Invariance group important for the interpretation of Bose-Einstein correlations

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From measured momentum distributions to inferences about the interaction regions

V stands for: information about the interaction region

- :-(Emission function $S(K, X)$:-) V
- :-) Distribution of relative distance $\mathcal{S}_K(\Delta\mathbf{x})$:-(V
- :-(Single particle density matrix $\rho(\mathbf{p}; \mathbf{p'}; t \gg t_f)$
  - :-) $\rho(\mathbf{p}; \mathbf{p'}; t_{fmax})$
  - :-($\rho(\mathbf{x}; \mathbf{x'}; t_f)$ or $W(\mathbf{K}, \mathbf{X}, t_f)$:-) V

Ambiguity in the determination of the density matrix elements from the measured momentum distributions

J. Karczmarczuk, *Nucl. Phys.* **B78**(1974)370.
A. Bialas and K. Zalewski, hep-ph/0501017.

In most models
$\rho(\mathbf{p}_1, \ldots, \mathbf{p}_k; \mathbf{p}_1, \ldots, \mathbf{p}_k) = \sum_P \prod_{i=1}^{k} \rho_1(\mathbf{p}_i; \mathbf{p}_{Pi})$

Example
$\rho(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p}_1, \mathbf{p}_2) = \rho_1(\mathbf{p}_1, \mathbf{p}_1)\rho_1(\mathbf{p}_2; \mathbf{p}_2) + |\rho_1(\mathbf{p}_1; \mathbf{p}_2)|^2$

All momentum distribution are invariant under the replacement of $\rho_1(\mathbf{p}; \mathbf{p}')$ by
$\rho_{1\alpha}(\mathbf{p}; \mathbf{p}') = e^{i\alpha(\mathbf{p})}\rho_1(\mathbf{p}; \mathbf{p}')e^{-i\alpha(\mathbf{p}')}$,
where $\alpha(\mathbf{p})$ is any real-valued function of $\mathbf{p}$.

Implications for the emission function

- $\rho(\mathbf{p}_1; \mathbf{p}_2) = \int d^4X \; S(K, X) e^{iqX}$
  - $K = \frac{1}{2}(p_1 + p_2); \quad q = p_1 - p_2; \quad \Rightarrow Kq = 0$
- $\alpha(\mathbf{p}) = bp + \frac{1}{2}cp^2 \Rightarrow \alpha(p_1) - \alpha(p_2) = bq + cKq$
- $\rho_\alpha(\mathbf{p}_1, \mathbf{p}_2) = \int d^4X' \; S(K, X') e^{iq(X'+b+cK)}$
  - $X = X' + b + cK$
- $\rho_\alpha(\mathbf{p}_1, \mathbf{p}_2) = \int d^4X \; S(K, X - b - cK) e^{iqX}$
- Thus $S_\alpha(K, X) = S(K, X - b - cK)$
- For $b = 0$: $\rho_\alpha = \rho$, but $S_\alpha(K, X) = S(K, X - cK)$

Implications for the density matrix elements $\rho(\mathbf{x};\mathbf{x})$ and for the Wigner function

- $\rho(\mathbf{K},\mathbf{q}) = \frac{1}{(\sqrt{2\pi\Delta^2})^3}\ \exp\left[-\frac{\mathbf{K}^2}{2\Delta^2} - \frac{1}{2}R^2\mathbf{q}^2\right]$
- $\alpha(\mathbf{p}) = \mathbf{b}\cdot\mathbf{p} + \frac{1}{2}c\mathbf{p}^2; \quad \Rightarrow \alpha(\mathbf{p}_1) - \alpha(\mathbf{p}_2) = \mathbf{b}\cdot\mathbf{q} + c\mathbf{K}\cdot\mathbf{q}$

- $\rho(\mathbf{x};\mathbf{x}) = \int dKdq\ e^{i\mathbf{q}\cdot\mathbf{X}}\rho(\mathbf{K},\mathbf{q})$
- $\rho_\alpha(\mathbf{x};\mathbf{x}) = \frac{1}{\sqrt{2\pi(R^2+c^2\Delta^2)}}\ \exp\left[-\frac{(\mathbf{x}-\mathbf{b})^2}{2(R^2+c^2\Delta^2)}\right]$

- $W(\mathbf{K},\mathbf{X}) = \int\frac{dq}{(2\pi)^3}\ e^{i\mathbf{q}\cdot\mathbf{X}}\rho(\mathbf{K},\mathbf{q})$
- $W_\alpha(\mathbf{K},\mathbf{X}) = \frac{1}{(2\pi R\Delta)^3}\ \exp\left[-\frac{\mathbf{K}^2}{2\Delta^2} - \frac{(\mathbf{X}+\mathbf{b}+c\mathbf{K})^2}{2R^2}\right]$

How to get the additional information?

