# Phi-phi back-to-back correlations in finite expanding systems

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# Outline

- Introduction
- Brief review of the formalism
- Hypotheses used for finite expanding system
- Non-relativistic treatment
- Illustration of results: $\phi\phi$ back-to-back pairs
- Summary and future plans

# Introduction

- Late 90's → Back-to-Back Correlations (BBC) among boson-antiboson pairs were shown to exist if the masses of the particles were modified in a hot and dense medium
  [Asakawa, Csörgo" & Gyulassy, P.R.L. 83 (1999) 4013].
- 2001 → It was shown that similar BBC existed among fermion-antifermion pairs with medium modified masses
  [Panda, Csörgo", Hama, Krein & SSP, P. L. B512 (2001) 49].

- **Some properties:**
  - Similar formalism for both bosonic (bBBC) and fermionic (fBBC) Back-to-Back correlations
  - Similar (and unlimited) intensity of fBBC and bBBC
  - Expected to appear for $p_T \leq 1 - 2\ \mathrm{GeV}/c$

# Formalism

- **Infinite medium**

$$H = H_0 - \frac{1}{2}\int d\vec{x}\, d\vec{y}\, \phi(\vec{x})\, \delta M^2(\vec{x}-\vec{y})\phi(\vec{y})$$ → In-medium Hamiltonian

$$H_0 = \frac{1}{2}\int d\vec{x}\,(\dot{\phi}^2 + |\nabla\phi|^2 + m^2\phi^2)$$ → Asymptotic (free) Hamiltonian, in the rest frame of matter

- Scalar field $\phi(x) \leftrightarrow$ quasi-particles propagating with momentum-dependent medium-modified effective mass, $m_*$, related to the vacuum mass, $m$, by

♦ $$m_*^2\left(\left|\vec{k}\right|\right) = m^2 - \delta M^2\left(\left|\vec{k}\right|\right)$$

- Consequently: $\Omega_k^2 = m_*^2 + \vec{k}^2 = \omega_k^2 - \delta M^2\left(\left|\vec{k}\right|\right)$

$\Omega_k$ → frequency of the in-medium mode with momentum $\vec{k}$

# Some comments (bosonic BBC):

- $b_k\,(b^\dagger{}_k)$ → in-medium annihilation (creation) operator
- $a_k\,(a^\dagger{}_k)$ → annihilation (creation) operator of the asymptotic quanta with 4-momemtum $k^\mu = (\omega_k, \vec{k})$ ; $\omega^2 = \sqrt{m^2 + \vec{k}^2} > 0$

They are related by the Bogoliubov transformation:

$$\begin{cases} a^\dagger{}_k = c_k\, b^\dagger{}_k + s_{-k}\, b_{-k} \\ a_k = c_k^*\, b_k + s^*_{-k}\, b^\dagger_{-k} \end{cases} \quad ; \quad \boxed{c_k = \cosh[f_k]} \; ; \; \boxed{s_k = \sinh[f_k]}$$

- $\boxed{f_k = \frac{1}{2}\ln(\omega_k\,/\,\Omega_k)}$ → squeezing parameter (the Bogoliubov transformation is equivalent to a squeezing operation)
- $a$-quanta → observed; $b$-quanta → thermalized in medium

# Full two-particle correlation ($\pi^0$'s)

$$\langle a^\dagger_{k_1} a^\dagger_{k_2} a_{k_1} a_{k_2} \rangle = \langle a^\dagger_{k_1} a_{k_1} \rangle \langle a^\dagger_{k_2} a_{k_2} \rangle + \langle a^\dagger_{k_1} a_{k_2} \rangle \langle a^\dagger_{k_2} a_{k_1} \rangle + \langle a^\dagger_{k_1} a^\dagger_{k_2} \rangle \langle a_{k_1} a_{k_2} \rangle$$

**NOTATION**

$$N_1(\vec{k}_i) = \omega_{k_1} \frac{d^3N}{d^3k} = G_c(\vec{k}_i, \vec{k}_i) \equiv G_c(i,i) = \omega_{k_1} \langle a^\dagger_{k_i} a_{k_i} \rangle$$ → **Spectra**

$$G_c(\vec{k}_1, \vec{k}_2) \equiv G_c(1,2) = \sqrt{\omega_{k_1} \omega_{k_2}} \langle a^\dagger_{k_1} a_{k_2} \rangle$$ → **Chaotic amplitude**

$$G_s(\vec{k}_1, \vec{k}_2) \equiv G_s(1,2) = \sqrt{\omega_{k_1} \omega_{k_2}} \langle a_{k_1} a_{k_2} \rangle$$ → **Squeezed amplitude**

$$C_2(\vec{k}_1, \vec{k}_2) = 1 + \frac{|G_c(1,2)|^2}{G_c(1,1) G_c(2,2)} + \frac{|G_s(1,2)|^2}{G_c(1,1) G_c(2,2)}$$

**HBT** **BBC**

$$\therefore \begin{cases} G_c(1,2) = \sqrt{\omega_k\,\omega_{k_2}}\left[\left\langle \left(c^*_{k_1} b^\dagger_{k_1}\right)\left(c_{k_2} b_{k_2}\right)\right\rangle + \left\langle \left(s_{-k_1} b_{-k_1}\right)\left(s^*_{-k_2} b^\dagger_{-k_2}\right)\right\rangle\right] \\ G_s(1,2) = \sqrt{\omega_k\,\omega_{k_2}}\left[\left\langle \left(s^*_{-k_1} b^\dagger_{-k_1}\right)\left(c_{k_2} b_{k_2}\right)\right\rangle + \left\langle \left(c_{k_1} b_{k_1}\right)\left(s^*_{-k_2} b^\dagger_{-k_2}\right)\right\rangle\right] \end{cases}$$

