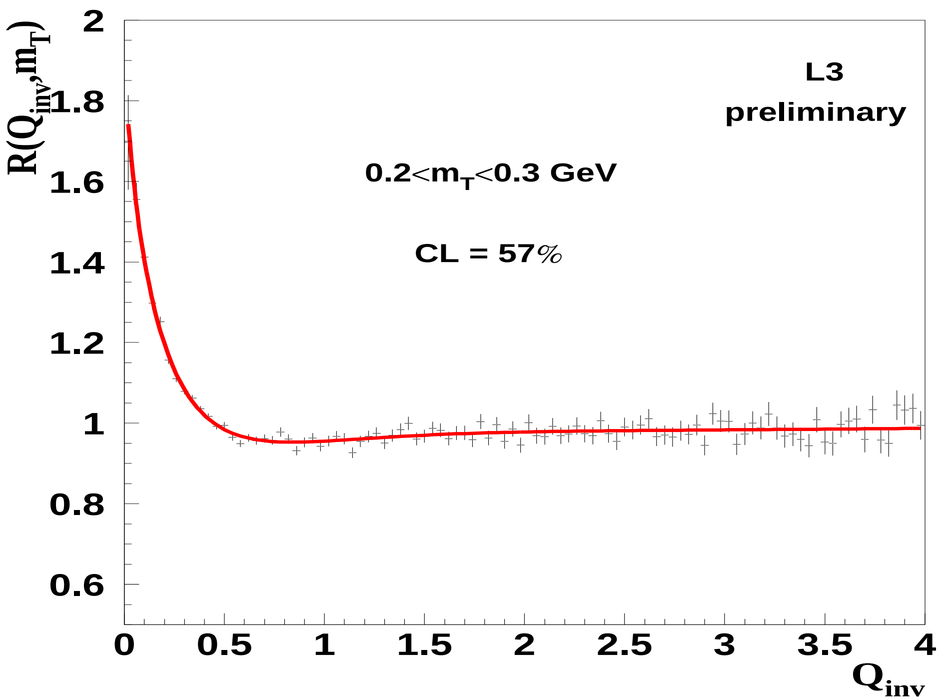
Then, R_2 depends on Q^2 and $\bar{m}_{\mathrm{T}}$

$$_2(Q^2, \bar{m}_{\mathrm{T}}) = \gamma \left[1 + \lambda \cos\left(\frac{\tau_0 Q^2}{\bar{m}_{\mathrm{T}}} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{\bar{m}_{\mathrm{T}}}\right)^{\alpha}\right) \exp\left(-\left(\frac{\Delta\tau Q^2}{\bar{m}_{\mathrm{T}}}\right)^{\alpha}\right)\right](1 + \delta Q)$$



$$Q^2, \bar{m}_T) = \gamma \left[1 + \lambda \cos\left(\frac{\tau_0 Q^2}{\bar{m}_T} + \tan\left(\frac{\alpha\pi}{2}\right) \left(\frac{\Delta\tau Q^2}{\bar{m}_T}\right)^{\alpha} \right) \exp\left(- \left(\frac{\Delta\tau Q^2}{\bar{m}_T} \right)^{\alpha} \right) \right] (1 + \delta Q)$$