Implications for the density matrix elements $\rho(\mathbf{x};\mathbf{x})$ and for the Wigner function

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How to get the additional information?

Application: explication of the dependence $r_T(M_T)$

T. Csörgö and J. Zimányi, *Nucl. Phys.* **A51**7(1990)588.
A. Bialas et al. *Phys. Rev.* **D62**(2000)114007.

- $K^\mu = \lambda X^\mu$ Hubble flow
- $X_0^2 - X_\parallel^2 = \tau^2; \quad K_0^2 - K_\parallel^2 = M_T^2; \quad \Rightarrow \lambda = \frac{M_T}{\tau}$
- $S = S_\parallel\, S_T; \quad S_T = \exp\left[-\frac{\mathbf{X}_T^2}{2r_T^2} - \frac{(\mathbf{K}_T - \lambda \mathbf{X}_T)^2}{2\delta_T^2}\right]$
- $\langle \mathbf{X}_T^2 \rangle = 2r_T^2$
  True under the assumption of position-momentum correlations as given above

Alternative interpretation of the data

- $S_T = \exp\left[-\frac{\phi_T^2}{2R_D^2} - \frac{(\mathbf{X}_T - \phi_T)^2}{2R_\phi^2}\right]$
- $R_\phi = \frac{r_T}{\sqrt{1+\mu^2}}$; $\quad R_D = \mu R_\phi$; $\quad \mu = \frac{r_T}{\tau\delta_T} M_T$; $\quad \phi_T = r_T \frac{\mu}{1+\mu^2}\frac{\mathbf{K}_T}{\delta_T}$
- $S_{\alpha T} = \exp\left[-\frac{\mathbf{K}_T^2}{2\delta_T^2} - \frac{\mathbf{X}_T^2}{2R_\phi^2}\right]$; $\quad cK_T = \phi_T \Rightarrow c = \frac{r_T\,\mu}{\delta_T(1+\mu^2)}$
- $\langle \mathbf{X}_T^2 \rangle = 2R_\phi^2 < 2r_T^2$ and M_T- dependent.
  "Measured" under the assumption of no momentum-position correlations

A more complicated example

- $d = 1; \quad \rho(K, q) = \frac{1}{\sqrt{2\pi\Delta^2}} \exp\left[-\frac{K^2}{2\Delta^2} - \frac{1}{2} R^2 q^2\right]$

- $\alpha(p) = \frac{4}{3a^3} p^3; \quad \Rightarrow \alpha(p_1) - \alpha(p_2) = \frac{4}{3a^3} q \left(4K^2 + \frac{q^2}{3}\right)$

- $S_\alpha = \frac{a}{\sqrt{2\pi\Delta^2}} \exp\left[-\frac{K^2}{2\Delta^2} + B\right] Ai(A)$

  where $Ai(\ldots)$ is the Airy function and

- $A = a\tilde{X} + \frac{\omega^4}{4}; \quad B = \frac{\omega^2}{2}\left[A - \frac{\omega^4}{12}\right]; \quad \omega = aR; \quad \tilde{X} = X - \frac{4}{a^3} K^2$

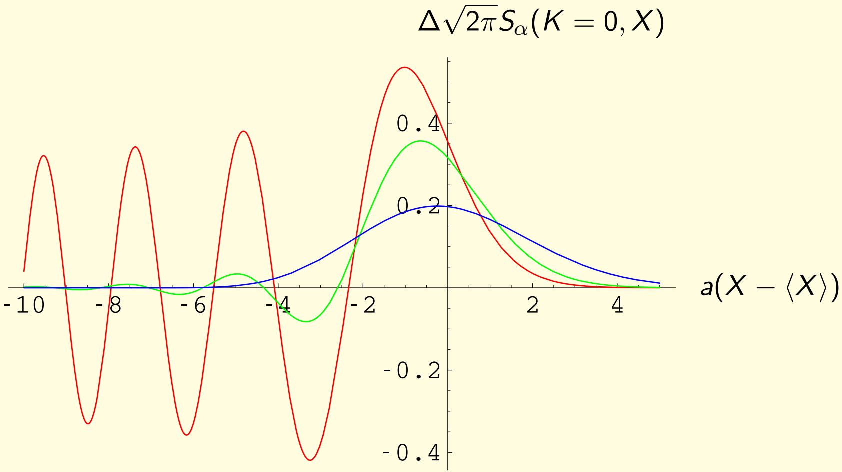

Fig. 1: Emission functions $S_\alpha(K=0,X)$ for: $aR=0$ (red), $aR=1$ (green) and $aR=2$ (blue). $\langle X\rangle$ is the average value of X at given aR and $K=0$.

Conclusions

- Data without additional assumptions tell us little about the interaction regions.
- The same fit can be obtained from models widely different in these additional assumptions and thus giving conflicting information about the interaction region.
- How to obtain and how to test the additional assumptions?