- **After performing the thermal averages →** $\langle \mathcal{O} \rangle = Tr(\hat{\rho}\mathcal{O})$
  - If the thermal $b$ gas freezes out suddenly at some time, at temperature $T$, the observed ***single-particle distribution*** for $a$ is

$$N_1(\vec{k}) = \frac{V}{(2\pi)^3}\,\omega_k\left[|c_k|^2\, n_k + |s_{-k}|^2\,(n_{-k}+1)\right] \equiv \frac{V}{(2\pi)^3}\,\omega_k\, n_1(\vec{k})$$

  - And the ***squeezed correlation function*** is given by

$$C_s(\vec{k},-\vec{k}) = 1 + \frac{\left| c_k\, s^*_k n_k + c_{-k} s^*_{-k}\,(n_{-k}+1)\right|^2}{n_1(\vec{k})\, n_1(-\vec{k})}$$

# Finite size medium moving with collective velocity

- For a hydrodynamical ensemble → amplitudes can be written as [ Makhlin & Sinyukov, N.P. A566 (1994) 598c ]:

$$G_c(1,2) = \frac{1}{(2\pi)^3}\int d^4\sigma_\mu(x)\, K^\mu_{1,2}\, e^{iq_{1,2}.x}\Big[\big|c_{1,2}\big|^2 n_{1,2} + \big|s_{-1,-2}\big|^2 \big(n_{-1,-2}+1\big)\Big]$$

$$G_s(1,2) = \frac{1}{(2\pi)^3}\int d^4\sigma_\mu(x)\, K^\mu_{1,2}\, e^{i2K_{1,2}.x}\Big[s^*_{-1,2}\, c_{2,-1}\, n_{-1,2} + c_{1,-2},1\, s^*_{-2,1}\big(n_{1,-2}+1\big)\Big]$$

- $\sigma^\mu \leftrightarrow$ hydrodynamical freeze-out surface
- Squeezing coefficient: $c_{i,j} = \cosh[f_{i,j}]$ ; $s_{i,j} = \sinh[f_{i,j}]$

$$f(i,j,x) = \tfrac{1}{2}\ln\Big[\big(K^\mu_{i.j}\, u_\mu(x)\big)/\big(K^{*\nu}_{i.j}\, u_\nu(x)\big)\Big] = \tfrac{1}{2}\ln\left[\frac{\omega_{k_i}(x)+\omega_{kj}(x)}{\Omega_{k_i}(x)+\Omega_{kj}(x)}\right]$$

  - » Two-particle momenta: $K^\mu_{i,j} = \frac{1}{2}(k_i + k_j)$ ; $q^\mu_{i,j} = (k_i - k_j)$
  - » $u^\mu$ → local flow vector at freeze-out

# Additional hypotheses

– $n_{i,j}$ → **Boltzmann limit of Bose-Einstein distribution:**

$$n_{i,j}(x) \sim \exp\left[-\left(K^{\mu}_{i,j}u_{\mu} - \mu(x)\right)/T(x)\right]$$

Hydro parameterization → $\dfrac{\mu(x)}{T(x)} = \dfrac{\mu_0}{T(x)} - \dfrac{\vec{r}^2}{2R^2}$

- Freeze-out:

Sudden freeze-out → $\int dt\, E_{i,j} e^{-2iE_{i,j}.\tau}\, \delta(\tau - \tau_0)\, d\tau_f = E_{i,j} e^{-2iE_{i,j}.\tau_0}$

Finite emission interval → $\int dt\, E_{i,j}\, F(\tau_f)\, e^{-iE_{i,j}(\tau-\tau_0)}\, d\tau_f = \dfrac{E_{i,j}}{[1 + (E_{i,j}\Delta t)^2]}$

(corresponding to: $F(\tau) = \dfrac{\theta(\tau - \tau_0)}{\Delta t} e^{-(\tau-\tau_0)/\Delta t}$ )

- Non-relativistic limit:

$$u^{\mu} = \gamma(1, \vec{v}) \quad ; \quad \vec{v} = \langle u \rangle \frac{\vec{r}}{R}$$

$$\gamma = (1 - \vec{v}^2)^{-1/2} \approx 1 + \tfrac{1}{2}\vec{v}^2 \quad [\mathcal{O}(v^2)]$$

- For large mass $m$ and small mass shifts, $(m_* - m)/m \ll m$
  → flow effects on squeezing parameter:

$$\mathcal{O}\left[\left(\frac{Kin.En.}{m}\right)\left(\frac{\delta M^2}{m^2}\right)\right]$$ → flow effects on $f_{i,j}$ are negligible

$\Rightarrow$ $c_{i,j}$ and $s_{i,j}$ → flow independent

- What about the volume $V$ ?
  - » $s_{i,i} = 0$ outside mass-shift region ($\delta$M=0)
  - » terms $\propto n_{i,j}$ → finite (hydro solution)