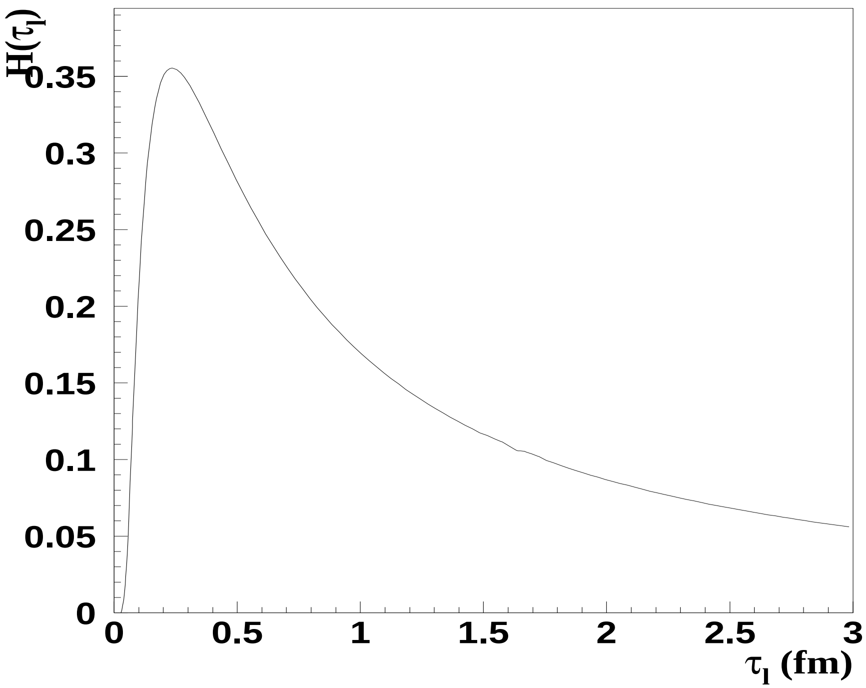
$$(Q^2, \bar{m}_\mathrm{T}) = \gamma\left[1 + \lambda\cos\left(\frac{\tau_0 Q^2}{\bar{m}_\mathrm{T}} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{\bar{m}_\mathrm{T}}\right)^{\alpha}\right)\exp\left(-\left(\frac{\Delta\tau Q^2}{\bar{m}_\mathrm{T}}\right)^{\alpha}\right)\right](1 + \delta Q)$$

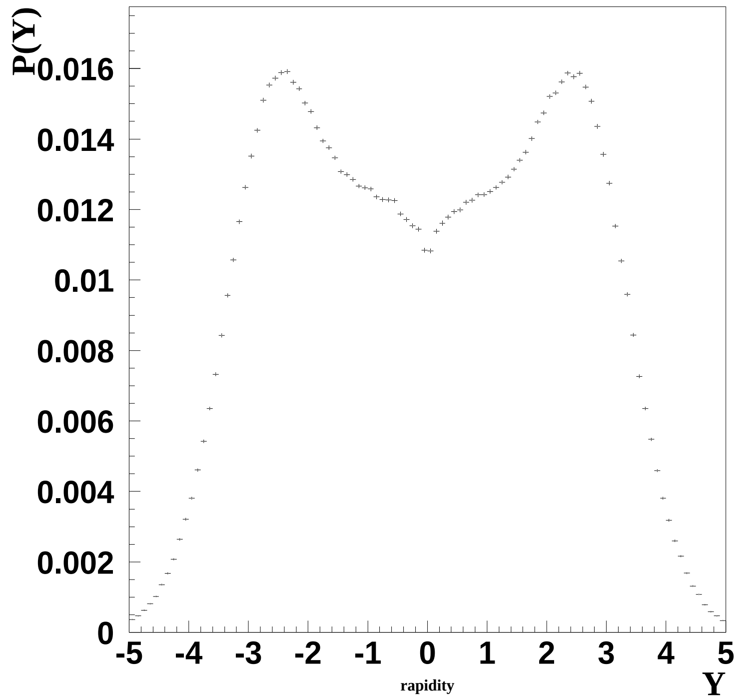


parameter	value
γ	0.97 $\pm$ 0.01
λ	1.03 $\pm$ 0.09
$\tau_0 (fm)$	0.002 $\pm$ 0.239
$\Delta\tau (fm)$	1.8 $\pm$ 0.3
α	0.36 $\pm$ 0.02
δ	0.004$\pm$0.003
χ^2/NDF	91/94