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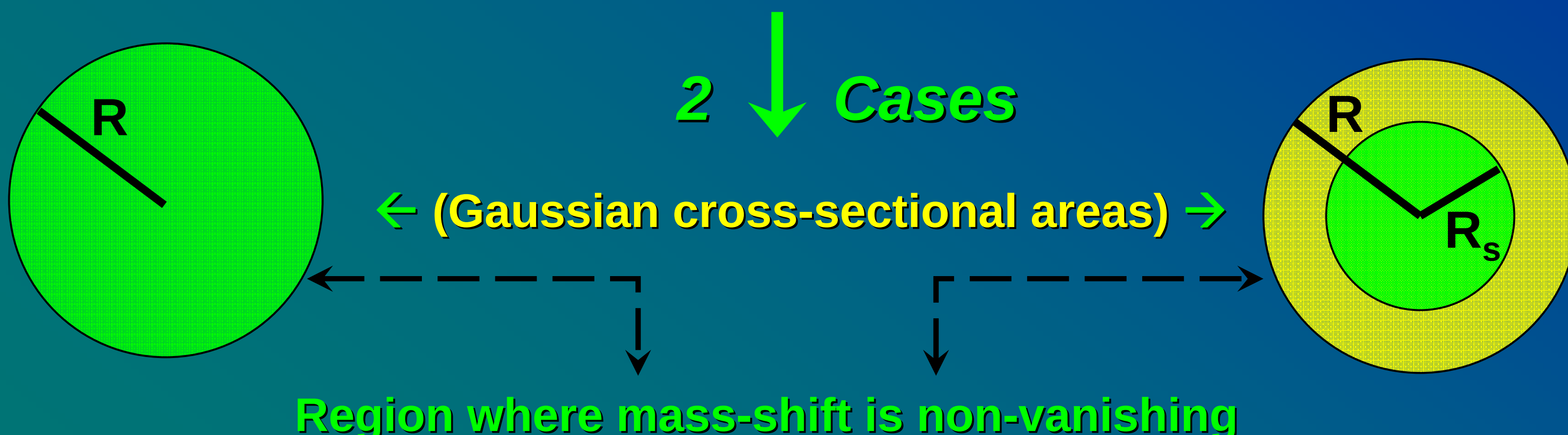
  - » → integration could be extended to infinity:

$\Rightarrow$ easiest $V$ profile: Gaussian $\approx \exp[-\vec{r}^{\,2}/(2R^2)]$

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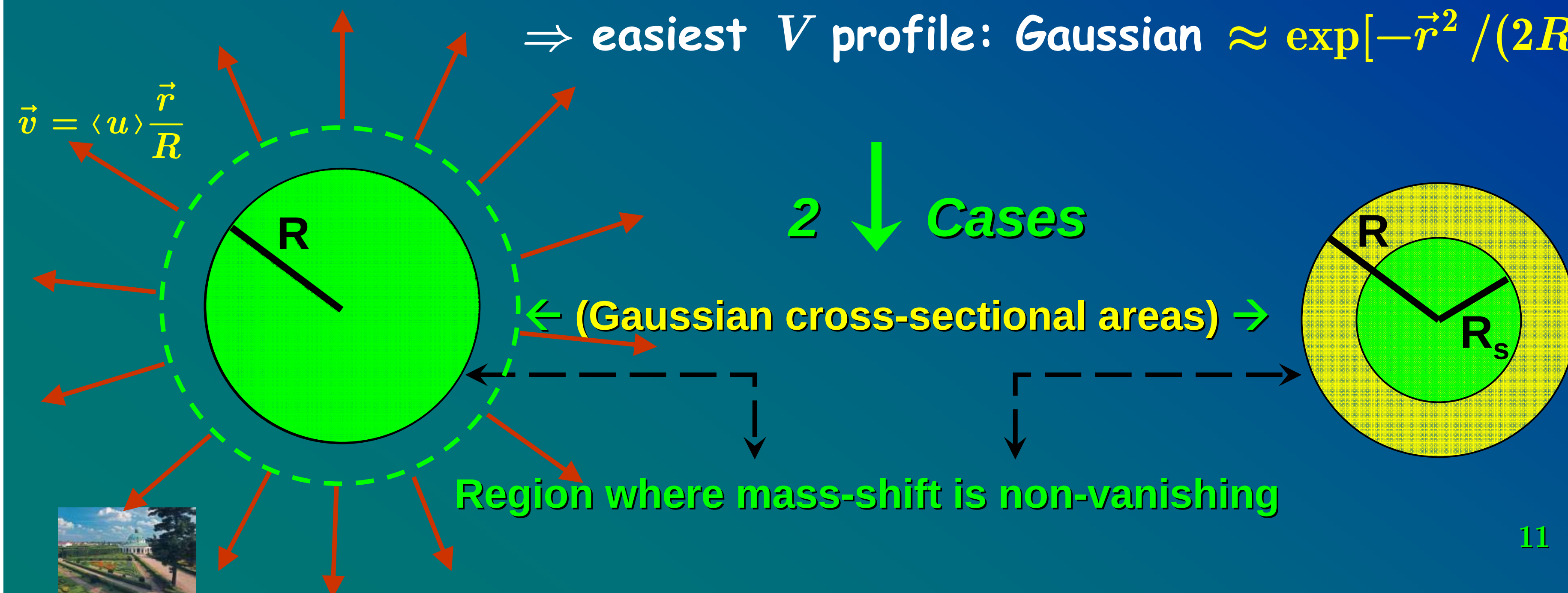
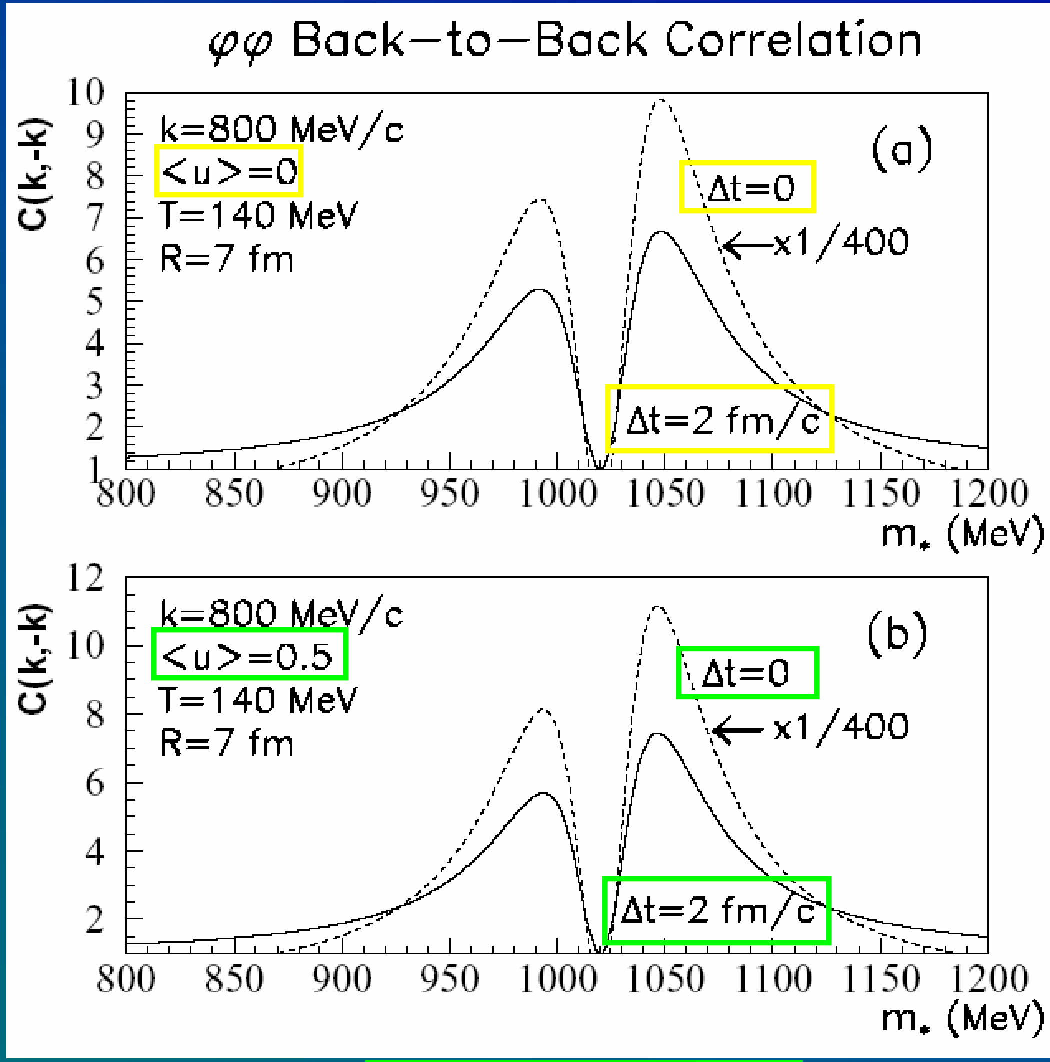