Results: $\Delta\tau \approx 2$ fm $\quad \tau_0 = 0$ fm $\quad \alpha \approx 0.4$

$$S(r,z,\tau) = I(r)G(\eta)H(\tau)$$





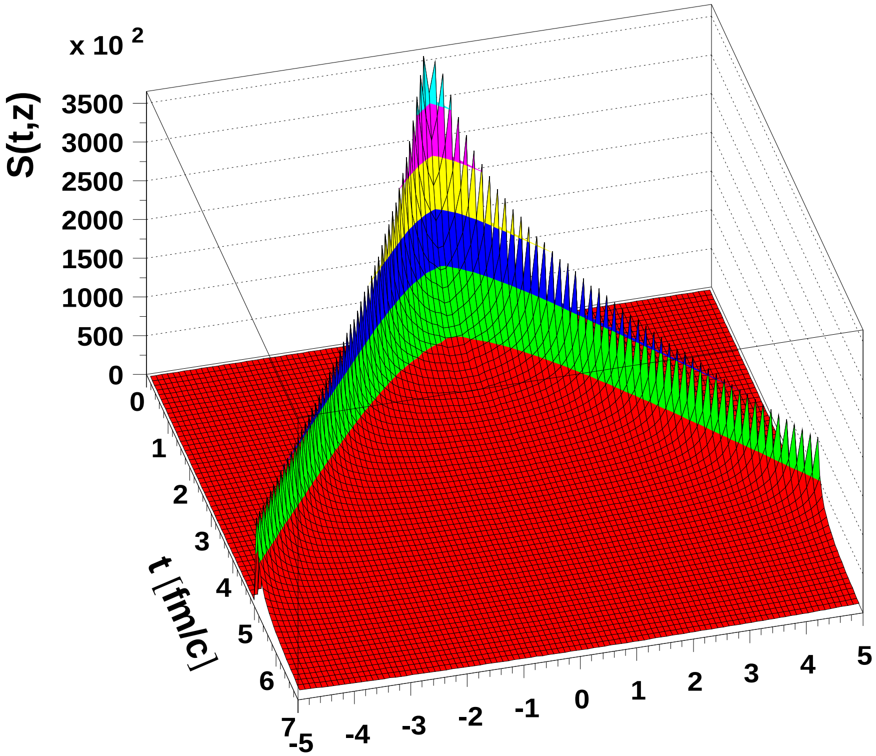
Proper-time distribution with $\alpha = 0.4$,

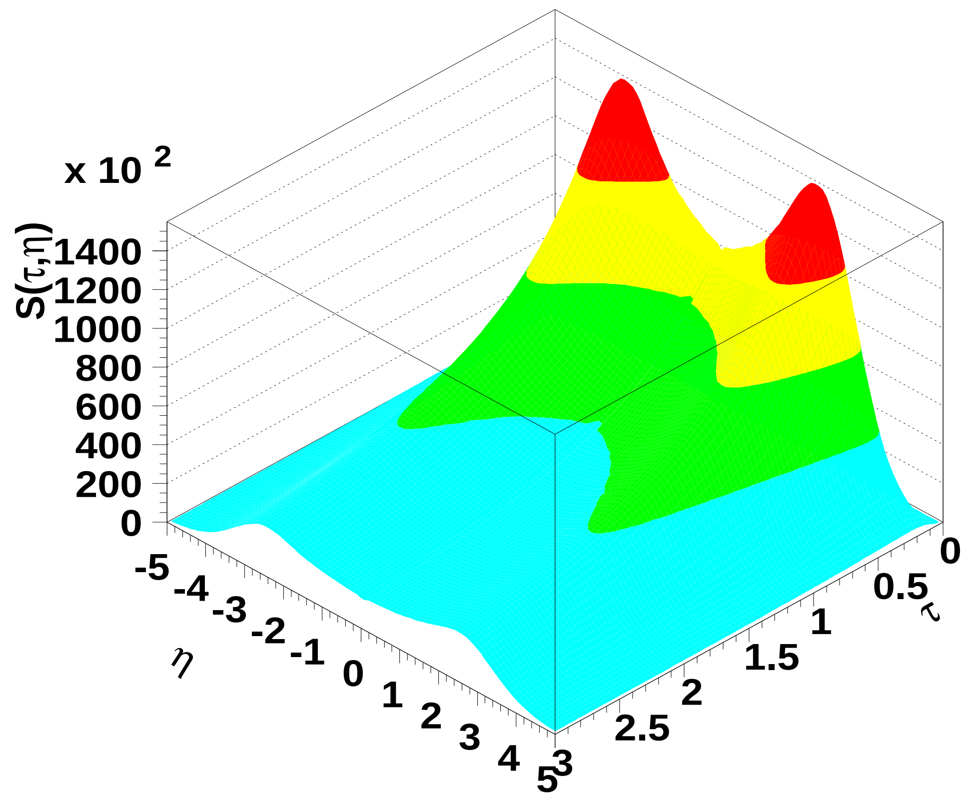
$\Delta\tau = 2$ and $\tau_0 = 0$.