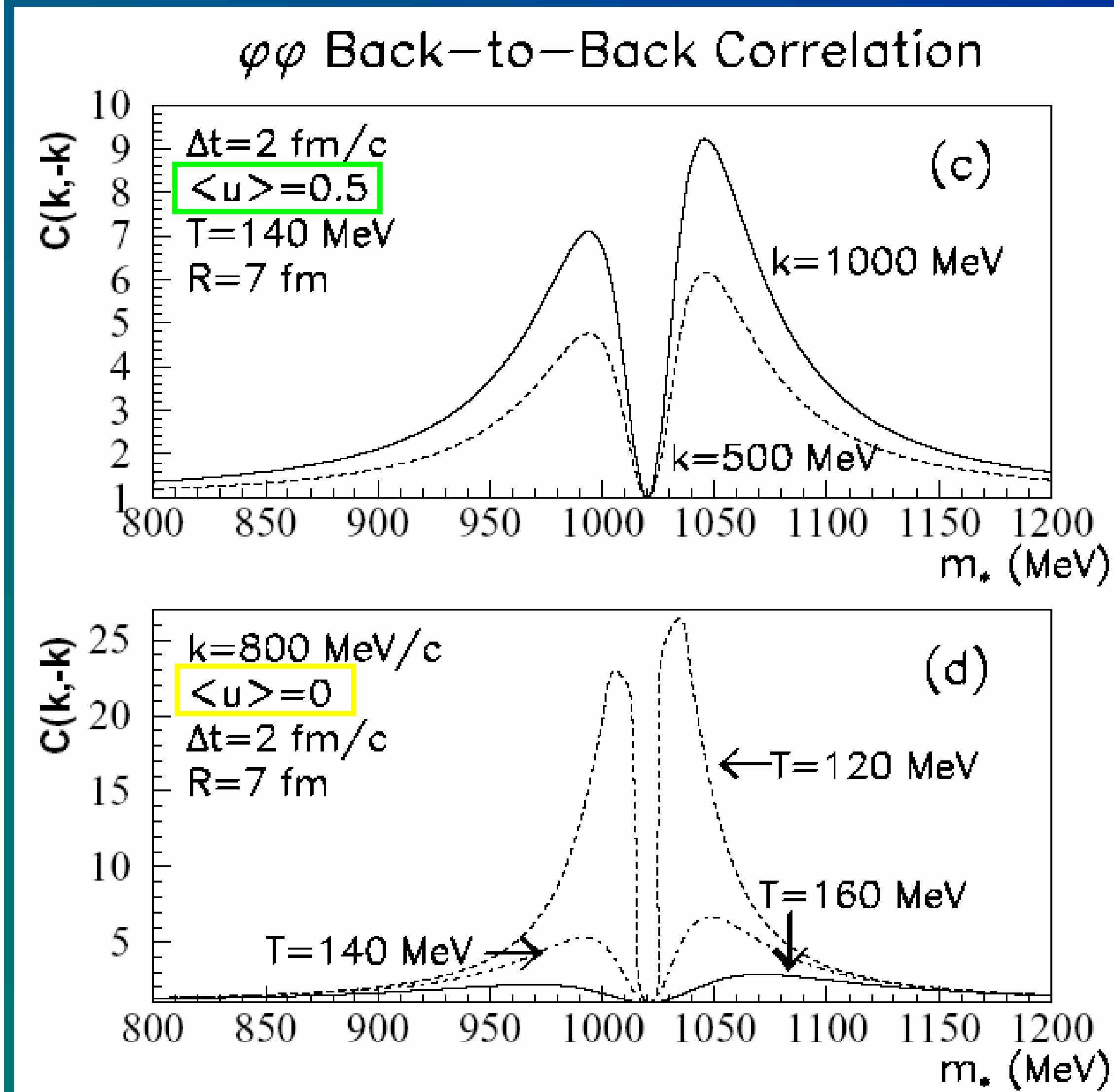
# Squeezing region of radius $R$

- **Flow, finite size & time**
  - Finite emission times → significantly suppress BBC signal intensity but it still is strong (already known)
  - Finite size systems → BBC signal $\propto$ size of mass-shifting region
  - For weak flow coupling → signal strength sensitive to momenta; BBC with $<u>=0.5$: comparable, weaker or even mildly enhanced than for $<u>=0$, depending on $k$




$m_\phi = 1020 MeV$

# Rôle of momenta and temperatures




- **Momenta ($<u>=0.5$):**
  - Increasing modulus of the back-to-back momenta of the pair enhances the effect

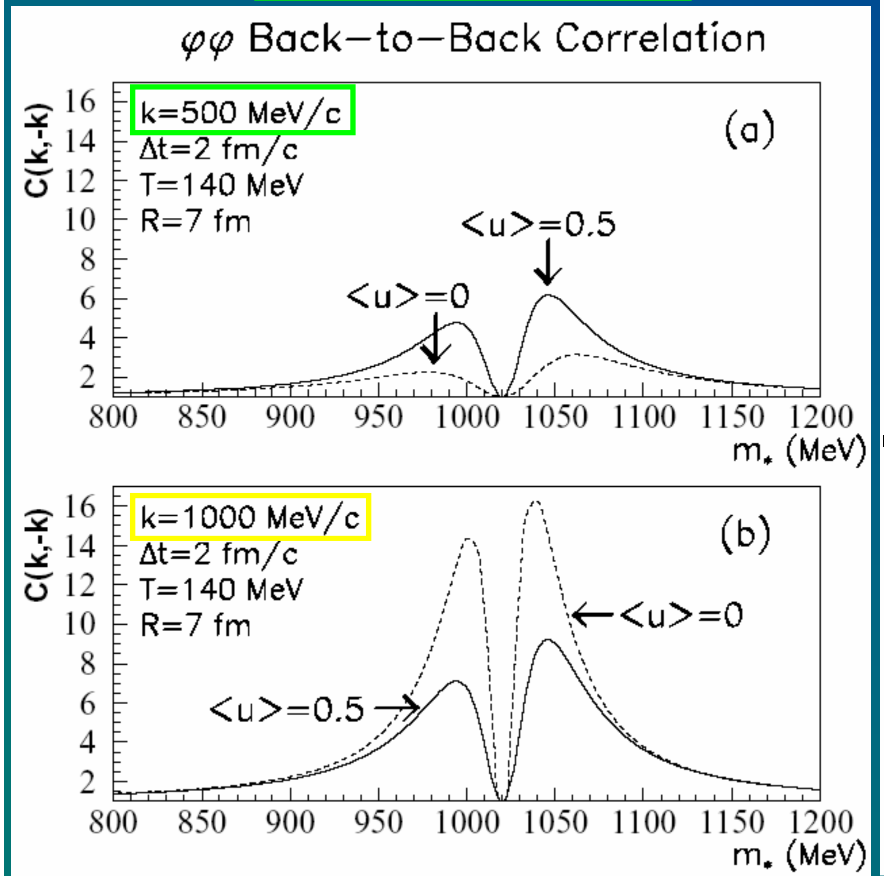
$$m_\phi = 1020 MeV$$

- **Temperature ($<u>=0$):**
  - BBC signal enhanced for lower emission (freeze-out) temperatures for finite sized systems emitting during finite time interval

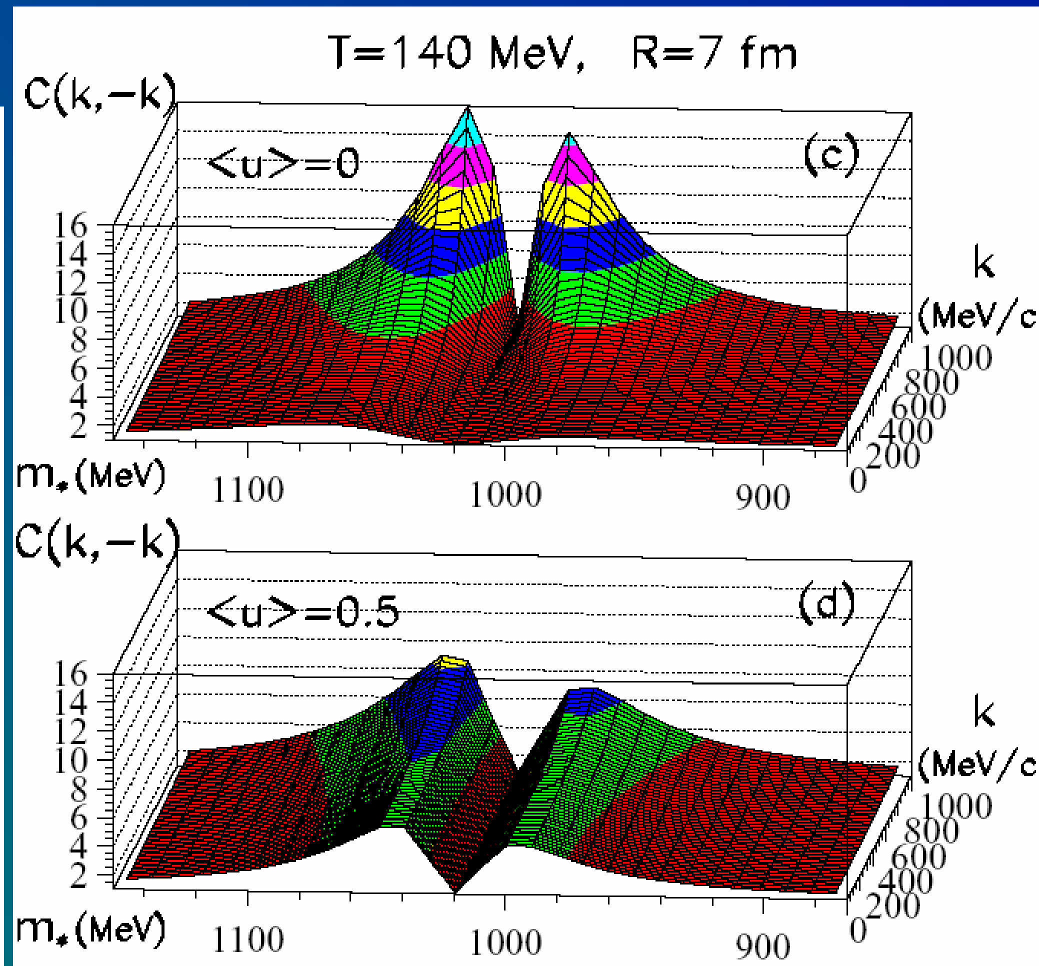
# Squeezing region of radius $R$

– $T, R, \Delta t$ fixed for two values of $k$ and $<u>$

$m_\phi = 1020 MeV$

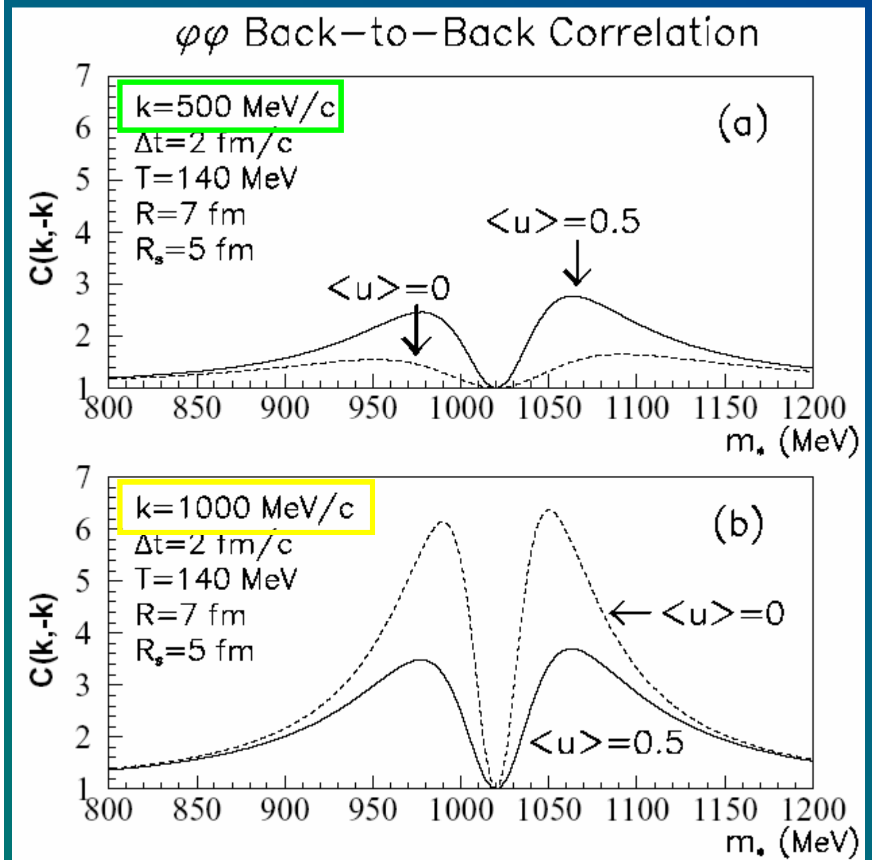



– Varying $k$ as well:

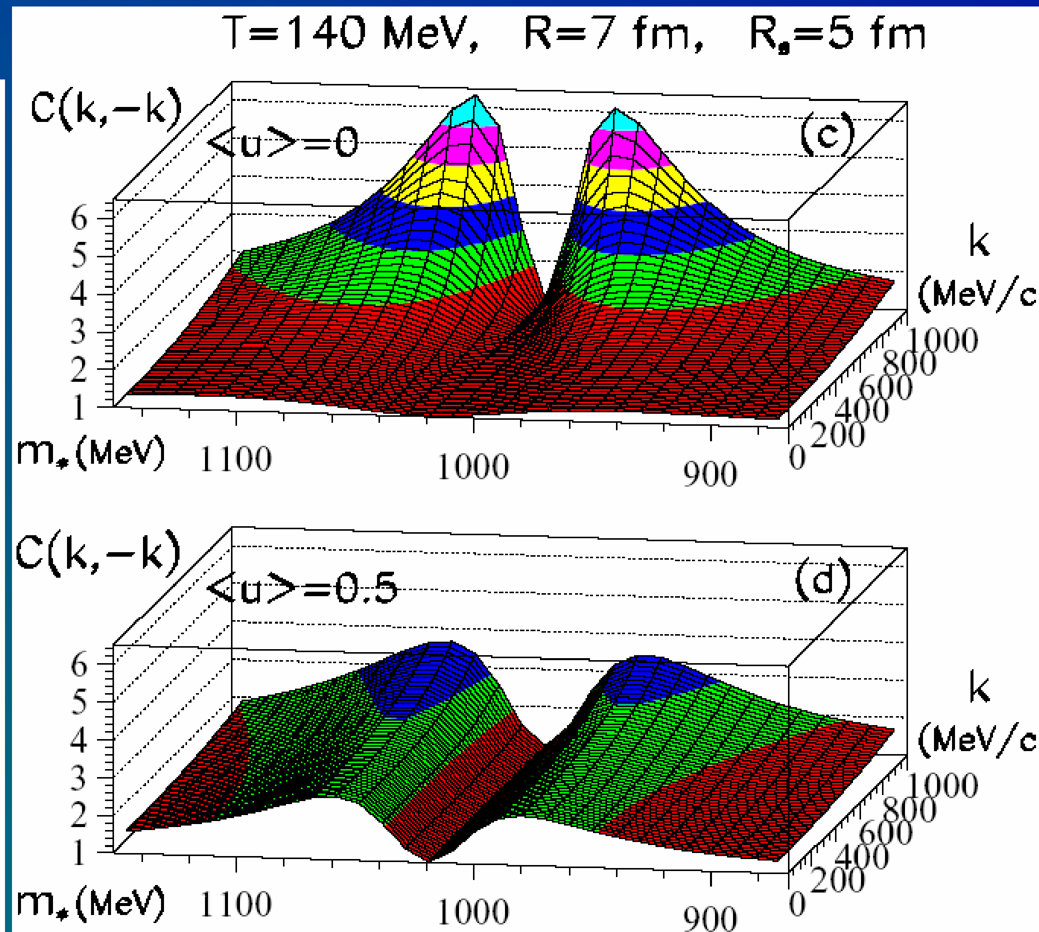


# Squeezing radius $R_s$, system radius $R$

- $T, R, \Delta t$ fixed for two values of $k$ and $<u>$




- Varying $k$ as well:




$m_\phi = 1020 MeV$

**Sensitive to size of squeezing region**

# Summary

- Main goal: revive discussions on Back-to-Back Correlation
- → Estimated strength of BBC signal for:
  - Finite size systems emitting during finite time intervals
  - Expanding system with radial flow & non-relativistic limit
  - Illustration → simplified situation of weak flow dependence of the BBC pair (close to central region) → $\phi\phi$ correlation
- For single ($R$) and two volumes ($R$ & $R_s$) cases:
  - BBC vs. $m_*$ and BBC vs. $m_*$ vs. $k$
    - Pronounced maxima around $m_* \approx m$
    - Similar behavior with and without flow
    - BBC signal:
      - » increases for increasing size of mass-shifting region
      - » sensitive to spread in emission interval
      - » sizeable effect for particular cases studied here

# Next

- Introduce model-based mass-shift
- Perform more realistic calculations with flow
- Search for kinematical regions optimizing BBC signal
- Estimate shape and width of the BBC around the direction $k_2 = -k_1 = k$
- Experimental feedback on acceptance → necessary
- ...