$$\eta = \frac{1}{2}\log\left(\frac{t+z}{t-z}\right) = y,$$

measured rapidity distribution.

$$S(r,z,\tau) = I(r)G(\eta)H(\tau)$$





$$S(t, r_z) = G(\eta)H(\tau)$$

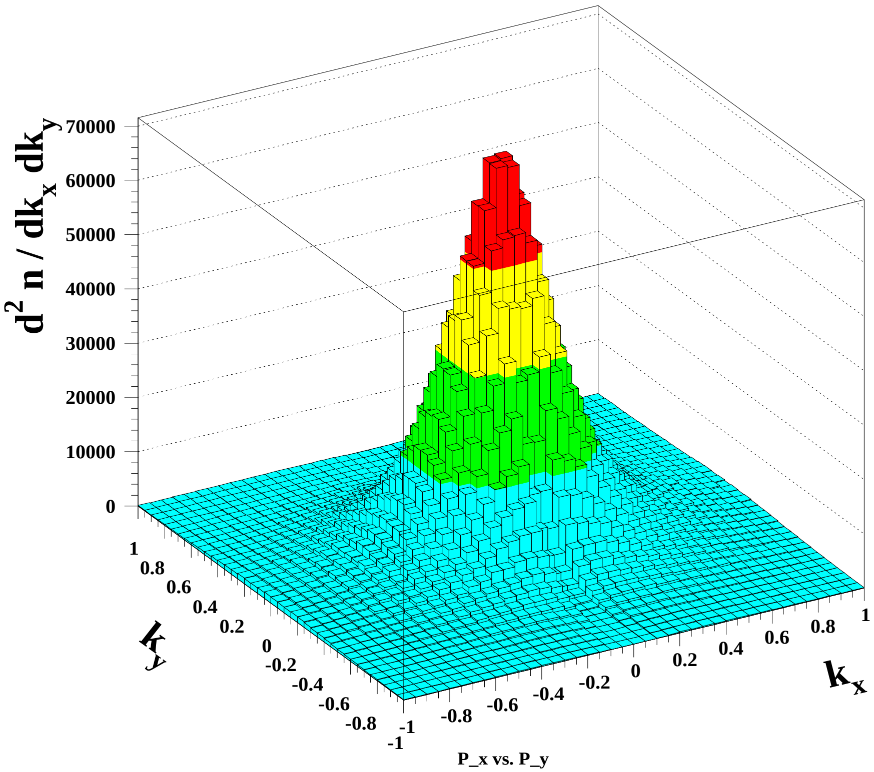
$$S(\tau,\eta) = G(\eta)H(\tau)$$

Movie equation

Using the τ-model, the particle production in coordinate space, as a function of the proper-time, can be reconstracted:

$$\frac{\mathrm{d}^4 n}{\mathrm{d}\tau \mathrm{d}^3 r} = \frac{m_t^3}{\tau^3} H(\tau) N_1 \left(k = \frac{\bar{m}_t r}{\tau_l} \right)$$

- $H(\tau)$ from BE correlations.
- $N_1(k)$ is the meausre single-particle spectrum.



Summary

- Good description of the BEC function is achived using Lévy stable distribution.
- Using τ-model source function can be reconstracted.
- Particle production starts immediadetly after the collision.
- It happens close to the light-cone and expanding.
- The source has a long tail.

Bose-Einstein VI

Transverse mass and rapidity are independent!