# 1st WPCF - Kroměříž

# See you at the 2nd WPCF!

# BACKUPS

# BBC function

- Momenta of the pair

$$k_2 = -k_1 = k$$

- Back-to-Back correlation function

$$C_s(k_1,k_2) = 1 + \left\{ |c_0||s_0| \left[ R^3 + 2\left( \frac{R^2}{(1+\frac{m^2\langle u\rangle^2}{m_*T})} \right)^{\frac{3}{2}} \exp\left( -\frac{m_*}{T} - \frac{k^2}{2m_*T} \right) \right] \right\}^2 \times$$

$$\left\{ |s_0|^2 R^3 + \left( |c_0|^2 + |s_0|^2 \right) \left( \frac{R^2}{(1+\frac{m^2\langle u\rangle^2}{m_*T})} \right)^{\frac{3}{2}} \exp\left( -\frac{m_*}{T} - \frac{k^2}{2m_*T} - \frac{R^2(m\langle u\rangle k/m_*)}{(1+\frac{m^2\langle u\rangle^2}{m_*T})RT} \right) \right\}^{-2}$$

# Correspondences

- **Bosonic BBC**

$$c_k = \cosh[f_k] \; ; \; s_k = \sinh[f_k]$$

$$\begin{cases} a^{\dagger}{}_k = c_k \, b^{\dagger}{}_k + s_{-k} \, b_{-k} \\ a_k = c_k \, b_k + s^*_{-k} \, b^{\dagger}_{-k_1} \end{cases}$$

$$\begin{cases} f_k \equiv r_k^{ACG} = \frac{1}{2} \log\left(\frac{\omega_k}{\Omega_k}\right) \\ \omega_k^2 = m^2 + \vec{k}^2 \\ \Omega_k^2 = \omega_k^2 - \delta M^2(|k|) \\ m_*^2 = m^2 - \delta M^2(|k|) \end{cases}$$

- **Fermionic BBC**

$$c_k = \cos[f_k] \; ; \; s_k = \sin[f_k]$$

$$\begin{pmatrix} a_{\lambda,k} \\ \tilde{a}^{\dagger}_{\lambda',-k} \end{pmatrix} = \begin{pmatrix} c_k & \frac{f_k}{|f_k|} s_k A \\ -\frac{f_k^*}{|f_k|} s_k^* A^{\dagger} & c_k^* \end{pmatrix} \begin{pmatrix} b_{\lambda,k} \\ \tilde{b}^{\dagger}_{\lambda',-k} \end{pmatrix}$$

$$A = [\chi^{\dagger}_{\lambda} (\sigma.\hat{k}) \tilde{\chi}_{\lambda'}] \; ; \; A^{\dagger} = [\tilde{\chi}^{\dagger}_{\lambda'} (\sigma.\hat{k})^{\dagger} \chi_{\lambda}]$$

$\chi_{\lambda'}$ → is a Pauli spinor

$$\tilde{\chi}_{\lambda'} = -i\sigma^2 \chi_{\lambda'} \; ; \; \hat{k} = \vec{k}/|\vec{k}|$$

$$\tan(2f_k) = -\frac{|k|\Delta M(k)}{\omega_k^2 - \Delta M(k) M}$$

$$m_*(k) = m - \Delta M(k)$$

$$\omega_k^2 = m^2 + \vec{k}^2 \; ; \; \Omega_k^2 = m_*^2 + \vec{k}^2$$

# Proton-Proton BBC

$$H = H_0 + H_I \quad ; \quad H_0 = \int d\vec{x} : \bar{\psi}(x)(-i\vec{\gamma}.\vec{\nabla} + M)\psi(x):$$

$$\psi(x) = \frac{1}{V} \sum_{\lambda,\lambda',\vec{k}} \left(u_{\lambda,\vec{k}} a_{\lambda,\vec{k}} + v_{\lambda',-\vec{k}} a^{\dagger}_{\lambda',-\vec{k}}\right) e^{i\vec{k}.\vec{x}}$$

$$\langle a^{\dagger}_{k_1} a^{\dagger}_{k_2} a_{k_1} a_{k_2} \rangle = \langle a^{\dagger}_{k_1} a_{k_1} \rangle \langle a^{\dagger}_{k_2} a_{k_2} \rangle - \langle a^{\dagger}_{k_1} a_{k_2} \rangle \langle a^{\dagger}_{k_2} a_{k_1} \rangle + \langle a^{\dagger}_{k_1} a^{\dagger}_{k_2} \rangle \langle a_{k_1} a_{k_2} \rangle$$

- Quasi-particle description of the system → medium effects are taken into account through a self energy function
- For spin-1/2 particle under mean field in many-body system:

$$\sum{}^{s} + \gamma^0 \sum{}^{0} + \gamma^i \sum{}^{i}$$ → to be determined from detailed calculation

- $\Sigma^S$ → notation: $\Sigma^s(k) = \Delta M(k)$
- $\Sigma^1$ → very small → negligible
- $\Sigma^0$ → weakly momentum dependent → locally thermalized medium: $\mu_* = \mu - \Sigma^0$ → do not need to be specified (results for net baryon number)
- Hamiltonian $H_1$ → describes a system of quasi-particles with momentum-dependent mass $M_* = m - \Delta M(k)$